Diffraction Peak in $p\cdot p\ (\bar{p}\cdot p)$ Scattering and Growth of the Total Cross Section

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As the expression of $\sigma_t$ for $p\cdot p$ scattering with which good fits to the data in the ISR energy range can be obtained, we consider two kinds of forms with the following properties: (i) $\sigma_t$ does increase like $\log^2 (s/s_0)$ and (ii) $\sigma_t$ tends to a finite value in the high energy limit. The scattering amplitude based on the dual absorptive model is modified by taking into account the effects of the growth of total cross section. It is shown that the experimental data for the slope parameter, $\sigma_t$ and $d\sigma/dt$ in forward $p\cdot p$ scattering at $s=40\sim 2800$ GeV$^2$ can be explained well in terms of the modified amplitudes in both cases (i) and (ii).

§ 1. Introduction

According to the ISR experiments$^3$ for elastic $p\cdot p$ scattering at small $|t|$, the slope parameter $b$ deviates markedly from the logarithmic extrapolation through the Serpukhov data.$^2$ In order to explain the result, we have previously studied the properties$^8$ of $b(0)$ at $t=0$ on the basis of the dual absorptive model$^4$ in which the imaginary part of the helicity non-flip amplitude at small $|t|$ can be expressed by

$$\text{Im} f(s, t) = A_P \exp \left( \frac{1}{2} B_P t \right) + A_R \exp \left( -\frac{1}{2} B_R t \right),$$

where $B_P = B_P(0) + 2\alpha_P' \log s$, $B_R = B_R(0) + 2\alpha_R' \log s$, $\alpha_P(0) = 1 + \alpha_P' t$, $\alpha_R(t) = 0.5 + \alpha_R' t$, $r = (5\sim 5.5) \text{ GeV}/c^{-1}$ at intermediate energies and $s$ is given in units of GeV$^2$. The subscripts $P$ and $R$ are added in order to indicate the contributions from the Pomeron and the $R$ ($\rho$, $\omega$, $A_2$ and $P'$) trajectories, respectively.

In the study$^3$ we have assumed that the total cross section $\sigma_t$ in the high energy region such as $s \geq 100$ GeV$^2$ remains flat, as a function of energy, approximately equal to 39 mb. According to the new ISR experiments,$^6$ however, the total cross section for $p\cdot p$ scattering increases by $\Delta \sigma_t = (4.1 \pm 0.7) \text{ mb}$ when the center-of-mass energy $(\sqrt{s})$ increases from 23 to 53 GeV, and an expression of the form

$$\sigma_t = \sigma_0 + \sigma_1 \left[ \log \left( s/s_0 \right) \right]^\nu$$

provides a good fit to the data over an energy range $100 \leq s \leq 2800$ GeV$^2$ with the following values of parameters:

$$\sigma_0 = (38.4 \pm 0.3) \text{ mb}, \quad \sigma_1 = (0.9 \pm 0.3) \text{ mb}, \quad s_0 = 200 \text{ GeV}^2$$

and $\nu = 1.8 \pm 0.4$.

(3)
Also it is said that the data\textsuperscript{5}) do not exclude a saturation of the total cross section at a large finite value in the infinite energy limit. In what follows we refer to as cases (i) and (ii) when $\sigma_t$ does increase like $[\log (s/s_0)]^\nu$ and tends to a finite value in the high energy limit, respectively.

The purpose of this paper is to study the properties of the diffraction peak in elastic $p\cdot p$ ($\bar{p}\cdot p$) scattering at high energy by taking into account the effects of the growth of $\sigma_t$ and to examine the difference between cases (i) and (ii). In § 2, we investigate the expression of $\sigma_t$ and determine the values of parameters so that the experimental results for $\sigma_t$ in $p\cdot p$ ($\bar{p}\cdot p$) scattering at $s=40 - 2800\text{ GeV}^2$ may be reproduced. By making use of the results in § 2, the slope parameter $b$ for $p\cdot p$ ($\bar{p}\cdot p$) scattering at $t=0$ is evaluated in § 3. In § 4, the differential cross sections for elastic $p\cdot p$ ($\bar{p}\cdot p$) scattering at $|t| = 0 - 0.35 (\text{GeV}/c)^2$ are estimated and it is shown that good fits to the data for $d\sigma/dt$ in $p\cdot p$ scattering can be obtained in both cases (i) and (ii). Finally we discuss the difference between the results for cases (i) and (ii).

§ 2. Total cross sections for $p\cdot p$ ($\bar{p}\cdot p$) scattering

First let us consider the case (i) in which $\sigma_t$ does increase like $[\log (s/s_0)]^\nu$ with $\nu \leq 2$. Cheng et al.\textsuperscript{6}) have given a simple model of hadron scattering which is based on the impact picture derived from quantum field theory and have shown that the model can lead to the results consistent with existing data for elastic $p\cdot p$ scattering in the forward direction.

Recently Leader and Maor\textsuperscript{7}) have shown that the growth of $\sigma_t$ is correlated with the break in $d\sigma/dt$ near $t = -0.08 (\text{GeV}/c)^2$, by using the following forms as the expressions of $\sigma_t$ and $f(s, t)$:

$$\sigma_t = C + D \log^2 (s/s_0) \quad (4)$$

and

$$f(s, t) = i \{ C \exp \left( \frac{1}{2} \beta_s t \right) + D \log^2 (s/s_0) \exp \left[ \frac{1}{2} b \sigma_0 \log^2 (s/s_0) \right] \}, \quad (5)$$

where $C$ and $D$ correspond, respectively, to $\sigma_s$ and $\sigma_1$ in Eq. (2). In their approach,

$$16\pi (d\sigma/dt) = |f(s, t)|^4, \quad (6)$$

and they have assumed that $\beta_s = \text{const.} = 10.8 (\text{GeV}/c)^{-2}$. This corresponds to the case of $\alpha\nu_a = 0$ in the expression (1).

We are interested in the expression (4), because it has a form similar to that suggested by Susskind et al.\textsuperscript{8}) on the basis of their model for parton-parton interaction. These authors have considered the so-called soft and hard collisions between hadronic constituents and have emphasized that $\sigma_{\text{soft}}$ at high energies becomes constant ($\approx 38.5\text{ mb}$), while $\sigma_{\text{hard}}$ has a $\log^2 (s/s_0)$-dependence, where $\sigma_{\text{soft}}$ and $\sigma_{\text{hard}}$ are the cross sections due to the soft and hard collisions, respectively.
In view of these results, we pay attention to the model of Leader and Maor in our phenomenological study of $p-p$ ($\bar{p}-p$) scattering.

On the basis of Eqs. (1), (2) and (5), we adopt the following form of $f(s, t)$ as an expression in case (i):

$$\Im f(s, t) = A_R s^{-(1/3)} \times \exp \left( \frac{1}{2} B_R t \right) J_0 \left( r \sqrt{1-t} \right) + C \times \exp \left( \frac{1}{2} B_p t \right) + D \log^2 \left( s/s_0 \right) \times \exp \left[ \frac{1}{2} B_t \log^2 \left( s/s_0 \right) \right].$$

(7)

Then,

$$\sigma_t = A_R s^{-(1/3)} + C + D \log^2 \left( s/s_0 \right).$$

(8)

Now let us briefly discuss the sign of $A_R$ by taking into consideration the value of $s_0$ in Eq. (3).

If the sign of $A_R$ is positive, the expression (8) leads to the following conclusion: When $s_0 = 200$ GeV$^2$, the value of $\sigma_t$ at $s = 20$ GeV$^2$ is larger than that of $\sigma_t$ at $s = 2000$ GeV$^2$. This is inconsistent with experimental data (see Fig. 1). Therefore, in case (i) we adopt $A_R$ with negative sign.

As is shown in Fig. 1(a), good fits to the data for $\sigma_t$ at $s = 40 - 2800$ GeV$^2$ can be obtained with

$$C = 38.4 \text{ mb}, \quad D = 0.9 \text{ mb}, \quad s_0 = 200 \text{ GeV}^2 \quad \text{and}$$

$$A_R = \begin{cases} A_{p'} + A_{d\bar{d}} - A_s - A_\sigma = -0.35C = -13.44 \text{ mb} & \text{for } p-p \text{ scattering,} \\ A_{p'} + A_{d\bar{d}} + A_s + A_\sigma = 1.4C = 53.76 \text{ mb} & \text{for } \bar{p}-p \text{ scattering.} \end{cases}$$

(9)

Note that the experimental results for $\sigma_t$ in $p-p$ scattering not only in the ISR energy range but also in a lower energy range ($40 \leq s \leq 200$ GeV$^2$) cannot be
reproduced without the third term in Eq. (7) or (8) when $A_R$ has a negative sign, because the $(A_R s^{-\ell/2} + C)$ in this case is an increasing function of energy in the range $40 \leq s \leq 200 \text{ GeV}^2$, in spite of the fact that the observed $\sigma_t$ decreases with energy in this $s$-region.

Next let us discuss the expression of $\sigma_t$ in case (ii). Theoretical studies based on the Gribov-Reggeon calculus have been made by several authors.\textsuperscript{9,10} Jacob\textsuperscript{10} has employed a form

$$\sigma_t = \sigma_0 [1 - \lambda / (\log p_L + a)]$$

and has shown that a good fit to the data for $\sigma_t$ in the ISR energy range can be obtained with the following values of parameters:

$$\sigma_0 = 60 \text{ mb}, \quad \lambda = 3 \quad \text{and} \quad a = 3.$$  \hspace{1cm} (11)

However, the experimental results for $\sigma_t$ in a lower energy region such as $p_L < 200 \text{ GeV}/c$ cannot be reproduced with the expression (10). In view of this fact, we pay attention to the increase of cross section at $p_L > 200 \text{ GeV}/c$

$$\Delta \sigma_t = \sigma_0 [1 - \lambda / (\log p_L + a)] - \sigma_0 [1 - \lambda / (\log p_0 + a)]$$

$$= \sigma_0 \lambda \log (p_L/p_0) / [(\log p_L + a) (\log p_0 + a)]$$

which can be derived from (10) and use this expression of $\Delta \sigma_t$ in our study of diffraction scattering, where $p_0 = 200 \text{ GeV}/c$. On the basis of Eqs. (1) and (12), we adopt the following forms as the expressions of $\sigma_t$ and Im$f(s, t)$ in case (ii):

$$\sigma_t = A_R s^{-\ell/2} + C$$

$$+ \theta (p_L - p_0) \{ \sigma_0 \lambda \log (p_L/p_0) / [(\log p_L + a) (\log p_0 + a)] \}$$

and

$$\text{Im } f(s, t) = A_R s^{-\ell/2} \exp (\frac{1}{2} B_{pt}) J_0 (r \sqrt{-t}) + C \exp (\frac{1}{2} B_{pt})$$

$$+ \theta (p_L - p_0) \{ \sigma_0 \lambda \log (p_L/p_0) / [(\log p_L + a) (\log p_0 + a)] \} \exp (\frac{1}{2} B't),$$

where

$$\theta (p_L - p_0) = \begin{cases} 0 & \text{for } p_L \leq p_0, \\ 1 & \text{for } p_L > p_0 \end{cases}$$

and $p_0 = 200 \text{ GeV}/c$.

We now think it necessary to make some discussion about the $\theta (p_L - p_0)$ function which is not analytic at $p_L = p_0$. Recently Cline et al.\textsuperscript{11} have made the following speculation: The production of partons is primarily responsible for the rise in the total $p-p$ cross section, analogous to the rise at lower energies near the pion production threshold. If this is the case, we wish to point out the possibility that the $\theta (p_L - p_0)$ function in Eq. (13) or (14) can be understood in terms of the threshold effect of parton production.
Since the observed $\sigma_1$ at $p_L = 20 - 200$ GeV/c decreases with energy, the sign of $A_R$ must be positive and the $(A_R s^{-1/2} + C)$ is a decreasing function of energy. In order to reproduce the experimental result for $\sigma_1$ at 1500 GeV/c, it may be necessary to modify more or less the value of $\sigma_1\lambda$ mentioned in Eq. (11).

As is shown in Fig. 1(b), good fits to the data for $\sigma_1$ at $s = 40 - 2800$ GeV$^2$ can be obtained with

$$A_R = \begin{cases} 10 \text{ mb} & \text{for } p-p \text{ scattering,} \\ 67 \text{ mb} & \text{for } \bar{p}-p \text{ scattering.} \end{cases}$$

Note that the value (210 mb) of $\sigma_1\lambda$ in Eq. (16) is consistent with $\lim_{p_L \to \infty} \sigma_1[1 - (p_L + a)] = 60$ mb, if $\lambda = 3.5$.

§ 3. Slope parameter for $p-p$ ($\bar{p}-p$) scattering at $t=0$

Using Eqs. (7) and (14), we can easily write down the expression of the slope parameter $b(0)$ defined by

$$b(0) = \frac{d}{dt} [\log (d\sigma/dt)]_{t=0}.$$

For case (i),

$$b(0) = \frac{\xi}{\zeta},$$

where

$$\xi = B_p + (A_R/C)s^{-1/2}(B_R + B_p + r^2/2) + (A_R/C)s^{-1}B_R + r^2/2) + (D/C)\log^2(s/s_0) + (D/C)\log^2(s/s_0),$$

$$\zeta = \left(1 + 2(A_R/C)s^{-1/2} + (A_R/C)s^{-1} + (D/C)\log^2(s/s_0)ight),$$

and

$$dJ_s(r^\sqrt{-t})/dt|_{t=0} = r^3/4.$$  

For case (ii),

$$b(0) = \frac{\xi'}{\zeta'},$$

where

$$\xi' = B_p + (A_R/C)s^{-1/2}(B_R + B_p + r^2/2) + (A_R/C)s^{-1}B_R + r^2/2) + \theta(p_L - p_0) \{ (A'/C)'B' + (A'/C)[(A_R/C)s^{-1/2}(B_R + B' + r^2/2) + B_R + B'] \},$$

$$\zeta' = \left(1 + 2(A_R/C)s^{-1/2} + (A_R/C)s^{-1} + (D/C)\log^2(s/s_0) \right),$$

and

$$A' = \sigma_1\lambda \log (p_L/p_0)/[\log (p_L + a) \log (p_0 + a)].$$
We have emphasized in the previous paper\textsuperscript{b} that in the dual absorptive model, $b(0)$ is larger (smaller) than $B_P$ when $A_R$ is positive (negative) and tends to $B_P$ in the high energy limit. As mentioned in § 2, $A_R$ has the positive and negative signs in cases (ii) and (i), respectively. Therefore, it is necessary to assume in case (ii) a large $\alpha p'$, in order to explain the energy dependence of $b(0)$ at $p_L = 20-200 \text{ GeV}/c$, where $B_P = B_P(0) + 2\alpha p' \log s$. In case (i), on the other hand, we may adopt a small $\alpha p'$. We show in Figs. 2 (a) and 2 (b) our results for the slope parameter $b(0)$ in cases (i) and (ii), respectively, when the parameters have the following values in addition to those in Eqs. (9) and (16):

For case (i),
\begin{align*}
B_R &= 1.0 + 1.8 \log s, \\
B_P &= 10.7 + 0.14 \log s, \\
B &= 2.0 (\text{GeV}/c)^{-2} \\
\frac{r^2}{2} &= 13 (\text{GeV}/c)^{-2}.
\end{align*}

For case (ii),
\begin{align*}
B_R &= 1.0 + 1.8 \log s, \\
B_P &= 6.8 + 0.8 \log s, \\
B' &= 8.3 (\text{GeV}/c)^{-2} \quad \text{and} \quad \frac{r^2}{2} = 13 (\text{GeV}/c)^{-2}. \\
\end{align*}

The dashed curve in Fig. 2(a) shows the previous results\textsuperscript{15} for $b(0)$ in $p$-$p$ scattering which have been obtained by using the expression (7) with
\begin{align*}
B_R &= 1.3 + 2.0 \log s, \\
B_P &= 8.9 + 0.54 \log s, \\
B &= 2.0 (\text{GeV}/c)^{-2} \quad \text{and} \quad \frac{r^2}{2} = 13 (\text{GeV}/c)^{-2},
\end{align*}
in addition to the values in Eq. (9).
It seems that in case (i), the experimental data for $b(0)$ in $p-p$ scattering can be explained well by the dashed curve rather than the solid curve (see Fig. 2(a)). As will be mentioned in § 4, however, it is difficult to obtain a good fit to the data for $d\sigma/dt$ at $|t|=0.2-0.35 (\text{GeV}/c)^2$ when we adopt the values of parameters given in Eq. (26).

Our main results for $b(0)$ are the following: 1) For both cases (i) and (ii), there is definitely shrinking of the diffraction peak but the rate of shrinking decreases with energy, so far as $p-p$ scattering at $s<2000 \text{ GeV}^2$ is concerned. This is of course consistent with experimental data. 2) In case (i), the $b(0)$ for $p-p$ ($\bar{p}-p$) scattering at $s=40-2800 \text{ GeV}^2$ is smaller (larger) than $B_p$. At $s\leq 3000 \text{ GeV}^2$, the $b(0)$ for $p-p$ scattering intersects the straight line $B_p=10.7+0.14 \log s$ and increases rapidly with energy. The slope parameter $b(0)$ not only in $\bar{p}-p$ but also in $p-p$ scattering at $s>3000 \text{ GeV}^2$ becomes larger than $B_p$. In case (ii), on the other hand, the slope parameters at $t=0$ for both $\bar{p}-p$ and $p-p$ scattering in a region from $s=40$ to $s=500 \text{ GeV}^2$ are larger than $B_p$. The $b(0)$ for $p-p$ ($\bar{p}-p$) scattering intersects the straight line $B_p=6.8+0.8 \log s$ at $s\approx 500 \text{ GeV}^2$ ($s\approx 2500 \text{ GeV}^2$). 3) The results shown in Figs. 2(a) and 2(b) enable us to say that the experimental data for $b(0)$ in $p-p$ ($\bar{p}-p$) scattering can be explained well by our model in which the effects of the growth of $t$ are taken into account.

§ 4. Differential cross sections for elastic $p-p$ ($\bar{p}-p$) scattering at small $|t|$

In this section we try to estimate the differential cross section $(d\sigma/dt)$ at small $|t|$ under the assumption that diffraction scattering can almost be described in terms of purely imaginary amplitude without helicity flip, that is

$$16\pi (d\sigma/dt) = |\text{Im} f(s,t)|^2$$

as in Ref. 7). In order to see the correlation between the behavior of $d\sigma/dt$ at small $|t|$ and the growth of $\sigma$, as well as the effect of the Regge amplitude due to the $R$-trajectories on $d\sigma/dt$, we study $p-p$ interactions at $s=2800$ and $462 \text{ GeV}^2$, since the last term in Eq. (7) or (14) would have a large (small) effect on diffraction scattering at $s=2800 \text{ GeV}^2$ ($s=462 \text{ GeV}^2$) and since experimental data$^{19}$ for $d\sigma/dt$ are available at these energies.

The differential cross sections at $|t|=0-0.35 (\text{GeV}/c)^2$ are calculated by using the expressions (7) and (14) with the values of parameters given in Eqs. (9), (16), (24) and (25). The results for $d\sigma/dt$ at $s=2800$ and $462 \text{ GeV}^2$ are shown in Figs. 3(a) and 3(b), respectively, and compared with the data.$^{19}$ The agreement is very good except for a small difference between the observed values of $d\sigma/dt$ at $|t|>0.2 (\text{GeV}/c)^2$ and the predicted ones in case (ii). When the differential cross section is estimated with the parameters in Eqs. (9) and (26), the theoretical $d\sigma/dt$ in case (i) is also too small at $|t|>0.2 (\text{GeV}/c)^2$ to repro-
duce the experimental result. However, it may be impossible to exclude such a solution, because our assumption of purely imaginary amplitude without helicity flip is not so good at $|t| > 0.2 \text{(GeV/c)}^2$ and the real part of $f(s, t)$ or the helicity flip amplitude may contribute considerably to $d\sigma/dt$ in this $t$-region.

In our study, the difference between the differential cross sections for $\bar{p}-p$ and $p-p$ scattering comes from the effects of the Regge amplitude $A_R \propto s^{-1/2} \times \exp \left( \frac{1}{2} B g \right) J_0 \left( r \sqrt{-t} \right)$ due to the $\omega$ and $\rho$ trajectories. Since the Bessel function $J_0 \left( r \sqrt{-t} \right)$ changes its sign at $t = -0.2 \text{(GeV/c)}^2$, the $\chi(t)$ defined by

$$
\chi(t) = \left[ \frac{d\sigma}{dt}(\bar{p}-p) - \frac{d\sigma}{dt}(p-p) \right]
$$

changes sign around this $t$-value. This is usually called the "crossover" phenomenon.

The contributions from the Regge amplitudes due to the $R$-trajectories decrease with energy because of the factor $s^{-1/2}$. This can be seen through the following results: $\chi(t=0)$ at $s = 462 \text{ GeV}^2$ is about 13 and 11 mb/(GeV/c)$^2$ for cases (i) and (ii), respectively, while $\chi(t=0)$ at $s = 2800 \text{ GeV}^2$ is about 6 and...
5 mb/(GeV/c)^2 for cases (i) and (ii), respectively.

It is well known that duality and the absence of exotic resonance lead to exchange degeneracy between the Regge trajectories with opposite signatures. Thus, it can be expected that \( A_0 \) for \( p-p \) scattering turns out to be zero, if there is no resonance in the \( p-p \) system. As is shown in Eq. (9) or (16), however, this is not the case, although the value of \( |A_0| \) for \( p-p \) scattering is much smaller than that for \( \bar{p}-p \) scattering. This would mean that in the description of \( p-p \) scattering, duality does not hold exactly but may be regarded as an approximately correct concept.

Finally, we wish to discuss briefly the difference between cases (i) and (ii). So far as our results for \( \sigma_b(0) \) and \( d\sigma/dt \) at \( s = 40 - 2800 \text{ GeV}^2 \) are concerned, it is difficult to determine which case is more promising. In order to discriminate between them, it may be necessary to examine the real part (Re \( f(s,t) \)) of the forward amplitude^40 which can be estimated by making use of the dispersion relation, since it can be expected that the properties of \( \sigma_b \) at \( s > 3000 \text{ GeV}^2 \) have a large effect on those of the Re \( f(s,t) \) in the ISR energy range through the dispersion relation.

References

14) For the predictions of \( \text{Re}(s,t)/\text{Im}(s,t) \) in case (i), see C. Bourrely and J. Fischer, CERN Report, Ref. TH. 1652-CERN, 12 April 1973.