Anomalies of Trace Identities and the Callan-Symanzik Equation in the Space-Time of Continuous Dimension

Akira UKAWA

Department of Physics, University of Tokyo, Tokyo 113

(Received January 18, 1974)

Anomalies of trace identities are investigated by generalization of the method of dimensional regularization in which the Lagrangian itself is defined in the world of continuous dimension. By means of setting the momentum transfer at the dilatation current equal to zero, the explicit form of the anomalies is determined and the Callan-Symanzik equation is derived.

§ 1. Introduction

In recent years the asymptotic behavior of Green’s functions in field theory has attracted people with renewed interest and has been investigated extensively in connection with the Bjorken scaling of electroproduction structure functions. The differential equation formulated by Callan$^3$ and Symanzik$^5$ has proved itself to be most powerful for such investigations. The asymptotic behavior, on the other hand, is closely related with the properties of Lagrangians under scale transformations since at asymptotic momenta mass terms may be neglected and scale invariance would become exact. Therefore, it is natural to think that the Callan-Symanzik equation might be a consequence of scale invariance. More precisely, we may ask if it is possible to derive the Callan-Symanzik equation from the “Ward”-type identities—to be referred to as trace identities from now on—*) expressing

\[ \partial \sigma D_\sigma = \theta \phi \],

where \( \theta \) and \( D_\sigma \) are the improved energy-momentum tensor and the dilatation current, respectively. The answer seems to be affirmative**) provided we use another set of identities—to be called the Ward identities in what follows—expressing conservation of energy and momentum:

\[ \partial \theta = 0 \].

Unfortunately, the trace identities derived naively by canonical reasoning do not

*) We follow the notation of Refs. 3) and 4).

**) Indeed, Callan$^3$ started with Eq. (1). His derivation is incomplete, however, in that the origin of his function \( f(\lambda) - \beta(\lambda) \) in the usual notation—is not clear at all. Symanzik, on the other hand, never used Eq. (1).
A. Ukawa
give the correct Callan-Symanzik equation. This is easy to understand if we only remember that introduction of some kinds of massive regulator field is unavoidable in the conventional regularization methods to render the perturbation expansions meaningful. Roughly speaking, a massive regulator field $\phi$ of mass $M$ contributes a term $-M^2 \phi^2$ to $\theta^1_\mu$ and the effect of the added term does not disappear as $M$ tends to infinity due to the factor $M^2$. In other words, the correct trace identities should contain an anomaly. Unlike the famous VVA triangle anomaly, this anomaly receives full contribution from higher order radiative corrections which, together with the complications caused by the presence of regulator fields, gives rise to difficulty in a direct perturbation theoretic derivation of the anomalous trace identities in the conventional regularization framework.

A way out of this difficulty is provided by the method of continuous dimension for which no regulator field is necessary. In usual applications of this idea, however, an analytic continuation to the continuous dimension $n$ of space-time is made after Feynman integrals are written down. This is clearly inadequate to treat scale invariance properly since even a scale invariant Lagrangian in the four-dimension becomes non-invariant in the $n$-dimension. Therefore, we must formulate the theory in the $n$-dimensional space-time from the very beginning at the Lagrangian level. Any coupling which is dimensionless in the four-dimension then acquires a certain dimension and contributes to the divergence of the dilatation current $\partial^\alpha D_\alpha$ in addition to the mass terms. Moreover, operator dimensions of fields are of course anomalous, resulting in other terms in $\partial^\alpha D_\alpha$. In this way we include all the possible sources inherent in the theory to violate scale invariance. Then the naive trace identities are true to all orders of perturbation theory. These identities can be further analyzed and give the correct trace identities with anomaly, which, combined with the Ward identities, reproduce the Callan-Symanzik equation.

In this paper we shall illustrate the general program mentioned above by performing detailed calculations for the simplest model, namely quartically self-coupled scalar field. Essential ingredients of our approach are completely contained in this example and extension to more general theories is straightforward.**

Some remarks on renormalization and unitarity in $n$-dimension are in order here. As for renormalization, we use the Bogoliubov-Parasiuk-Hepp procedure since the $R$-operation and its interpretation by formal counter terms work in any dimension if we define the superficial degree of divergence of a graph by its value in the four-dimension. Unitarity is not so simple since it is not clear whether the Landau-Cutkosky rule can be extended to $n$-dimension or not. We do not, however, make explicit use of unitarity, and this question, though interesting,

---

* The Ward identities should not have such an anomaly since it expresses the homogeneity of space-time.
** Theories which explicitly depends on $\gamma_5$ matrix are excluded.
In § 2 we give our model and write down the Ward and trace identities. In § 3 the trace identities are analyzed for a special configuration of external momenta and the Callan-Symanzik equation is derived. Discussions on the results are given in § 4.

§ 2. Energy-momentum tensor, dilatation current, Ward and trace identities

We consider the model specified by the Lagrangian

\[ L = \frac{1}{2} (\partial^\mu \phi \partial_\mu \phi - \mu^2 \phi^2) - \frac{\lambda}{4!} \phi^4 \]

\[ + (Z_s - 1) \frac{1}{2} (\partial^\mu \phi \partial_\mu \phi - \mu^2 \phi^2) + Z_\delta \mu^2 \phi^2 - (Z_1 - 1) \frac{\lambda}{4!} \phi^4 \]  \hspace{1cm} (3)

where \( \phi \) is a renormalized scalar field, \( \mu \) and \( \lambda \) are the renormalized mass and coupling constant, respectively, and the dimension of space-time is chosen to be \( n \). From this Lagrangian, we can construct the improved energy-momentum tensor \( \theta_{\mu\nu} \):

\[ \theta_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left( \frac{1}{2} \partial^\sigma \phi \partial_\sigma \phi - \frac{1}{2} \mu^2 \phi^2 - \frac{\lambda}{4!} \phi^4 \right) \]

\[ + \frac{d_\phi}{2(n-1)} (\partial^\rho \phi \partial_\rho \phi - \partial_\rho \partial_\rho) \phi^2 + \text{counter terms} \]  \hspace{1cm} (4)

and the improved dilatation current \( D_\mu \):

\[ D_\mu = x^\nu \theta_{\mu\nu}. \]  \hspace{1cm} (5)

If we use the canonical commutation relation of field \( \phi \), the dilatation current thus constructed satisfies the following commutation relation:

\[ i \left[ \int d^{n-1}x D_\mu(x), \phi(y) \right]_{x^* \to y^*} = \left( y \frac{\partial}{\partial y} + d_\phi \right) \phi(y) \]  \hspace{1cm} (6)

which at the same time defines the operator dimension \( d_\phi \) of field \( \phi \). Divergence of the dilatation current can also be calculated:

\[ \partial^\mu D_\mu = \mu^2 \phi^2 + \gamma_\phi (\partial^\mu \phi \partial_\mu \phi - \mu^2 \phi^2) + (4 - n - 4\gamma_\phi) \frac{\lambda}{4!} \phi^4 + (Z_s - 1) \mu^2 \phi^2 - Z_\delta \mu^2 \phi^2 \]

\[ + (Z_1 - 1) \gamma_\phi (\partial^\mu \phi \partial_\mu \phi - \mu^2 \phi^2) + (Z_1 - 1) (4 - n - 4\gamma_\phi) \frac{\lambda}{4!} \phi^4. \]  \hspace{1cm} (7)

Here \( \gamma_\phi \) denotes the deviation of dimension \( d_\phi \) from its free field value \( (n-2)/2 \):

\[ d_\phi = \frac{n-2}{2} + \gamma_\phi. \]  \hspace{1cm} (8)
Equations (4) \sim (8) are straightforward generalizations of canonical theory of scale invariance given in Ref. 4) with due account of requisite modifications mentioned in \S 1.

Two questions should be raised at this point. First; are \( \theta_{n*}, D_n \) and \( \partial^* D_n \) finite\(^*) \) operators? This question is partly answered by use of the Ward and trace identities to be derived shortly.\(^**\) Second; does the canonical commutation relation (6) agree with that calculated by the Bjorken-Johnson-Low limit?\(^12,13\) This question is deeply connected with the first one and we have not reached any conclusion yet. The commutator (6), therefore, should be taken as an assumption in this paper.

Now we define three types of Green's functions:

\[ \Gamma_{\mu\nu}^{(N)}(q; p_1, \ldots, p_N) = \int d^nx d^n x_1 \cdots d^n x_N e^{i(qx + p_1 x_1 + \cdots + p_N x_N)} \langle T^*(\theta_{\mu\nu}(x)\phi(x_1)\cdots\phi(x_N)) \rangle_0, \]

(9)

\[ \Gamma^{(N)}(q; p_1, \ldots, p_N) = \int d^nx d^n x_1 \cdots d^n x_N e^{i(qx + p_1 x_1 + \cdots + p_N x_N)} \langle T(\partial^* D_n(x)\phi(x_1)\cdots\phi(x_N)) \rangle_0, \]

(10)

\[ G^{(N)}(p_1, \ldots, p_N) = \int d^nx d^n x_1 \cdots d^n x_N e^{i(p_1 x_1 + \cdots + p_N x_N)} \langle T(\phi(x_1)\cdots\phi(x_N)) \rangle_0, \]

(11)

where \( T \) is the time-ordered product and the \( T^* \)-product in Eq. (9) is defined as follows:

\[ \langle T^*(\theta_{\mu\nu}(x)\phi(x_1)\cdots\phi(x_N)) \rangle_0 = \langle T(\theta_{\mu\nu}(x)\phi(x_1)\cdots\phi(x_N)) \rangle_0 \]

\[ -i \frac{d}{d_n} N(q_{\mu*} - n_{\mu} n_n) \langle T(\phi(x_1)\cdots\phi(x_N)) \rangle_0 \sum_{t=1}^{N} \delta^n(x-x_t) \]

(12)

\( (n_n \) is an \( n \)-dimensional generalization of the time-like unit vector \( n_n = (1, 0, 0, 0) \)).

Making use of the canonical commutation relation of field \( \phi \), Eqs. (1) and (2) can be converted into the Ward identities

\[ q^{\mu*} \Gamma_{\mu\nu}^{(N)}(q; p_1, \ldots, p_N) = -i \sum_{t=1}^{N} (p_t + q) G^{(N)}(p_1, \ldots, p_t + q, \ldots, p_N) \]

(13)

and the trace identities

\[ q^{\mu*} G^{(N)}(q; p_1, \ldots, p_N) = \Gamma^{(N)}(q; p_1, \ldots, p_N) - id^N_{\phi} \sum_{t=1}^{N} G^{(N)}(p_1, \ldots, p_t + q, \ldots, p_N). \]

(14)

\(^*) By "finite" we mean the absence of unwanted poles at \( n=4 \).

\(^**\) The original proof of Ref. 3) that those identities are finite is not correct since \( \partial^* D_n \) contains many types of terms other than mass terms if infinities of perturbation theory are properly taken into account.
The crucial point in the following analysis is that, though derived by canonical reasoning, both the Ward and trace identities must be true as it stands to all orders of perturbation theory. This is because we have avoided the occurrence of troublesome infinities by going to the n-dimension and at the same time included all the effects of scale invariance breaking (the latter fact is explicit in Eq. (7)).

The absence of poles at \( n = 4 \) in \( \Gamma^{(N)} \) and \( \Gamma^{(N)} \) when \( q = 0 \) follows naturally from these two identities. We differentiate Eq. (13) with respect to \( q \) and then set \( q = 0 \):

\[
\Gamma^{(N)}(0; p_1, \ldots, p_N) = -i \left[ (N-1) g_{\mu \nu} + \sum_{i=1}^{N} p_{\mu i} \partial_{p_{\nu i}} \right] G^{(N)}(p_1, \ldots, p_N). \tag{15}
\]

Since renormalizability of our model assures that \( G^{(N)} \) has no poles at \( n = 4 \), Eq. (15) tells us that this is also the case with \( \Gamma^{(N)}(0; p_1, \ldots, p_N) \), which, together with Eq. (14), is sufficient to guarantee the same for \( \Gamma^{(N)}(0; p_1, \ldots, p_N) \).

§ 3. Anomalous trace identities and the Callan-Symanzik equation

The Green's function \( \Gamma^{(N)}(q; p_1, \ldots, p_N) \) introduced in § 2 can be further simplified when \( q = 0 \).

Let us consider the Green's function

\[
G^{(N)}(p_1, \ldots, p_N) = Z^{N/2} G_0^{(N)}(p_1, \ldots, p_N), \tag{16}
\]

where \( G_0^{(N)} \) denotes the unrenormalized Green's function to be expanded in powers of the unrenormalized coupling constant \( \lambda_0 \) using \( i(p^2 - \mu_0^2)^{-1} \) as the free propagator (\( \mu_0 \) is the unrenormalized mass). A Feynman diagram \( G \) of order \( m \) in the unrenormalized perturbation expansion contributes the following term to \( G_0^{(N)} \):

\[
G_0^m = (-i\lambda_0)^m \Gamma_0 \int \frac{d^Dk}{(2\pi)^D} \prod_{r=1}^{N_{\text{int}}} \frac{i}{q_r^2 - \mu_0^2} \prod_{i=1}^{N} \frac{i}{p_i^2 - \mu_0^2} \tag{17}
\]

with \( \Gamma_0 \) being the combinatorial factor, \( k_1(1 \leq i \leq L) \) the loop momenta and \( N_{\text{int}} \) the number of internal lines. Let us consider how we can construct from (17) Feynman integrals of \( \Gamma^{(N)}(0; p_1, \ldots, p_N) \). To see what happens, we write \( \partial^a D_a \) by unrenormalized quantities,

\[
\partial^a D_a = \mu_0^2 \phi_0^2 + \gamma_1 (\partial^a \phi_0 \partial_a \phi_0 - \mu_0^2 \phi_0^2) + (4 - n - 4\gamma_1) \lambda_0 \phi_0^4 \tag{18}
\]

(\( \phi_0 \) is the unrenormalized field). The first term \( \mu_0^2 \phi_0^2 \) is the unrenormalized mass insertion term and we denote the contribution of this term to \( \Gamma^{(N)} \) by \( \Delta \Gamma_u^{(N)} \). The effect of the second term \( \gamma_1 (\partial^a \phi_0 \partial_a \phi_0 - \mu_0^2 \phi_0^2) \) is replacement of each propagator \( i(Q^2 - \mu_0^2)^{-1} \) \( (Q = q_r, 1 \leq r \leq N_{\text{int}} \) or \( = p_i, 1 \leq i \leq N) \) by

\[
i \frac{i}{Q^2 - \mu_0^2} 2\gamma_1 (Q^2 - \mu_0^2) \frac{i}{Q^2 - \mu_0^2} = 2i\gamma_1 \frac{i}{Q^2 - \mu_0^2}. \]
This amounts to the multiplication of $G_0^m$ by $2i\gamma_\phi (N_{\text{int}} + N)$. As for the third term $i(4-n-4\gamma_\phi) (-i\lambda_0/4! \phi_0)$, we have only to replace every vertex $-i\lambda_0$ of $G_0^m$ by $i(4-n-4\gamma_\phi) (-i\lambda_0)$ one by one, therefore we get $i(4-n-4\gamma_\phi)m$ times $G_0^m$. These two contributions add up to

$$Z_{s-N/2} [2i\gamma_\phi (N_{\text{int}} + N) + i(4-n-4\gamma_\phi) m] G_0^m = Z_{s-N/2} [\tau_\phi N + (4-n) \lambda_0 \frac{\partial}{\partial \lambda_0}] G_0^m. \tag{19}$$

(We have used the relation $2N_{\text{int}} + N = 4m$ and $m\lambda_0^m = \lambda_0 (\partial / \partial \lambda_0) \lambda_0^m$.) By means of summation over all possible diagrams, Eq. (19) gives

$$\Gamma^{(N)}(0; \rho_1, \ldots, \rho_N)$$

$$= \Delta \Gamma^{(N)}(0; \rho_1, \ldots, \rho_N) + iZ_{s-N/2} [\tau_\phi N + (4-n) \lambda_0 \frac{\partial}{\partial \lambda_0}] G_0^{(N)}(\rho_1, \ldots, \rho_N)$$

$$+ i \left[ N \left( \tau_\phi + \frac{4-n}{2} \lambda_0 \frac{\partial}{\partial \lambda_0} \ln Z_{s} \right) + (4-n) \lambda_0 \frac{\partial \lambda}{\partial \lambda_0} \frac{\partial}{\partial \lambda} \right] G^{(N)}(\rho_1, \ldots, \rho_N). \tag{20}$$

In this equation, it is important to note that the derivatives with respect to $\lambda_0$ are taken with fixed $\mu_0$. Since $\mu / \mu_0$, $\lambda / \mu_0^{4-n}$ and $Z_i$ depend on $\mu_0$ and $\lambda_0$ only through the combination $\lambda_0 / \mu_0^{4-n}$, it is possible to rewrite Eq. (20) in the following way;

$$\Gamma^{(N)}(0; \rho_1, \ldots, \rho_N) = Z^{-1} \Delta \Gamma^{(N)}(0; \rho_1, \ldots, \rho_N)$$

$$+ i \left[ (1-Z^{-1}) \mu \frac{\partial}{\partial \mu} + [(4-n) \lambda - Z^{-1} \beta(\lambda)] \frac{\partial}{\partial \lambda} \right]$$

$$+ (1-Z^{-1}) N_{\gamma_\phi} \frac{\partial}{\partial \lambda_0} \frac{\partial}{\partial \lambda}$$

$$G^{(N)}(\rho_1, \ldots, \rho_N), \tag{21}$$

where we have introduced the multiplicative factor $Z$ and the function $\beta(\lambda)$ by

$$Z = 1 / \frac{\partial \ln \mu}{\partial \ln \mu_0} \lambda_0^{6x},$$

$$\beta(\lambda) = \mu \frac{\partial \lambda}{\partial \mu} \lambda_0^{6x} \tag{22}$$

and defined $\Delta \Gamma^{(N)}$ by

$$\Delta \Gamma^{(N)}(q; \rho_1, \ldots, \rho_N) = Z \Delta \Gamma^{(N)}(q; \rho_1, \ldots, \rho_N).$$

Moreover we have identified the up-to-now undetermined anomalous dimension $\gamma_\phi$ as
Anomalies of Trace Identities and the Callan-Symanzik Equation

\[ \Gamma_\phi = \mu \frac{\partial \ln Z_\phi}{\partial \mu} \]

(23)

(These forms for \( Z, \beta (\lambda) \) and \( \Gamma_\phi \) agree with their usual definitions.) Substituting the expression of \( \Gamma^{(N)} \) from Eq. (21) in the trace identities (14) for \( q = 0 \), we get the anomalous trace identities:

\[
g^{\ast \ast} \Gamma^{\langle \langle \ast \rangle \rangle} (0; p_1, \ldots, p_N) = Z^{-1} \Delta \Gamma^{(N)} (0; p_1, \ldots, p_N) + i \left[ (1 - Z^{-1}) \mu \frac{\partial}{\partial \mu} + \left[ (4 - n) \lambda - Z^{-1} \beta (\lambda) \right] \frac{\partial}{\partial \lambda} \right. \\
\left. - N \left[ \frac{n - 2}{2} + Z^{-1} \nu \right] \right] G^{(N)} (p_1, \ldots, p_N).
\]

(24)

This is the final result of our graphical analysis. Unfortunately, the anomalous term has a rather complicated structure and the meaning of these identities is not clear in such a form. To simplify these identities (and this modification leads us to the Callan-Symanzik equation as we shall see below), we use Eq. (15) which gives

\[
g^{\ast \ast} \Gamma^{\langle \langle \ast \rangle \rangle} (0; p_1, \ldots, p_N) = -i \left[ (N - 1) n + \sum_{i=1}^{N} p_i \frac{\partial}{\partial p_i} \right] G^{(N)} (p_1, \ldots, p_N) = 0,
\]

(25)

which simply represents that \( G^{(N)} \) has the mass dimension \( N(n/2 + 1) - n \). Eliminating the \( \sum_{i=1}^{N} p_i (\partial / \partial p_i) \) term from (25) and (26) and then equating the resultant expression for \( g^{\ast \ast} \Gamma^{\ast \ast} \) with the right-hand side of Eq. (24), we get the simplified form which is nothing other than the Callan-Symanzik equation:

\[
\left[ \mu \frac{\partial}{\partial \mu} + \beta (\lambda) \frac{\partial}{\partial \lambda} + N \gamma_\phi \right] G^{(N)} (p_1, \ldots, p_N) = -i \Delta \Gamma^{(N)} (0; p_1, \ldots, p_N).
\]

(27)

For comparison, we write down the conventional form of the Callan-Symanzik equation:

\[
\left[ \mu \frac{\partial}{\partial \mu} + \beta (\lambda) \frac{\partial}{\partial \lambda} - N \gamma_\phi \right] \bar{G}^{(N)} (p_1, \ldots, p_N) = -i \Delta \Gamma^{(N)} (0; p_1, \ldots, p_N),
\]

(28)

where \( \bar{G}^{(N)} \) and \( \Delta \Gamma^{(N)} \) denote the one-particle-irreducible parts of \( G^{(N)} \) and \( \Gamma^{(N)} \), respectively. In order to establish the equivalence of Eqs. (27) and (28), let us recall that Symanzik\(^{14}\) derived Eq. (28) by differentiating

\[
\Gamma^4 (p_1, \ldots, p_{2N}; m^2, g) = Z(s)^{-s} \Gamma (p_1, \ldots, p_{2N}; m^2(s), g(s))
\]

(29)

with respect to \( s \) and then setting \( s = 0 \). The negative sign in front of \( N \gamma_\phi \) in

\( ^{14} \) For the notation, see Ref. 14. His \( \Gamma^4 (p_1, \ldots, p_{2N}; m^2, g) \) corresponds to our \( \bar{G}^{(N)} (p_1, \ldots, p_{2N}) \).
Eq. (28) originates from the factor $Z(s)^{-N}$ in Eq. (29). If we apply his method to the full Green's function $G^{(s)}$, $Z(s)^{-N}$ in Eq. (29) should be replaced by $Z(s)^N$, which explains the positive sign in front of $N\gamma_\phi$ in Eq. (27). The distributive property of the operator $\mu(\partial/\partial \mu) + \beta(\lambda) (\partial/\partial \lambda) + N\gamma_\phi$, noted by Callan\textsuperscript{1} provides still another way to check the equivalence, since the full Green's functions $G^{(s)}$ can be expressed as sums of certain products of the one-particle-irreducible parts. Essentially, $G^{(s)}$ is obtained from $\overline{G}^{(s)}$ by attaching the propagator $[\overline{G}^{(s)}]^{-1}$ to each external leg. Since (1) it is the inverse of $\overline{G}^{(s)}$ attached to the external legs, and (2) the operator $\mu(\partial/\partial \mu) + \beta(\lambda) (\partial/\partial \lambda) + N\gamma_\phi$ is distributive, there should be (and there really is) a difference by $2N\gamma_\phi$ between the constant terms on the left-hand sides of Eqs. (27) and (28).

§ 4. Discussion and summary

In the previous section, we have derived the Callan-Symanzik equation (27) for the full Green's functions from the anomalous trace identities. However, the proof of these identities is not essentially altered, if we take one-particle irreducible parts of $\Gamma_{\mu}^{(m)}$, $\Gamma^{(s)}$ and $G^{(s)}$. Therefore, the Callan-Symanzik equation (28) for such proper vertices can also be derived from the anomalous trace identities. This fact constitutes the third proof of the equivalence of Eqs. (27) and (28).

More important is the fact that our proof clarifies the key role played by the term $(4-n)(\lambda/4!) \phi^4$ in $\partial^s D_\mu$. Without this term, the $\beta(\lambda)$ term would never have shown up and identification of $\gamma_\phi$ would have been impossible. Because of the factor $4-n$, the effect of such a term cannot easily be taken into account if we stick to the four-dimension. This fact convinces us of the inevitability of introducing the continuous dimension at the Lagrangian level and at the same time proves the power of such a formulation. Now we turn to the weak point of our proof, namely the commutator (6). We should check whether the commutator determined by the Bjorken-Johnson-Low limit of the quantity

$$ \int d^n x e^{i x \cdot p} \langle \phi | T^s (D_\mu (x) \phi (0)) | 0 \rangle $$

agrees with the right-hand side of Eq. (8) or not. Unfortunately, a general discussion seems difficult without recourse to the trace identities (14) originally derived with the very aid of Eq. (6). If we use the trace identities (14), a simple calculation shows that the coefficient of $q_0^{-1}$ of (30) becomes $d_\phi$ plus $\Gamma^{(s)}(q, p, -p-q) G^{(s)}(p+q)$ in the limit $q_0 \to \infty$, $q = 0$. However, even the finiteness of $\Gamma^{(s)}(q, p, -p-q)$ is not clear when $q = 0$, because of the non-soft operator $\gamma_\phi \partial_\mu \phi \partial_\mu \phi$ in $\partial^s D_\mu$. In fact, lower order calculations show that a marvelous cancellation is expected among the contributions from $\gamma_\phi \partial_\mu \phi \partial_\mu \phi - \mu^2 \partial_\mu \phi$ and from $(4-n)(\lambda/4!) \phi^4$ in order to make $\Gamma^{(s)}(q, p, -p-q)$ finite. If such a cancellation indeed occurs, then $\Gamma^{(s)}(q, p, -p-q)$ would behave at most like some powers of $\ln(-q^2)$ and the BJL limit value would coincide...
with the canonical one. Moreover the cancellation mechanism would also work for \( \overline{J}^{(n)}(q; p_1, \ldots, p_N) \), and therefore, both \( \overline{J}^{(n)}_{\alpha\beta} \) and \( \overline{J}^{(n)} \) would be finite. Everything would be consistent in such a case. These points remain to be further investigated.

For summary, we have formulated the generalization of the method of dimensional regularization suitable for the study of scaling properties of Green’s functions. Making use of this formulation, we have determined the form of the anomaly in the trace identities and clarified its connection with the Callan-Symanzik equation.

Acknowledgement

The author would like to thank Professor K. Nishijima for useful criticisms and careful reading of the manuscript.

References

3) C. G. Callan, Jr., S. Coleman and R. Jackiw, Ann. of Phys. 59 (1970), 42.

Note Added:

After completing this work, it came to the author’s attention that D. A. Akyeampong and R. Delbourgo developed a similar idea of introducing anomalies via dimensional regularization (ICTP preprint, ICTP/72/33).