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The Physics of Limits

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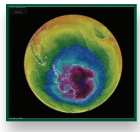

This image signals that this contribution is a featured part of the special collection on "Climate Change and Sustainability in Intro Physics."

Physics offers a solid foundation upon which to build simple yet powerful perspectives on challenges we face inhabiting a finite planet. This work offers suggestions for framing the human endeavor in the contexts of space, time, energy, and ecology, as well as hinting at productive order-of-magnitude calculations that students can use to gain powerful insights into constraints within which we must operate.

Tools from the physical sciences can be employed to clarify demands associated with expansion of modernity within a finite resource environment. Our complex world can appear to be primarily governed by human constructs in political, legal, ethical, and social domains. Yet underneath it all is an uncompromising biophysical foundation upon which all other constructs rest. It is as easy to neglect this ever-present backdrop as it is to take for granted the air we breathe.

Physics has the benefit of being able to cut through the baffling tangle of human fabrications to offer stark and incontrovertible truths that can help us appreciate a bigger picture. Often, order-of-magnitude estimates are sufficient to expose important features of the world, without having to obsess over distracting minutiae. This paper sketches a variety of examples that may inspire the application of similar approaches to related questions.

Setting the stage

Spatial perspectives

Textbook graphics are incapable of conveying astronomical scales, as the ink dots (or pixels) on the page become too small to perceive or even produce. As a result, students often have a distorted sense of the extraterrestrial environment, which can lead to unrealistic expectations. Grasping the true magnitude might provide a sense of insignificance and humility that could be important in defining how we are to interact with the world—treating Earth as a rare oasis that we are lucky to inhabit as guests.

Myriad possibilities for representing scales can be used, but here I suggest scaling the Sun to a 1-mm grain of sand.

This is easy to visualize and already humbling. Using this scale produces the following results:

- The solar system is the size of a bedroom.
- The brilliantly luminous sand grain in the middle of the bedroom contains 99.85% of the mass within.
- The few scattered specks of dust (planets) are hard to even notice.
- Humans have never traveled farther than 0.25 mm from microbe-sized Earth on this scale, and mere microns in the last 50 yr.
- The next star is another luminous sand grain 30 km away: a whole day's walk.

One feature to emphasize is that we need not be precise in this exercise: doing so could diminish the simple impact. While some of these elements may be obvious to instructors, classroom surveys I have conducted over the years reveal that most students are stunned to realize our limited travel range—imagining that we have continued going to the moon (and beyond) since the Apollo era. It puts the *far* more distant Mars into useful context.

One point to stress is how much empty space is out there. Entertainment often gives the false impression of not being able to turn around without bumping into another planet, nebula, or space ship. The space environment is also dim. Surface brightness is conserved, so that no matter how close one gets to a nebula, for instance, it does not appear to get any brighter than what we see from here.

It is also possible to explore extensions of this scale to galactic and intergalactic domains, but intuition breaks: one single scale is not adequate to simultaneously comprehend the solar system and universe, as the dynamic range exceeds our intuitive abilities.

On a related note, density can be a useful way to characterize our unusual circumstance. The universe as a whole is roughly 30 orders of magnitude less dense than the environment we consider to be ordinary. Figure 1 illustrates the spectrum. Direct human experience tends to span fewer than four orders of magnitude—from the rarefied atmosphere atop Mt. Everest to dense materials like gold and platinum.

Temporal perspectives

Not only is our species isolated to a speck of dust around a speck of sand slowly swirling in an unfathomably sparse swarm of 100 billion other sand grains (stars) in just one of a comparably large number of isolated galaxies, our temporal existence is also minuscule on a cosmological scale.

A common approach to temporal perspectives is to scale some reference interval to a day, a year, a lifetime, or something similar for which we have a direct sense. Table I offers some examples, but many variations are possible. Note that the temporal inputs in the table are not always precise—and do not need to be. Thus, some rounding liberty has been taken to make results more memorable and intuitive.

No matter how one slices it, the glaring lesson is that our modern age is a transient flash: an alarming eruption, or a glitch, even. I often relate it to a fireworks show: dazzling, impressive, awe-inspiring, unlike anything that has ever

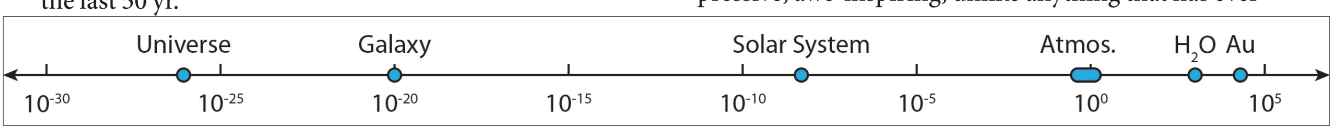

Fig. 1. Densities in the universe, in kg/m^3. Direct human experience is confined to the right-hand end of the spectrum. The atmosphere slug represents Mt. Everest to sea-level air density.

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Table I. Temporal scales.

Phenomenon	Age	Lifetime	Year	Day
Universe	13.8 Gyr	70 yr		
Solar system	4.5 Gyr	23 yr		
Life on Earth	3.5 Gyr	18 yr		1 d
Mammal "rule"	65 Myr	4 mo		25 min
First humans	3 Myr	6 d	1 yr	1 min
Homo sapiens	300 kyr	12 h	1 mo	7 s
Civilization	10 kyr	25 min	1 d	0.25 s
Fossil fuels	200 yr	30 s	35 min	5 ms
70% drop in vertebrates[1]	50 yr	8 s	10 min	1 ms

happened, then over.

One could contemplate exploring timelines graphically, as a nested set of one-dimensional time axes. As is the case for spatial scales, the large dynamic range prohibits a meaningful (linear) presentation on a single scale, which in some ways is the whole point. Logarithmic presentations cleverly "solve" this problem, but betray our intuition and mute exactly the point that is sought.

One mindset that can lead to poor decisions for a pleasant future is that the universe has been building toward *us*: that we are the culmination of a magnificent process of cosmology and evolution—rendered meaningful only by dint of its resulting in human civilization. The spatial and temporal perspectives might motivate students to question the validity of such self-importance. Surely, for instance, vast tracts of time still lie in the future: "now" is not the endpoint (Table I is guilty of conveying this impression as well). This becomes relevant because a key step in good decision-making is to recognize that it's not all about us, here and now: we're part of a larger, ancient ecosystem and would ultimately fare best as subordinate partners who value the far future.

Energy perspectives

Physics "owns" energy (and space and time if we're being greedy), so energy and power domains offer rich opportunities for gaining new understanding. It is useful to start on a personal scale to which every student can directly relate. Nutrition labels quantify the energetic content of the food we eat—usually in kilojoules or kilocalories (called Calories in the U.S.). Taking a standard daily diet to be 2000 kcal, the resulting 8.4 MJ spent over 86,400 s (1 d) works out to very nearly 100 W—a convenient and memorable number. In the past, we could relate this to a 100-W light bulb, but that loses meaning in a post-incandescent age (the lumen is a far more appropriate measure and should have been used from day one). A refrigerator—time averaged—is close to the mark, or a television, when turned on.

It is relatively easy to track down annual energy expenditures of individual countries or of the whole world. For instance, the U.S. Energy Information Agency produces an annual review of energy use.[2] In 2021, the U.S. consumed 97.33 "quads" (quadrillion Btu, or $97.33 \times 1.055 \times 10^{18}$ J) of energy. Dividing by the U.S. population and seconds in a year, this computes to a per-capita energy demand of 9800 W. This is approximately 100 times the individual's metabolic power, leading to the useful and provocative realization that Americans each (on average) employ 100 energy servants distributed across the land. It is therefore no wonder that we live such comfortable lives compared to those of the distant past. As fossil resources wane and net energy declines, we can expect to lay off some of these servants.

Another angle that gets students' attention is to explore what a continuation of historical growth rates in energy demand would require, physically. Applying the mathematically convenient—and reasonably representative—rate of 2.3% growth per year equates to a factor of 10 per century. Sparing details here, the result is that we would use

- the entire solar energy budget arriving at Earth (at 100% efficiency, impossibly) in 400 yr;
- the entire output of the Sun in all directions in 1400 yr;
- and the output of all stars in the Milky Way galaxy in 2500 yr (still short in Table I).

The thermodynamic consequences of this absurdity can also be examined, by which it is revealed that waste heat from all this energy production on Earth would rival climate change in a century and be 10 times worse a century after that, eventually boiling Earth's oceans in about 400 yr.[3] It can be tricky to convey to students that this is not a *prediction* of what *will*—or even *can*—happen, but a tool to understand *limits* to what is possible and what *cannot* transpire.

A focus on the role of fossil fuels in setting apart our current age is highly valuable, because it can help us understand this transient phase and that our recent experience is not a "new normal" that we can blithely assume applies to the indefinite future. A few points to consider:

- Fossil fuels were generated over $\sim 10^8$ yr and consumed on a timescale of $\sim 10^2$ yr, so that depletion is about one million times faster than "replacement." A representative value for the rate at which solar energy is sequestered in fossil fuels is tens of megawatts (one millionth present human demand of 18 TW). This is, relatably, the scale of electricity demand for a moderately large college campus. Fossil fuels are far from sustainable on a global scale.
- Filling a single car's fuel tank in a few minutes translates to energy delivery at a rate of about 15 MW—again comparable to a college campus demand or the entire globe's average rate of replacing fossil fuels. This fill-up of one car is comparable to 3000 homes running a whole-house air conditioner.
- The dominance of the U.S. in the world in the second half of the 20th century is closely related to the fact that the U.S. used about 70% of global oil in 1950 and over 80% of global natural gas (methane). This provides a literal definition of the term "superpower." Pursuit of prior glory is not simply a matter of political choices: the physical backing is just not there to do so (see section 7.2.1 of Ref. 4).

- It is not by chance that the Industrial Revolution coincided in both space and time with the adoption of fossil fuel use. In fact, the causality direction is rather clear: fossil energy fueled the Industrial Revolution. A student project exploring the timing, location, and pace of inventions will convey a sense that fossil fuels made this whole affair possible—for better or worse. Likewise, the Green Revolution in agriculture is a story of fossil fuels that has contributed to a soaring human population.
- Fossil fuels are so integral to our global civilization and technology that we rely on them not only for transportation in a global supply chain, but for making concrete and steel, and any alternative energy technology of consequence: nuclear, hydroelectric, wind, solar, etc. We ought to be careful assuming we can continue today's practices once fossil fuels are no longer available. It is in this sense that fossil fuels are taken for granted like the air we breathe—and ironically at the same time poisoning that very air.

Fossil fuels are a one-time resource that transformed the human footprint on our planet. We imagine a transition to renewable energy, but the resource burden does not disappear, and in fact becomes larger and ongoing in the material domain.[5] This century is one of reckoning with that reality: the future is not guaranteed to continue providing energy and materials at the rate to which we have so quickly become accustomed during this phase of rapid inheritance spending.

Ecological perspectives

As hinted in the last row of Table I, vertebrate counts are down to less than a third of what they were 50 yr ago.[1] Insects are also in rapid decline. The pattern of loss was already firmly established well before climate change began to manifest significantly. Like climate change, these are symptoms of our sudden industrial dominance of the planet and its habitats and ecosystems. Deforestation, extractive industries, urbanization, agricultural practices, and simple population dynamics have made life much harder for many nonhuman species on the planet, driving an alarming number to extinction. At this point, 96% of mammal mass on the planet is in the form of humans (36%) and our livestock (60%), leaving a tiny toehold of ~2% for land mammals and ~2% for marine mammals.[6]

Plots of almost any human-related metric over the last 1000 yr look like hockey sticks (e.g., population, energy, materials extraction, or gross domestic product; see Ref. 11), while plots of ecological metrics (e.g., forest cover and animal counts/mass) are rapidly plummeting. Table II offers an opportunity for students to exercise skill in curve fitting. As one possibility, an exponential of the form $A\{1 - \exp[(t - t_0)/\tau]\}$, where t is the year and τ is measured in years, can be analytically forced to go through the three data points in Table II for each of the three series (see the Appendix for guidance). The last two rows in the table indicate the result of the forced fit. The interpretation of the parameter t_0 is the year at which the fit line hits zero and the (modeled) resource is gone. While physics does not directly address ecological concerns, we can

Table II. Wild mammal mass and forest cover over time, together with fit parameters. Mammal data are from Refs. 6–8; forest data are from Refs. 9 and 10; I guess that half of forest cover was old growth (i.e., primeval, or virgin) in 1900 (it's 24% of forest today).

Year/Parameter	Wild Mammal Mass (Mt of C)	Forest Cover	Old Growth
–8000	15	57%	57%
1900	10	48%	24%
2018	3	38%	9%
τ	135 yr	158 yr	315 yr
t_0	2048	2191	2072

understand exponential curves and their ultimate limitations in a finite space.

Relatedly, it is instructive to interpret human population dynamics in terms of birth rates (global average: 18.1 births per 1000 people per year), death rates (7.7 per 1000 people per year), and the net rate as exponential or logistic progressions, as well as casting in terms of births, deaths, and net added people per second (4.5, 1.9, and 2.6, respectively).

Order-of-magnitude estimations

A number of crude but informative calculations can shed light on our challenges. Some have already been presented above. Here are a few others worth consideration, just as a taste.

A classic exercise is to compute how much sunlight arrives at Earth (based on the solar constant of 1360 W/m^2): in what amount of time (hours) do we receive our annual dose of societal energy? How much land area would have to be covered by 15% efficient solar panels? How does this compare to roof area? What about paved area? It sounds simultaneously trivial and impossibly daunting depending on the questions asked. How large would a battery have to be to provide days of energy to smooth out intermittency? What about seasonal storage? Do we have enough proven or estimated reserves of materials to construct such a battery? How much pumped storage would be required, and what would this mean in terms of reservoir area, flow rates, and concrete required?

On another front, if we were to replace our 18-TW energy habit by burning biomass in some form, what is the relevant timescale? In other words, dividing the energy content in all biomass (land and sea) by power indicates how long it would take to burn it all (and thus how rapidly it would all have to be replaced to be sustainable). Estimates of total biomass are about 500 Gt (dry), which incidentally stacks to 1 mm high if spread across Earth's surface (life is thin and precious). At an energy density of about 4 kcal/g, we get a timescale of 15 yr to burn it all. It takes longer than that to replace forests, so reliance on biomass can be ruled out as a sustainable approach, at the present scale of energy demand.

Although physics does not often veer into financial concerns, it is an illuminating exercise to try estimating the value of a barren planet at market prices for dirt, rock, water, copper, aluminum, silver, gold, etc. While any such approach will have obvious flaws, the result tends to be many orders of magnitude larger than the $\$10^{14}$ global annual budget. And this is before considering the priceless web of life that has evolved into interconnected ecosystems of exceeding complexity—for which we cannot conceivably conjure a substitute. This offers a sharp perspective on how we make decisions: those based on money—like many or most decisions we make—risk missing the *real* value. A similar cute exercise finds that Earth's animals are worth more than their weight in gold.[12] While also possessing flaws, the many orders-of-magnitude difference between the market value of a newt and its gold-equivalent value at least justifies asking whether our valuation schemes are biophysically appropriate.

Other productive paths

Leveraging the wide-boundary purview of physics onto important questions, we can make progress in asking big questions like, what practices will be possible to maintain for tens of thousands of years—if we want our civilization to be in its infancy, for instance? Continued mining is likely a dead end at today's scale for even hundreds of years more. Is recycling possible on such scales? Simple estimates of fractional recovery and turnover lifetime can quickly produce insightful conclusions on what the far future has in store.

As alluded to earlier, seeing the fossil fuel bonanza as a temporary fireworks show or a transient pulse is a powerful way to stimulate thought about central questions of our time. Is today unusual? Can tomorrow be the same or bigger/better? What role can humans play on this planet in the long term? Confronting these big questions—perhaps more often associated with philosophy rather than physics—can be informed by physics. All of this takes place on a physical stage, and the limits of a finite planet must ultimately prevail over unconstrained imagination.

I have performed many physics calculations over the years to better understand our challenges—many of which are represented on the *Do the Math* blog and in a general education textbook on the matter.[4] I am hoping that this article might stimulate similar fruitful explorations.

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Appendix: Fit solution

Table II suggests an analytic fit to the data of the form $A\{1-\exp[(t-t_0)/\tau]\}$, where A is the value at time $t=-\infty$, t_0 is the time at which the function hits zero, and τ is the time constant. Having three points, we can force a (nearly) exact fit by first approximating the early point at $t=-8000$ as being at $t=-\infty$. Labeling the three data points as A_1, A_2, and A_3 at times $t_1 \approx -\infty$, $t_2 = 1900$, and $t_3 = 2018$, we first set $A = A_1$ to correspond to the early data point, then force the following exact conditions:

$$A_1\{1-\exp[(t_2-t_0)/\tau]\} = A_2,$$

$$A_1\{1-\exp[(t_3-t_0)/\tau]\} = A_3.$$

By manipulating these two relations, it is possible to arrive at a solution for τ and t_0:

$$\tau = \frac{t_3 - t_2}{\ln\frac{A_1-A_3}{A_1} - \ln\frac{A_1-A_2}{A_1}},$$

$$t_0 = t_2 - \tau \ln\frac{A_1 - A_2}{A_1}.$$

Our treating $t_1 = -8000$ as $t_1 = -\infty$ results in a fractional error smaller than $\sim 10^{-14}$ at the early data point.

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