GEOPHYSICS, VOL. 42, NO. 3 (APRIL 1977); P. 514–521, 10 FIGS.

MICROWAVE BRIGHTNESS SPECTRA OF LAYERED MEDIA

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Many natural media are layered at the scale of microwaves. Examples include frozen soil over moist soil, ice over water, and snow over soil. The microwave brightness spectra of such media may exhibit interference patterns. Such patterns have been observed for emission from fresh-water ice but not for emission from snow or from seasonally frozen soil. Three factors which determine whether interference is detectable are the spectral resolution of the radiometer, the uniformity of layer thickness, and the distinctness of interfaces between layers. Analyses of these factors show that: (1) The radiobrightnesses of layered media vary sufficiently slowly with wavelength that radiometers designed for the radio-astronomy bands provide adequate spectral resolution; (2) a variability of layer thickness greater than 15 percent of the free-space wavelength in the area viewed by the radiometer will effectively eliminate an interference pattern; and (3) a diffuse interface, whose thickness is 15 percent of the free-space wavelength, is transparent to microwaves so that effects of the interface will not appear in the radiobrightness spectrum.

INTRODUCTION

The sum of the intensities, at a particular wavelength, of an object's thermal radiance and the sky radiance it reflects is called the object's brightness temperature at that wavelength. The 8–14 micron (or thermal infrared) brightness temperature of most natural objects can be used to infer the thermal temperature of the object. This is possible because thermal infrared emissivities of most natural surfaces are relatively independent of composition and of structure. Microwave (1 mm to 1 m) emissivities of soil, rock, or vegetation, in contrast, vary strongly with free water content and with geometric factors such as view angle, polarization, surface roughness, and internal structure. Therefore, the objective of most microwave brightness studies in geophysical remote sensing is to derive moisture content or something about one or more of the geometric factors. A combination of microwave and infrared brightness temperatures (e.g., England and Johnson, 1975) can be used to obtain microwave emissivities. These emissivities can then be interpreted in terms of the particular problem. For example, Hoekstra and Delaney (1974), Schmugge et al (1974), and Poe and Edgerton (1974) have discussed deriving moisture content from microwave emissivity.

Thermal microwave radiation originates within the emitting rock, soil, ice, or snow, and not at these materials' optical surfaces. The effective depth of the radio source is nearly proportional to wavelength. Therefore, variations in composition with depth, or the presence of structural elements such as internal or volume scatterers, may produce diagnostic radiobrightness spectra. That is, spectral radiobrightness may indicate the depth to a compositional interface (Blinn et al, 1972; England and Johnson, 1975), or may allow an estimate of the grain-size distribution of a scattering medium (England and Johnson, 1976). The theory of emission from uniformly layered media was presented by Stogryn (1970), and theories of emission from a half-space containing isotropic scatterers and from a layered medium containing Rayleigh scatterers were presented by England (1974, 1975). The relative effects of nonuniform layering,

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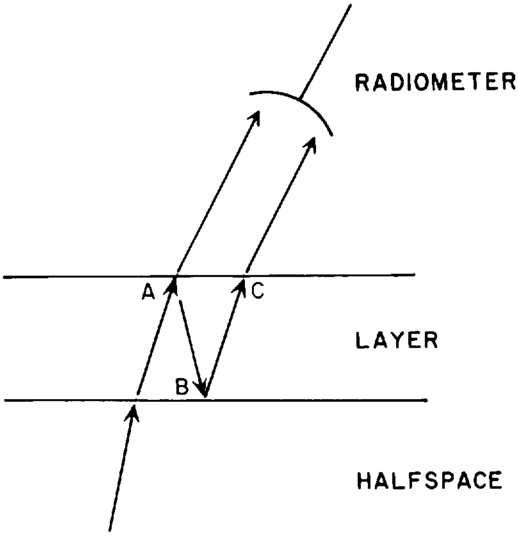


FIG. 1. Schematic of radio emission from a two-layer medium. Path ABC represents a multiple reflection that interferes with the primary ray.

scattering, and variations of the relative permittivity were discussed by England (1976).

The perceived radiobrightness spectrum of a layered medium is affected by the bandwidth of the radiometer, by lateral variations in the thickness of layers within the field-of-view of the radiometer, and by the depth rate-of-change of the dielectric properties at the interfaces between layers. These three effects are examined in this report for the simple case of a layer over a half-space. The extrapolation to multiple layered media is straightforward.

RADIOBRIGHTNESS OF TWO-LAYER MEDIA

Radio emission originating within or traversing the layer shown in Figure 1 is multiply reflected (path ABC) and interferes with itself so that the radiobrightness of the two-layer medium is partially determined by the two-way path length within the layer.

If the surface and interface within the area viewed by the radiometer are plane-parallel, the radiobrightness spectrum shows a sinusoidal interference pattern. Similarly, if the thickness of the layer varies smoothly over distances that are large with respect to the radiometer's field-of-view, the radiobrightness at a particular wavelength varies sinusoidally along a traverse. Blinn et al (1972) have demonstrated the former phenomenon for variable-thickness layers of sand over a metal plate. We have observed the latter effect in the radiobrightness of lake ice (Figure 2).

The analytical expression for the specific wavelength microwave emissivity e_p of two layers whose surface and interface are plane-parallel is

$$e_p = 1 - \bar{A}^t_p A^t_p, \tag{1}$$

where p refers either to vertical (V) or to horizontal (H) polarization, and $\bar{A}^t_p$ is the complex conjugate of the total reflection coefficient A^t_p. This coefficient is

$$A^t_p = \frac{A^1_p + A^2_p e^{-2ik\delta/\mu} e^{-4\beta\delta/\mu}}{1 + A^1_p A^2_p e^{-2ik\delta/\mu} e^{-4\beta\delta/\mu}}, \tag{2}$$

where A^1_p and A^2_p are the Fresnel reflection coefficients

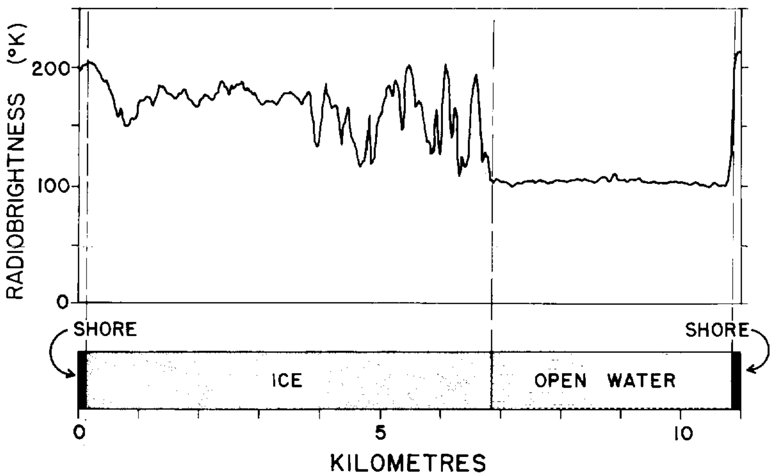


FIG. 2. Radiobrightness profile across the partially ice-covered American Falls Reservoir near Pocatello, Idaho. Interference within the ice is responsible for the 70 degree variation in brightness temperature over ice.

at the surface and interface, respectively, δ is the thickness of the upper layer, k is wave number, μ is the direction cosine with respect to vertical of the ray's path within the upper layer, and 2β is the dielectric absorption. The Fresnel reflection coefficients are (Stratton, 1941)

$$A_V^1 = \frac{\mu/\cos\phi - n_1^*}{\mu/\cos\phi + n_1^*},$$

$$A_H^1 = \frac{n_1^*\,\mu/\cos\phi - 1}{n_1^*\,\mu/\cos\phi + 1},$$

and

$$A_V^2 = \frac{\cos\phi_2/\mu - n_2^*/n_1^*}{\cos\phi_2/\mu + n_2^*/n_1^*},$$

$$A_H^2 = \frac{n_2^*\cos\phi_2/n_1^*\,\mu - 1}{n_2^*\cos\phi_2/n_1^*\,\mu + 1}, \qquad (3)$$

where ϕ is the view-angle, with respect to vertical, of the radiometer, and ϕ_2 is the equivalent view-angle in the half-space. The complex index of refraction n_m^* of the layer, where $m = 1$, or of the half-space, where $m = 2$, for a nonconducting medium, is

$$n_m^* = n_m - i\kappa_m,$$

$$n_m = \sqrt{\epsilon_m}\left[\frac{1 + (1 + \tan^2\Delta_m)^{1/2}}{2}\right]^{1/2},$$

and

$$\kappa_m = \sqrt{\epsilon_m}\left[\frac{-1 + (1 + \tan^2\Delta_m)^{1/2}}{2}\right]^{1/2}, \qquad (4)$$

where ϵ_m and $\tan\Delta_m$ are the relative permittivity and dielectric loss tangent, respectively. The ϕ_m and μ are related by Snell's law:

$$n_m \sin\phi_m = n_1[1 - \mu^2]^{1/2}. \qquad (5)$$

The dielectric absorption is

$$2\beta_m = \frac{2\pi\sqrt{\epsilon_m}}{\lambda_0}\tan\Delta_m, \qquad (6)$$

where λ_0 is free space wavelength.

SPECTRAL RESOLUTION

The sensitivity or temperature resolution ΔT_b of a microwave radiometer is (Tiuri, 1966),

$$\Delta T_b = \frac{T_s}{\sqrt{\Delta f \tau}}, \qquad (7)$$

where Δf is predetection bandwidth, τ is integration time, and T_s is total system noise. In an airborne operation, the integration time is limited by the speed v and altitude D of the aircraft, and by the beamwidth $\Delta\phi$ of the radiometer. The potential angular resolu-

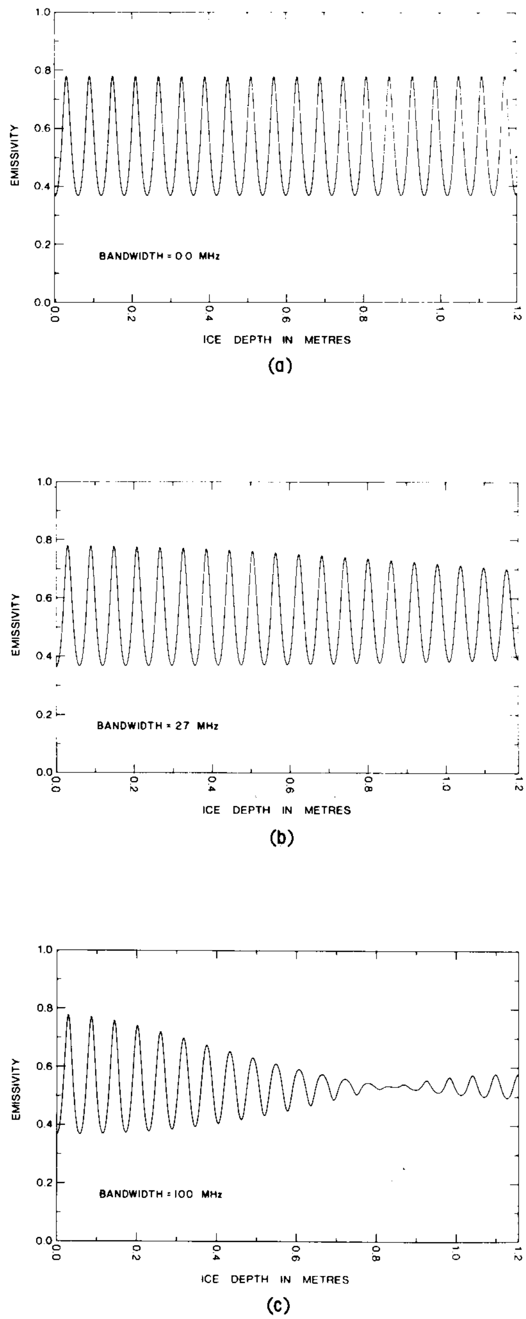


FIG. 3. The bandwidth dependence of the radiobrightness of lossless ice over water. A theoretical ice whose relative permittivity and loss tangent are 3.2 and 0.0, respectively, is used to illustrate the effects of receiver bandwidth upon a calculated radiobrightness. The center frequency f_0 is 1.4 GHz and the bandwidths are 0.0 MHz (a), 27 MHz (b), and 100 MHz (c). The radiometer is viewing the nadir.

tion of a nontracking radiometer is realized only if

$$\tau < \frac{D\Delta\phi}{v}, \tag{8}$$

so that the uncertainty of the observed radiobrightness is

$$\Delta T_b > T_s \sqrt{\frac{v}{\Delta f D \Delta\phi}}. \tag{9}$$

A typical noise temperature for the 21 cm radiometer used by the U.S.G.S. is 1600°K. The beamwidth of the radiometer's antenna is 15 degrees, so that the temperature resolution achieved in an aircraft traveling 40 m/sec (82 knots) at an altitude of 150 m is 1.6 $(\Delta f)^{-1/2}$ degrees, where Δf is in megahertz. It is desirable to resolve radiobrightness to a few tenths of a degree. Our airborne microwave radiometer, therefore, must have a bandwidth of tens of megahertz.

Large bandwidths spectrally average the radiobrightness. The effective emissivity of a two-layer medium becomes

$$e_p = 1 - \frac{1}{\Delta f}\int_{f_0-(\Delta f/2)}^{f_0+(\Delta f/2)} \tilde{A}'_p A'_p df, \tag{10}$$

for a radiometer with a rectangular passband of width Δf centered on frequency f_0. Figures 3a, b, and c illustrate the effect upon emissivity of bandwidths of 0.0, 27, and 100 MHz at a center frequency of 1.4 GHz. The critical parameter is the phase relationship, $2k\delta/\mu$ [equation (2)], for wave numbers corresponding to the band edges. An instrumentally induced null, such as the one shown in Figure 3c, occurs when the wavelength of the path ABC in Figure 1 is an integral multiple of $f_0/\Delta f$ wavelengths. For the situation shown in Figure 3c, layer thickness could not be easily derived from a radiobrightness spectrum. Most radiometers, however, utilize the frequency bands that are assigned to radio astronomy because these bands are relatively free of cultural noise. The smallest $f_0/\Delta f$ for bands in the 1 to 40 GHz range is about 52 (1.4 GHz band). Figure 3b illustrates this case and shows that spectral averaging becomes significant only for layers that are tens of wavelengths thick. Most layers in natural media, when this thick, are nearly opaque to microwaves. Generally, therefore, radiobrightness of natural media varies sufficiently slowly with frequency that the spectral resolution of most microwave radiometers is adequate.

NONUNIFORM LAYER THICKNESS

The spectral distortion discussed in the last section

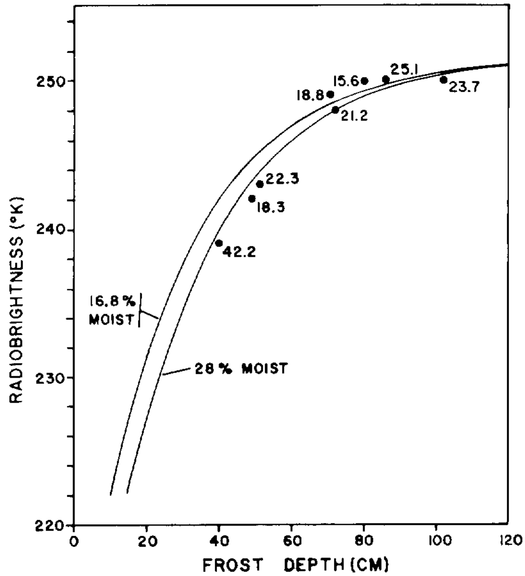


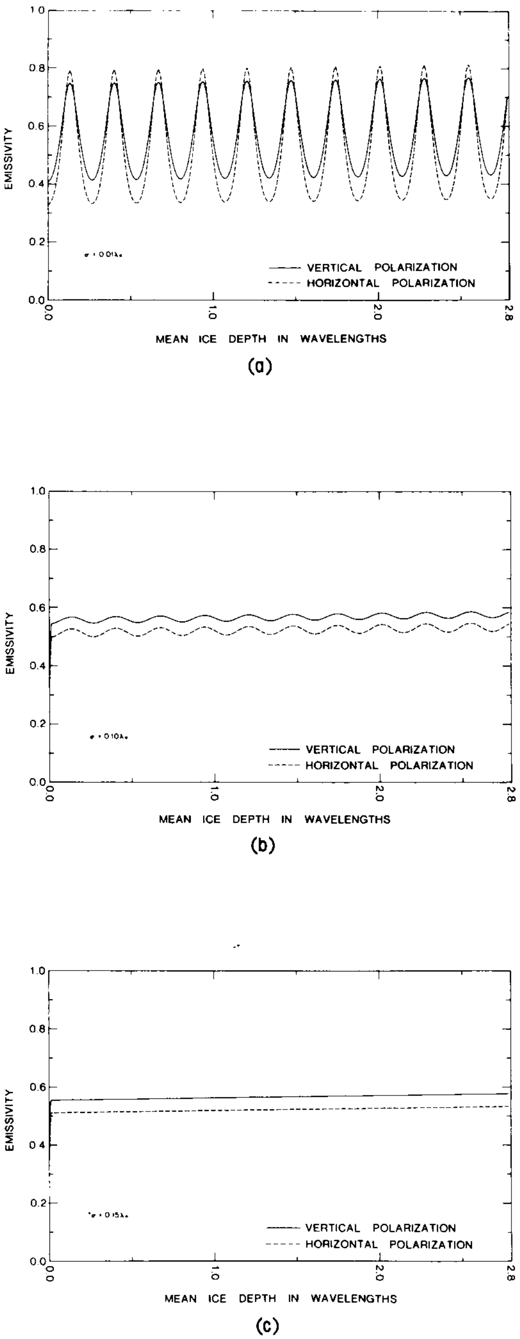
FIG. 4. Radiobrightness of seasonally frozen, sandy soil. The data are from England and Johnson (1975). The plotted points have labels designating moisture contents in weight fraction of the experimentally determined points. The theoretical curves are based upon equation (11), the spectrally averaged mathematical model.

is a product of the limited spectral resolution of a radiometer. A similar effect is obtained if the thickness of the upper layer, illustrated in Figure 1, is not uniform within the instantaneous field-of-view, or footprint, of the radiometer. At any point on the surface of the emitting medium, the interference phenomenon could be observed but the cumulative effect of nonuniform layering within the footprint is to average the spectral variation. For example, Figure 4 shows theoretical (curves) and observed (points) radiobrightnesses at 21.4 cm wavelength of frozen sandy soil overlying moist sandy soil for various moisture contents and frost depths. The theoretical models assume a nonuniform layer so that variations caused by interference are averaged. The computation becomes simpler because phase can be ignored. Rather than equations (1) and (2), the emissivity becomes

$$e_p = (1 - \tilde{A}_p^1 A_p^1)\left\{\frac{1 - \tilde{A}_p^2 A_p^2 e^{-4\beta\delta/\mu}}{1 - \tilde{A}_p^1 A_p^1 \tilde{A}_p^2 A_p^2 e^{-4\beta\delta/\mu}}\right\}. \tag{11}$$

Edgerton et al (1971) used an equivalent treatment to explain the radiobrightness of snow over soil.

Equation (11) applies only for sufficiently nonuniform layer thicknesses. Intermediate cases can be



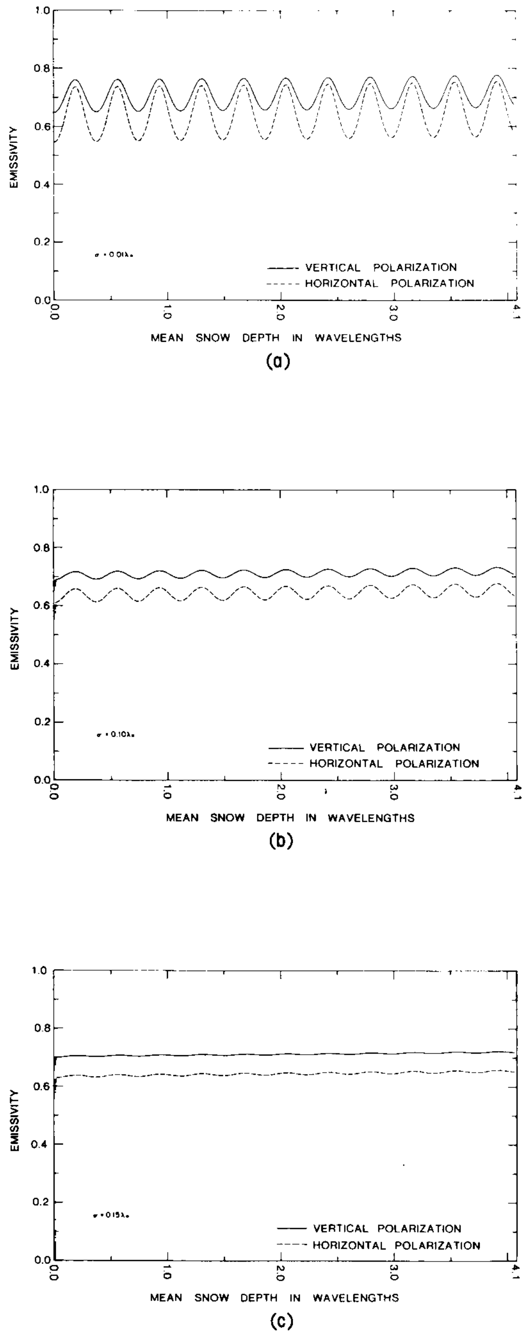


FIG. 5. Calculated emissivity of ice over fresh water. The dielectric properties for the ice are $\epsilon = 3.2$, $\tan\delta = 0.0009$, and those for the water are $\epsilon = 77.2$, $\tan\delta = 0.17$. The one sigma values of the Gaussian distributions are 0.01 (a), 0.10 (b), and 0.15 (c) free space wavelengths. The view angle ϕ is 30 degrees.

FIG. 6. Calculated emissivity of snow over moist soil. The dielectric properties for the snow are $\epsilon = 1.5$, $\tan\delta = 0.0009$, and those for the moist ground are $\epsilon = 20.0$, $\tan\delta = 0.13$. The one sigma values of the Gaussian distributions are 0.01 (a), 0.10 (b), and 0.15 (c) free space wavelengths. The view angle ϕ is 30 degrees.

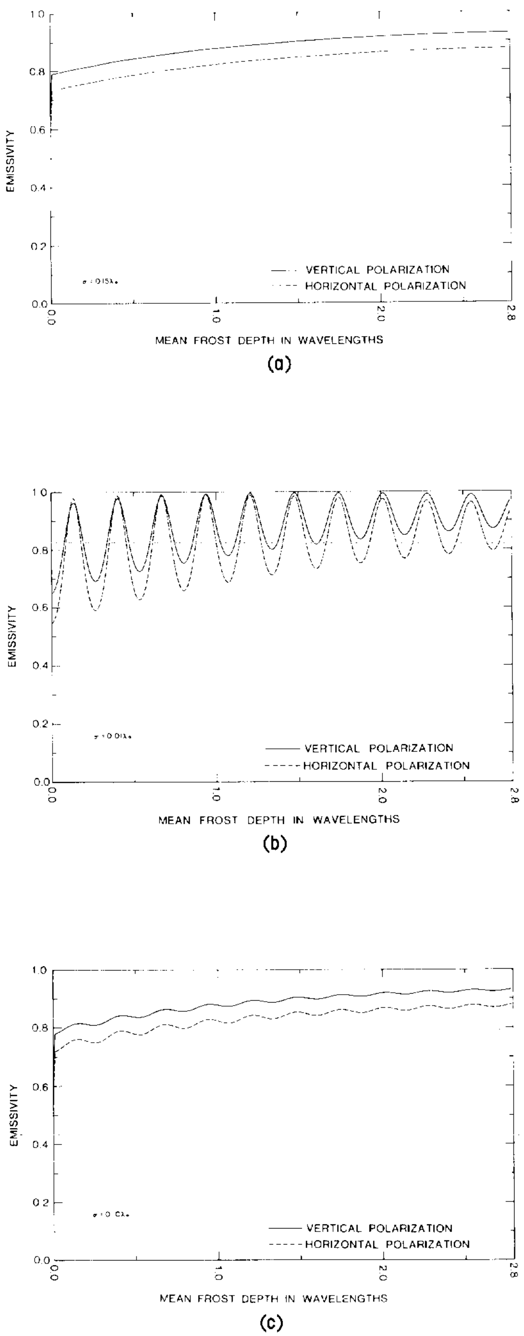


FIG. 7. Calculated emissivity of frozen soil over moist soil. The dielectric properties of the frozen soil are $\epsilon = 3.2$, $\tan \delta = 0.13$. The one sigma values for the Gaussian distributions are 0.01 (a), 0.10 (b), and 0.15 (c) free space wavelengths. The view angle ϕ is 30 degrees.

examined by postulating a truncated Gaussian distribution of thicknesses about a specific thickness δ so that the emissivity can be written

$$e_p = 1 - \frac{\int_0^{\infty} \tilde{A}'_p A'_p e^{-(z-\delta)^2/2\sigma^2} dz}{\int_0^{\infty} e^{-(z-\delta)^2/2\sigma^2} dz}, \tag{12}$$

where z is depth. Figures 5, 6, and 7 show the calculated emissivities of ice over fresh water, snow over moist ground, and frozen soil over moist soil. There is little spectral distortion in any case where $\sigma < 0.01$ of the free space wavelength and there is little evidence of interference in any case where $\sigma > 0.15$ of the free space wavelength. That is, whenever the variability in layer thickness exceeds 15 percent of the free space wavelength, interference can be discounted. Conversely, layer thickness can be reliably derived from the interference spectrum only if the layer is uniformly thick to significantly less than 10 percent of the free space wavelength.

DIFFUSE INTERFACE

The variation of radiobrightness with layer thickness is a result of the dielectric contrast between the layer and the half-space. If that interface were sufficiently diffuse, i.e., if the dielectric properties varied sufficiently slowly with depth, microwaves would be refracted but not reflected so that the radiobrightness would be unaffected by the compositional change at the interface. The radiobrightness, in this case, would be equivalent to that of a half-space filled with the same material as the upper layer of Figure 1 even though some, or even most, of the radiation originated within the lower layer.

Preliminary studies of seasonally frozen clay soil show little variation of radiobrightness with frost depth (Figure 8). One explanation is that, because clay freezes over a range of temperatures rather than at 0°C, the frost line in clay is diffuse and does not reflect 21.4 cm microwaves. The problem can be modeled as shown in Figure 9 where we assume a linear approximation to the transition between complex indices of refraction. A premise in this construct is that media where such a diffuse transition might occur would be sufficiently irregular that interference could be ignored. The emissivity of the model in Figure 9 was obtained by dividing the transition zone into 49 layers and computing the effective reflection coefficient A_p^2 for a 50 layer medium (e.g., Wait, 1962). Phase was preserved in the derivation of A_p^2

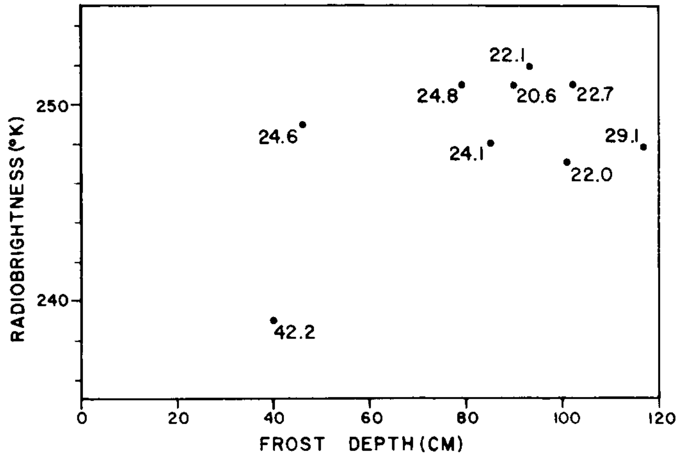


FIG. 8. Field measurements of the radiobrightness of seasonally frozen clay soil. The data are from England and Johnson (1975). The values at the data points designate moisture content in weight fractions. The upper left-hand data point has been transferred from Figure 4 as properly belonging with the clay set (England and Johnson, 1975). There does not appear to be a correlation between frost depth and radiobrightness.

but equation (11), where phase was ignored, was used to compute the total emissivities.

Figure 10 shows emissivity as a function of transition zone thickness Δ for clay soil that is frozen to a depth of 0.2 free space wavelengths. The emissivity of 0.78 at $\Delta = 0.0$ corresponds to the 21.4 cm radiobrightness of a 15 percent moist, sandy soil (distinct frost line) that is frozen to a depth of 4.3 cm. As the transition zone thickens, the emissivity increases to 0.90, the value for a half-space filled with frozen soil. For frozen clay, a transition zone that is 3.2 cm thick (15 percent of the 21.4 cm free space wavelength) is sufficient to explain our observations. Many radiometer systems employ wavelengths of a few millimeters, or centimeters. Transition zones in natural media whose thicknesses approach or exceed 15 percent of these wavelengths are common. The interpretation of millimeter and short centimeter spectra in terms of the variation of dielectric properties with depth may often be impossible.

CONCLUSIONS

Microwave radiobrightness spectra reveal vertical compositional structure only if certain criteria are met: (1) The bandwidth of the radiometer is sufficiently narrow to preclude artifically induced nulls, (2) layers within the footprint of the radiometer can be assumed either to be uniform in thickness to a few percent of the free space wavelength or to be nonuniform in thickness to greater than 15 percent of the free space wavelength, and (3) compositional interfaces are sharp to less than a few percent of the free space wavelength. Radiometers are generally designed to utilize the radio astronomy bands and these bands are sufficiently narrow to insure adequate spectral resolution. With the notable exception of lake ice, most geologic media are not uniformly layered over areas as large as an airborne radiometer's field-of-view, which typically has a diameter of tens or hundreds of meters. In these cases, interference phe-

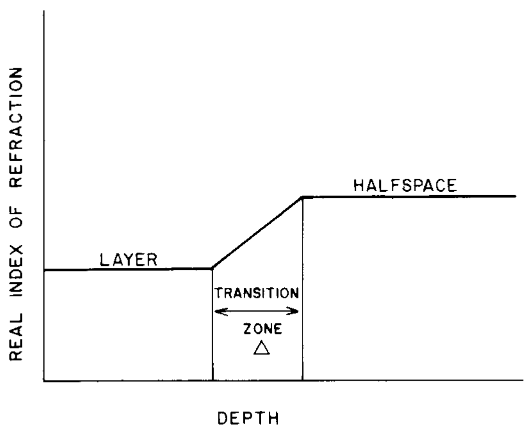


FIG. 9. Index of refraction for a two-layer medium with a linear transition between layers. The computation assumes a linear transition for both the real and the imaginary parts of the index of refraction.

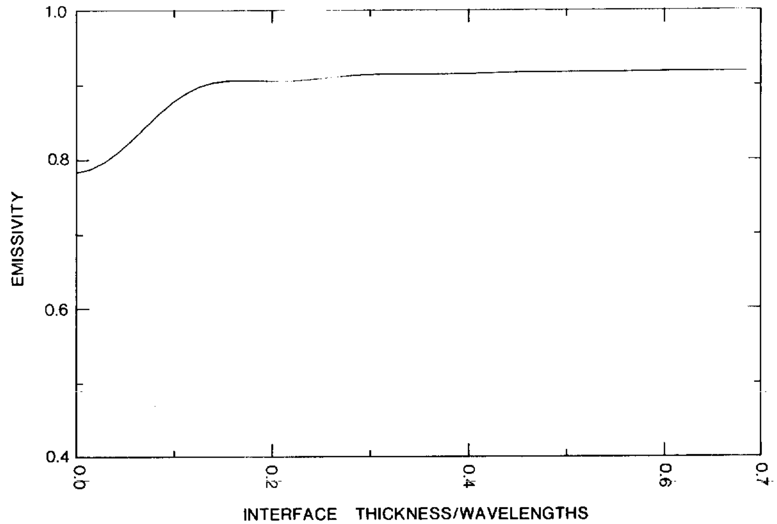


FIG. 10. Calculated radiobrightness of a two-layer medium as a function of the interface thickness. The model is of a layer of frozen soil 0.2 free space wavelengths thick lying upon a half-space of moist soil. The radiometer views nadir. As the interface between frozen and moist soil thickens, the radiobrightness approaches that of a half-space of frozen soil. That is, the interface becomes transparent to microwaves.

nomena at microwave frequencies are absent. Some compositional transitions in rock, soil, ice, or snow may be sufficiently diffuse with respect to the microwave wavelength that these media will appear compositionally homogeneous, particularly to shorter wavelength microwaves.

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