Estimation of current-induced pile groups scour using a rule-based method

N. Ghaemi, A. Etemad-Shahidi and B. Ataie-Ashtiani

ABSTRACT

Scour phenomenon around piles could endanger the stability of the structures placed on them. Therefore, an accurate estimation of the scour depth around piles is very important for engineers. Due to the complexity of the interaction between the current, seabed and pile group; prediction of the scour depth is a difficult task and the available empirical formulas have limited accuracy. Recently, soft computing methods such as artificial neural networks (ANN) and adaptive neuro-fuzzy inference system (ANFIS) have been used for the prediction of the scour depth. However, these methods do not give enough insight into the generated models and are not as easy to use as the empirical formulas. In this study, new formulas are given that are compact, accurate and physically sound. In comparison with the other soft computing methods, this approach is more transparent and robust. Comparison between the developed formulas and previous empirical formulas showed the superiority of the developed ones in terms of accuracy. In addition, the given formulas can be easily used by engineers to estimate the scour depth around pile groups. Moreover, in this study, design factors are given for different levels of acceptable risks, which can be useful for design purposes.

Key words | model tree, pile group, probabilistic design, scour depth, soft computing

N. Ghaemi
A. Etemad-Shahidi (corresponding author)
School of Civil Engineering,
Iran University of Science and Technology,
Tehran,
Iran,
P.O. Box 16765-163
E-mail: *a.etemadshahidi@griffith.edu.au*

A. Etemad-Shahidi
Griffith School of Engineering,
Gold Coast Campus,
Griffith University,
QLD, 4222

B. Ataie-Ashtiani
Department of Civil Engineering,
Sharif University of Technology,
Tehran,
Iran

INTRODUCTION

Pile groups are widely used in practice to support hydraulic and marine structures. Generally, this type of structure can significantly reduce construction costs, compared with spread footer structures when sediment scour is a consideration. Scour around the piles is one of the most important aspects of the structure stability and the reliable design of structures requires accurate prediction of the maximum scour depth around them. The process of the scour around a group of piles is fundamentally complex because it depends on many influential variables such as the arrangement of the piles, pile geometry, the sediment and flow properties.

Although current-induced scour around single piles has received considerable attention in the past (e.g. Laursen & Toch 1956; Hancu 1971; Shen 1971; Breusers et al. 1977; Breusers & Raudkivi 1991; Melville 1997; Melville & Chiew 1999; Melville & Coleman 2000; Richardson & Davis

2001), comparatively limited studies are available on current-induced scour around pile groups.

Hannah (1978) investigated scour around different arrangements of pile groups. He studied two-pile tandem and two-pile side by side arrangements in steady current condition and proposed a relationship between the pile group scour depth and spacing between the piles. Breusers (1972) reported hydraulic-model experiments on the scour at two-leg and three-leg offshore platforms due to wave and current. A series of experiments on current-induced scour around pile groups was conducted by Salim & Jones (1998). From these experiments, they studied the effect of the pile's arrangement on the scour depth. They noticed that the scour depth decreases with the increase of spacing between the piles. Scour around various arrangements of pile groups was also studied by Sumer et al. (2005). The main focus of their study was on scour depth variation in

different configurations (two-pile tandem, two-pile side by side, 2×2 square group, 3×3 square group, 5×5 square group and circular group arrangements). They found that the scour depth increases with the number of piles in a square pile group with $G/D = 4$, where G is the distance between piles, and D is the pile's diameter.

The above mentioned researchers examined different pile arrangements but did not propose any formula to predict the scour depth around pile groups. A series of experiments on scour around piles was conducted by the US Federal Highway Administration (FHWA), reported in HEC–18 (Richardson & Davis 2001). They presented an empirical formula for estimation of the scour depth around single piles. They modified this formula to predict the scour depth around a group of piles. From these experiments, they found that the relative scour depth (S/D, where S is the equilibrium scour depth) is governed by Froude number (Fr) and h/D where h is the flow depth directly upstream of the pile. They also noticed the effect of pile shape, angle of attack and bed material characteristics on the scour depth.

Ataie-Ashtiani & Beheshti (2006) also conducted a series of experiments on the scour around various arrangements of pile groups. They modified the formula presented by Richardson & Davis (2001) by adding a correction factor. This factor was obtained for pile groups aligned to the flow using results from their experiments. Their studies showed that S/D decreases as the gap to diameter ratio (G/D) increases. They also investigated the relationship between S/D and number of piles in line with flow (m) and number of piles normal to the flow (n).

In brief, due to the complexity of the scour process, a general model has not yet been presented to estimate the scour depth around a group of vertical piles. Hence, a transparent and robust model can be very useful for this purpose. One of the most common soft computing methods, artificial neural networks (ANN) has been used for scour estimation (e.g. Liriano & Day 2001; Kambekar & Deo 2003; Bateni & Jeng 2007; Zounemat-Kermani *et al.* 2009; Ghazanfari *et al.* 2010; Kazeminezhad *et al.* 2010). Khosronejad *et al.* (2003) used feed-forward multi-layer perceptron (MLP) for scour depth estimation around pile groups. Bateni & Jeng (2007) also used adaptive neuro-fuzzy inference system (ANFIS)

to predict the scour depth around pile groups. They compared their results with those of previous studies and showed that ANN was more accurate than existing empirical formulas. In their study, geometrical parameters, such as G/D and number of piles, were not considered and they only used a single pile arrangement. Recently, Zounemat-Kermani *et al.* (2009) estimated the scour depth around pile groups with different arrangements using ANN and ANFIS. They used experimental data of Ataie-Ashtiani & Beheshti (2006) and showed that the scour depth is mainly governed by G/D. However, the application of the model trees for the prediction of scour depth around a group of piles has not yet been investigated.

The main purpose of this study is to develop a robust and transparent model for the prediction of the scour depth around pile groups with different pile arrangements. Hence, M5', as one of the algorithms of Model Tree, is used for developing predictive and simple formulas. Unlike most of the soft computing methods, which are opaque, M5' algorithm can present formulas that are physically understandable and sound. Previously, M5' has been used successfully to predict sediment transport (Bhattacharya *et al.* 2007), significant wave height (Etemad-Shahidi & Mahjoobi 2009), breakwater response (Etemad-Shahidi & Bonakdar 2009; Bonakdar & Etemad-Shahidi 2011; Etemad-Shahidi & Bali 2011) and prediction of pile group scour due to waves (Etemad-Shahidi & Ghaemi 2011). The main advantage of M5' is that it yields transparent and compact formulas while other soft computing methods act like a black box. In addition this method needs less trial and error (Etemad-Shahidi & Ghaemi 2011). In fact, M5' can be considered as an advanced technique which combines a new data mining approach to split the domain with the multivariate linear regression method to obtain the formulas. In this study we also used the multi linear regression (MLR) method for the prediction of pile group scour depth.

This paper is outlined as follows: a background is given about the studies conducted on current-induced scour depth around pile groups. Model Tree and M5' algorithm are then described. The MLR method is described next. The used data set and modeling approach are then presented followed by the results and discussion section. Finally, the summary and conclusion are presented.

BACKGROUND

The local scour around piles occurs due to the interaction of flow, sediment and structure (Figure 1). When the quantities of arriving sediments are equal to the quantities of leaving sediments, dynamic equilibrium occurs. This depth is called equilibrium scour depth (Melville & Sutherland 1988). Scour around a single round pile depends on three classes of parameters, i.e. the flow characteristics, the sediment properties and the pile geometry. Thus, the functional relationship between equilibrium scour depth and its independent variable is (Ettema et al. 1998):

$$S = f(\mu, \rho, U, U_c, h, g, D, d_{50}) \tag{1}$$

where μ is the fluid dynamic viscosity, ρ is the fluid density, U is the average velocity of approach flow, U_c is the critical value of U associated with initiation of motion of particles on the bed surface, h is the flow depth, g is the gravitational acceleration and d_{50} is the sediment mean diameter. With the help of dimensional analysis, these independent variables can be presented in the following dimensionless form:

$$S/D = f(U/U_c, \text{Fr}, h/D, D/d_{50}, \text{Re}) \tag{2}$$

where Fr is Froude number and Re is the pile Reynolds number. These are defined as:

$$\text{Fr} = \frac{U}{\sqrt{g\,D}} \tag{3}$$

$$\text{Re} = \frac{\rho\,U\,h}{\mu} \tag{4}$$

The dimensionless parameters used in Equation (2) account for the effects of different physical processes that
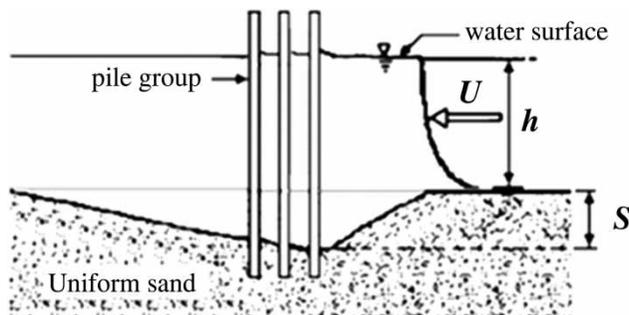

Figure 1 | Flow and local scour at a pile group (Zounemat-Kermani et al. 2009).

occur during the scour, such as flow-structure interaction, flow-seabed interaction and sediment transport. In Equation (2) the Reynolds number, Froude number and depth to diameter ratio account for the flow pattern around the piles. U/U_c describes the flow effects on the seabed process and D/d_{50} describes the interaction between pile and bed sediment.

For scour around a pile group, distance between piles, number of piles parallel to the flow and number of piles normal to the flow are also important. Thus, the dimensionless function for scour around a pile group can be written as:

$$S/D = f(U/U_c, \text{Fr}, h/D, D/d_{50}, \text{Re}, G/D, n, m) \tag{5}$$

Researchers have suggested several empirical methods to estimate the scour depth around a pile group. In the following these methods are briefly explained.

A commonly used empirical formula was provided by FHWA (Richardson & Davis 2001), which recommends:

$$S/D = 2.0\,K_1\,K_2\,K_3\,K_4\,(h/D)^{0.35}\,\text{Fr}^{0.43} \tag{6}$$

where K_1 is the correction factor for pier nose shape, K_2 is the correction factor for angle of attack, K_3 is the coefficient based on the channel bed condition and K_4 is the correction factor for armoring by bed material size. In HEC-18, it is recommended that the effective width of an equivalent full depth pile group (D^*) should be used in the above mentioned formula to predict the scour depth. D^* is the product of the projected width of the piles onto a plane normal to the flow multiplied by a spacing factor and a number of aligned rows factor:

$$D^* = D_{\text{proj}}K_G K_m \tag{7}$$

where D_{proj} is sum of the non-overlapping projected widths of piles, K_G is the coefficient for pile spacing and K_m is the coefficient for the number of aligned rows (Richardson & Davis 2001).

Another method for the prediction of the pile groups scour depth was given by Salim & Jones (1998). In this method, the scour depth is calculated using Equation (6)

assuming an equivalent solid pile group with piles touching each other at the same skew angle to the flow direction; and then this scour depth is multiplied by correction factors for spacing and angle of attack on the pile group. The spacing correction factor suggested by Salim & Jones (1998) did not consider the number of piles normal to the flow and the number of piles in line with the flow. This procedure was intended to be conservative when the number of piles normal to the flow is three or more (Ataie-Ashtiani & Beheshti 2006).

Ataie-Ashtiani & Beheshti (2006) suggested a new correction factor for pile groups aligned to the flow using results from different experimental data by including the effects of the numbers of piles parallel and perpendicular to the flow direction. This correction factor is:

$$K_{\text{Gmn}} = 1.11 \frac{m^{0.0396}}{n^{0.5225} \, (G/D)^{0.1153}} \tag{8}$$

In this method, the maximum scour depth is calculated by multiplying the correction factor (K_{Gmn}) and scour depth obtained for the equivalent solid pier using Richardson & Davis (2001) formula.

Another method was recommended by Sheppard & Glasser (2004). This method is in fact the procedure for single piles, modified for a group pile. In this method, the effective diameter is used in the single pile formula developed by Sheppard *et al.* (2004). The magnitude of the effective diameter is such that the scour depth at a circular pile with this diameter is the same as the scour depth at the pile group for the same sediment and flow conditions (Sheppard & Glasser 2004). By using this method the problem of computing equilibrium scour depth at the pile group is reduced to determining the value of the effective diameter of the pile group.

MODEL TREES (MTS)

The M5 algorithm is one of the most common algorithms from the decision tree family. The M5 tree model was created by Quinlan (1992) and then his idea was modified by Wang & Witten (1997) as a method called M5'. In this rule-based model, first a regression tree is built by dividing the sample space into several regions. Figure 2 shows the structure of a decision tree with two input parameters, in which the method of dividing the sample space is depicted.

This is to minimize the intra-subset variations in values from the root to the node and through the branch. This variation is measured by the standard deviation of the values coming from the root to the node. The standard deviation reduction is calculated by testing each attribute at the node and the attribute that maximizes the standard deviation reduction is chosen. The splitting process is stopped when output values of all samples that reach a node have minor variations or a small number of samples remain. Reduction in the standard deviation is calculated by the following equation:

$$\text{SDR} = \text{sd}(I) - \sum_i \frac{|I_i|}{|I|} \times \text{sd}(I_i) \tag{9}$$

In this equation, I is the set of records that reach the node, I_i are the ones that are the result of splitting the node according to the chosen parameter and sd is the standard deviation. After building the tree, a linear regression model is created for each inner node. This model is constructed according to the values related to that node and all the attributes that are used in the sub-tree rooted at that node. If the estimated error for the linear model at the sub-tree root is less than or equal to the mean error for the sub-tree, then the sub-tree is pruned.

After the pruning process, discontinuity may occur between the adjacent linear models at the leaves of the pruned tree. In the process of smoothing the tree in M5, the final model in a leaf is created from combining the produced model in that leaf with the available models in the path from the root to the leaf. In the smoothing process, the estimated value in each leaf is filtered in the reverse path form the leaf to the root. In each node, this value is combined with the predicted value from the linear model in that node using the following equation:
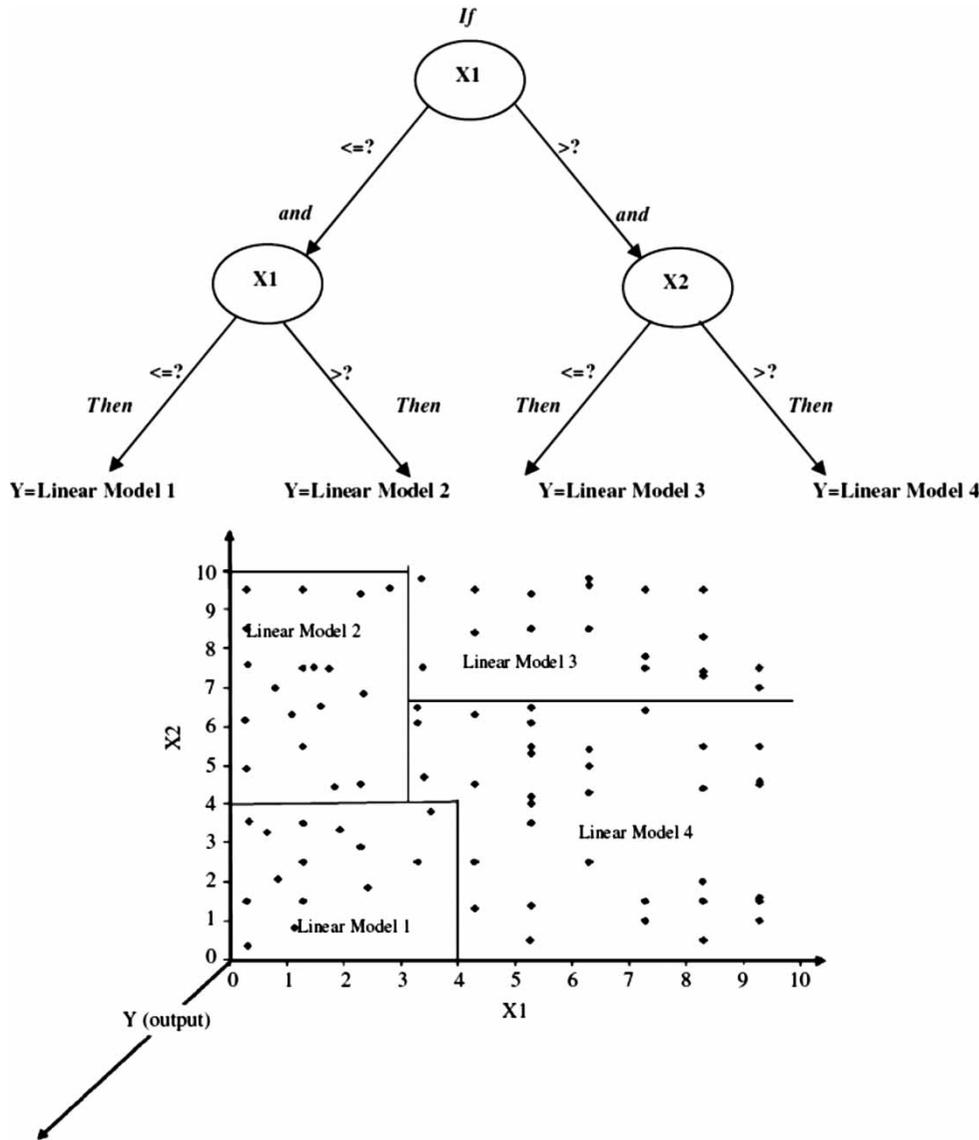
$$P' = \frac{lp + kq}{l + k} \tag{10}$$

Figure 2 | The M5′ model tree and splitting the input space $X_1 \times X_2$ by its algorithm (Etemad-Shahidi & Mahjoobi 2009).

where P' is the prediction passed up to the next higher node, p is the prediction passed to this node from below, q is the value predicted by the model at this node, l is the number of training instances that reach the node below and k is a constant (Wang & Witten 1997). Finally, the MT yields a set of linear multivariable equations (rules).

In summary, M5 splits the space into subspaces based on standard deviation reduction and this splitting of data will continue until the divisions do not significantly increase the prediction accuracy. Finally, the MT yields a linear multivariable equation for the bottommost subspace.

MULTI LINEAR REGRESSION (MLR)

MLR is a method for modeling the linear relationship between a dependent variable (Y) and one or more independent variables (X_i). The performance of MLR is based on minimizing the sum of squares of differences of observed and predicted values. Superiority of this method is that it is relatively simple and easy to implement. However, this method has limitations in predicting the phenomena with a nonlinear relationship or with multiple criteria. The following equation describes the relationship between the

dependent variable and independent variables in the MLR method:

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_i X_i \qquad (11)$$

where α is called the intercept and the β_i are slopes or regression coefficients (Zounemat-Kermani 2012).

DATA SET AND MODELING

Data set used

To investigate the scour depth around pile groups and develop a new prediction method, different experimental data sets were used including Hannah (1978), Coleman (2005), Ataie-Ashtiani & Beheshti (2006) and Amini *et al.* (2011) data sets. All of these experiments were conducted in clear water conditions. Ataie-Ashtiani & Beheshti (2006) used eight types of pile group arrangements, including the 2×1, 1×2, 2×2, 2×3, 2×4, 3×2, 1×3 and 1×4 ones. Data sets that were selected from the experiments of Hannah (1978) had side by side (2×1) and tandem (1×2) arrangements. In Coleman (2005) experiments, pile groups with 2×4 and 3×8 arrangements were used. Amini *et al.* (2011) used four types of pile group arrangements, including the 2×2, 2×4, 3×4 and 3×5 ones. The arrangements of piles in these experiments are shown in Figure 3 and the characteristics of gathered data are given in Table 1. The whole data set contains 156 data points which were divided into two parts for training (70%) and testing (30%) in a random way.

Input and output parameters

In this study, dimensionless parameters were used to build the model. This was performed to extend the results from experimental studies to real conditions. Here Re, Fr, U/U_c, h/D, D/d_{50}, G/D, n and m were employed for developing the model. The output parameter of the model was the
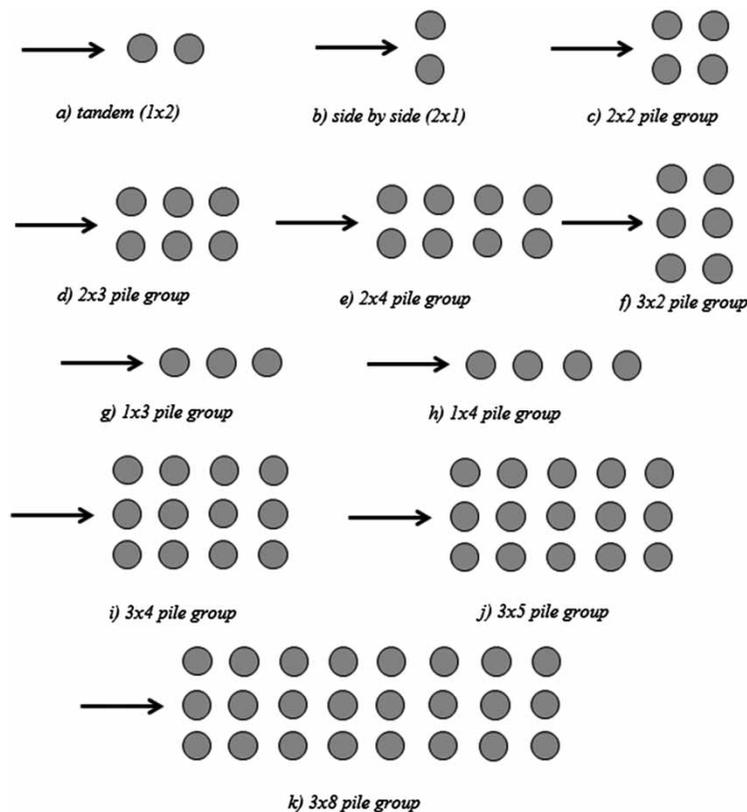

Figure 3 │ Configurations of the used pile groups.

Table 1 | Characteristics of the gathered data

Data sets from	University	Number of data	Experimental condition	Number of arrangements
Hannah (1978)	Univ. of Canterbury, New Zealand	16	Clear water	2
Coleman (2005)	Univ. of Auckland, New Zealand	3	Clear water	2
Ataie-Ashtiani & Beheshti (2006)	Sharif Univ. of Technology, Iran	100	Clear water	8
Amini et al. (2011)	National Hydraulic Research Institute of Malaysia (NAHRIM)	37	Clear water	4

scour depth which was normalized by the pile diameter (S/D).

Table 2 presents the ranges of these parameters. As shown, the considered arrangements in this study cover a wide range of G/D from 0.15 to 20, m from 1 to 8 and n from 1 to 3. These wide ranges of parameters help the model to learn the relation between the arrangements of the pile groups and the scour depth correctly. Moreover, h/D numbers vary from 1.5 to 25, Fr from 0.12 to 0.45 and S/D from 1.1 to 6.65.

M5′ model

The M5' model tree only presents a linear relationship between input and output variables, while the relationship between governing parameters and the scour depth is not necessarily linear. Thus, the model was developed with *ln (inputs)* and *ln (output)* to overcome this constraint (Bhattacharya et al. 2007; Etemad-Shahidi & Ghaemi 2011). Different combinations of governing parameters were used

Table 2 | Range of the parameters used for the prediction of current-induced scour around pile groups

Parameters	Min	Max	Mean	Std. Dev
Fr	0.125	0.447	0.309	0.086
Re	8.1×10^3	2.2×10^5	3.6×10^4	3.8×10^4
U/U_c	0.650	0.970	0.815	0.088
h/D	1.545	25	3.661	2.928
D/d_{50}	16.327	187.059	43.248	29.713
G/D	0.150	20	2.319	2.506
M	1	8	2.987	1.339
N	1	3	2.013	0.622
S/D	1.136	6.650	2.348	0.580

for building the model and only the simplest and the most accurate model is described here. Table 3 gives the input and output parameters for building the tree model.

Using logarithm of dimensionless numbers, the following formulas were obtained using M5':

$$\ln(S/D) = -0.09\ln(G/D) + 0.32\ln(h/D) + 0.35\ln(\text{Fr}) + 0.37\ln(n) + 0.72 \text{ for } \ln(G/D) < -0.22 \quad (12-a)$$

$$\ln(S/D) = -0.06\ln(G/D) + 0.26\ln(h/D) + 0.37\ln(\text{Fr}) + 0.07\ln(n) + 1.12 \text{ for } -0.22 < \ln(G/D) < 1.13 \quad (12-b)$$

$$\ln(S/D) = 0.45\ln(h/D) + 0.41\ln(\text{Fr}) + 0.62 \text{ for } \ln(G/D) > 1.13 \quad (12-c)$$

After the transformation, the formulas become:

$$\frac{S}{D} = 2.06 \cdot \left(\frac{G}{D}\right)^{-0.09} \cdot \left(\frac{h}{D}\right)^{0.32} \cdot \text{Fr}^{0.35} \cdot n^{0.37} \quad \text{for } \frac{G}{D} \langle 0.8 \quad (13-a)$$

$$\frac{S}{D} = 3.06 \cdot \left(\frac{G}{D}\right)^{-0.06} \cdot \left(\frac{h}{D}\right)^{0.26} \cdot \text{Fr}^{0.37} \cdot n^{0.07} \quad \text{for } 0.8 \leq \frac{G}{D} \leq 3.1 \quad (13-b)$$

Table 3 | Input and output parameters of M5' model

Input parameters	$\ln(\text{Fr}), \ln(h/D), \ln(G/D), \ln(m), \ln(n)$
Output parameters	$\ln(S/D)$

$$\frac{S}{D} = 1.86 \cdot \left(\frac{h}{D}\right)^{0.45} \cdot \text{Fr}^{0.41} \quad \text{for} \quad \frac{G}{D} \rangle 3.1 \qquad (13-c)$$

Figure 4 shows the M5' data splitting diagram.

MLR model

The MLR model for predicting scour depth using train data was created and then its accuracy was evaluated by test data. The relationship obtained using the MLR is presented in Equation (14).

$$\frac{S}{D} = 2.09 \cdot m^{0.03} \cdot n^{0.14} \cdot \frac{G}{D}^{-0.14} \cdot \frac{h}{D}^{0.38} \cdot \text{Fr}^{0.34} \qquad (14)$$

DISCUSSION OF THE RESULTS

These equations are compact and have good interpretability. According to Equation (13) by increasing Fr scour depth
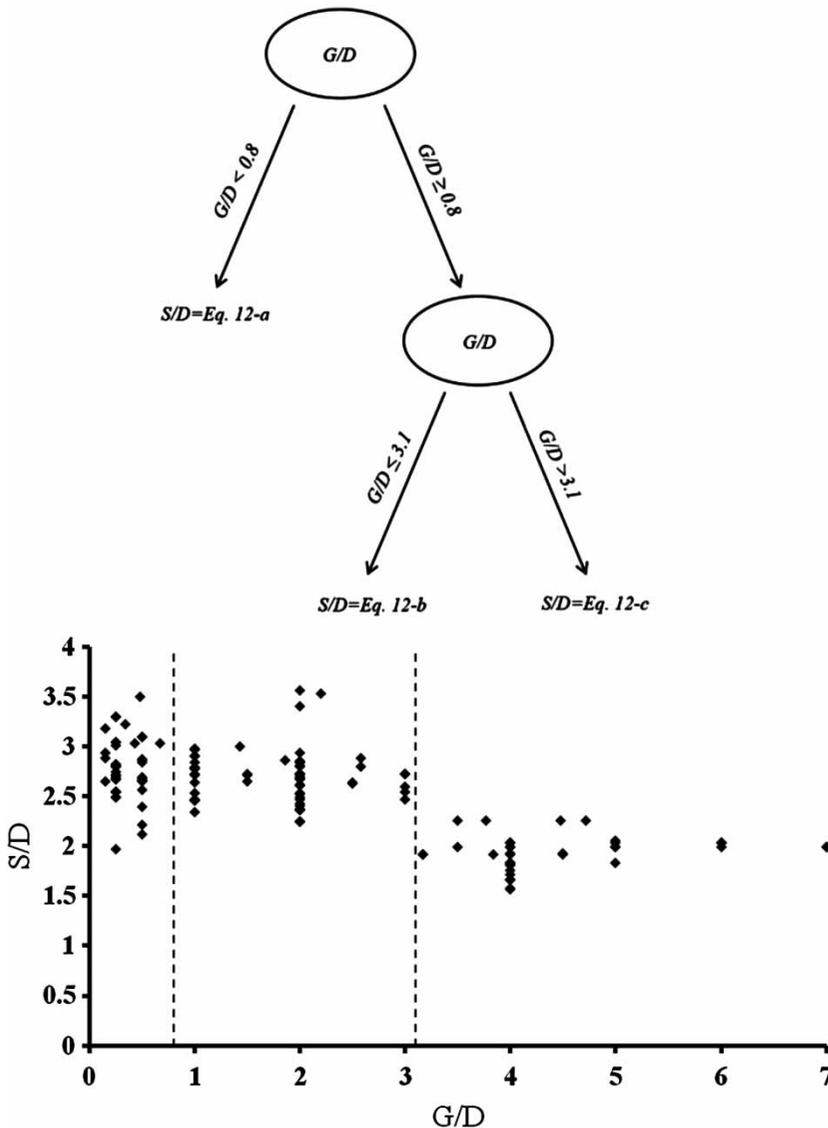

Figure 4 | The M5' data splitting diagram.

increases and when Fr becomes zero scour does not occur. This result is physically sound and is in line with the results of laboratory studies (see also Richardson & Davis 2001). Equation (13) also shows that the scour depth increases by increasing h. This behavior occurs because by increasing water depth, the difference between the pressure at the water surface and bed increases, causing a stronger down flow in front of the piles. This down flow acts as a water jet and causes the scour depth to increase.

Equation (13) shows that the scour depth decreases as the G/D increases. A similar trend was also seen in Ataie-Ashtiani & Beheshti (2006). According to the proposed formulas, it is clearly seen that by increasing G/D, the effect of this parameter decreases. This is due to the fact that the pile influence on each other decreases as the distance between the piles increases. In the other words, when the piles are installed with relatively large distances between them, the generated wake flow behind the first pile has less effect on the second pile. According to Ataie-Ashtiani & Beheshti (2006), the scour depth decreases when $G/D > 2$ and for large values of G/D it reaches its single-pile. Similarly Sumer & Fredsoe (1998) stated that when G/D becomes very large, there would be no interaction between piles, each pile would act as a single pile and there would be a limit for the scour depth. Equation (13) is in line with this physical behavior and when G/D is large, this parameter has no effect on the scour depth.

Moreover, Equation (13) shows a physically sound relationship between the pile group scour depth and arrangement of piles. In this equation, scour is not a function of m. This behavior is also seen in the study of Richardson & Davis (2001). This is because in the used data set, the flow

is normal to the piles. According to Richardson & Davis (2001), in these situations piles aligned with the current attack have no effect on the scour depth. The other point is the direct relationship between scour depth and the number of piles which are normal to the current (n). This direct relationship is also seen in Ataie-Ashtiani & Beheshti's (2006) and Richardson & Davis' (2001) formulas. This equation is physically sound, because by increasing the effective diameter of a pile group; the scour depth increases (Bayram & Larson 2000). Moreover, it should be noted that by increasing G/D in Equation 13, the effect of n on scour depth decreases. This finding also makes sense because by increasing the distance between piles, each pile acts as a single one. A summary of previous empirical formulas and the proposed ones are given in Table 4.

The accuracy of the results was compared with those of previous empirical formulas. In order to compare their performances quantitatively, parameters such as the correlation coefficient (CC), root mean square error (RMSE), scatter index (SI), index of agreement (I_a) and the discrepancy ratio (DR) were calculated as follows:

$$CC = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum (x_i - \overline{x})^2 \sum (y_i - \overline{y})^2}} \tag{15}$$

$$RMSE = \sqrt{\frac{1}{N} \sum (x_i - y_i)^2} \tag{16}$$

$$SI = \frac{\sqrt{\frac{1}{N} \sum (x_i - y_i)^2}}{\overline{x}} \tag{17}$$

Table 4 | Various approaches to estimate the current-induced scour depth around a group of piles

Author	Formula
Richardson & Davis (2001)	$S/D^* = 2.0\, K_1 K_2 K_3 K_4 \, (h/D^*)^{0.35} \, \mathrm{Fr}^{0.43}$
Ataie-Ashtiani & Beheshti (2006)	$S/D^* = 2.0\, K_1 K_2 K_3 K_4 \, (h/D^*)^{0.35} \, \mathrm{Fr}^{0.43} \times 1.11 \dfrac{m^{0.0396}}{n^{0.5225} \, (G/D)^{0.1153}}$
MLR	$S/D = 2.09\, m^{0.03} \, n^{0.14} \, (G/D)^{-0.14} \, (h/D)^{0.38} \, \mathrm{Fr}^{0.34}$ $S/D = S/D = 2.06\, (G/D)^{-0.09} \, (h/D)^{0.32} \, \mathrm{Fr}^{0.35} \, n^{0.37}, \quad G/D < 0.8$
Model Tree	$S/D = 3.06\, (G/D)^{-0.06} \, (h/D)^{0.26} \, \mathrm{Fr}^{0.37} \, n^{0.07}, \quad 0.8 \le G/D \le 3.1$ $S/D = 1.86\, (h/D)^{0.45} \, \mathrm{Fr}^{0.41}, \quad G/D > 3.1$

$$I_a = 1 - \frac{\sum (x_i - y_i)^2}{\sum (\,|x_i - \overline{x}| + |y_i - \overline{y}|)^2} \qquad (18)$$

$$DR = \frac{1}{N} \sum \frac{y_i}{x_i} \qquad (19)$$

where x_i and y_i denote the measured and the predicted values, respectively. N is the number of measurements. $\overline{x}$ and $\overline{y}$ are the corresponding mean values of the measured and predicted parameters.

The correlation coefficient, denoted by CC, is a measure of the strength of the straight-line or linear relationship between two variables (predicted and observed values). The CC takes on values ranging between $+1$ and -1. The correlation is $+1$ in the case of a perfect direct linear relationship and -1 in case of an inverse linear relationship, and the values in between indicate the degree of linear relationship between predictions and observations values. A CC of zero means the there is no linear relationship between the variables. It should be recognized that even if the correlation is close to 1, the predicted and observed values may not match each other; they only tend to vary similarly.

The RMSE measures the difference between values predicted by a model and the values actually observed from the experiments that are being modeled. Values near zero indicate a close match. The SI is in fact the normalized dimensionless form of the RMSE.

The I_a is also used to quantitatively describe the accuracy of model outputs. An efficiency of 1 $(I_a = 1)$ corresponds to a perfect match between predictions and observations.

The DR is one of the statistical indices that is used in this study. A DR value of close to 1 indicates a higher accuracy. A DR value of greater than 1 indicates that the model over predicts and DR values smaller than 1 indicate underestimation of the scour depth.

These statistical parameters (of the test data) are given in Table 5. As shown, the formulas derived by the model tree approach are more accurate than the previous formulas. According to Table 5, DR statistics show that M5' formulas presented in this study outperform other methods with a marginal overestimation and the Richardson & Davis

Table 5 | Performance indices of various approaches to predict the current-induced scour depth

Approach	Error measures				DR (Mean ± Std)
	RMSE	SI (%)	CC	I_a	
Richardson & Davis (2001)	1.02	44	0.28	0.24	1.34 ± 0.40
Ataie-Ashtiani & Beheshti (2006)	0.38	17	0.72	0.83	1.06 ± 0.19
MLR	0.38	16	0.74	0.85	1.03 ± 0.19
Model Tree	0.30	13	0.85	0.91	1.06 ± 0.13

(2001) formula overestimates the scour depth by about 34% on average. Figure 5 presents the comparison between the measured and predicted values of S/D for test data by all approaches. It shows that the traditional formulas are more conservative in comparison to the proposed ones. However, it is impossible to know the uncertainty or safety factor incorporated in them.

To have a safe, technically correct and economic design, engineers need a reliable prediction of scour depth. To assess the reliability of the used model, box plots can be used (Etemad-Shahidi & Ghaemi 2011). These plots are given using a design factor which can be directly related to the acceptable or desired risk level (Figure 6). As shown, the Richardson & Davis (2001) formula is more conservative and has the highest uncertainty (wider box plot). This is mainly because this formula is derived for design purposes and does not consider the arrangements of the piles properly. Figure 6 also shows that the box plot of the formulas developed in this study is narrower than those of others, an indicator of a higher level of confidence.

As mentioned before, Richardson & Davis' (2001) formula is more conservative than others and has the lowest median (0.85), while Ataie-Ashtiani & Beheshti's (2006) formula is very close to one and just slightly overestimates the scour depth. MLR is more accurate than them and its median is nearly equal to one.

SUMMARY AND CONCLUSION

In this study, the M5' model tree was used for the prediction of current-induced scour depth around pile groups. A series
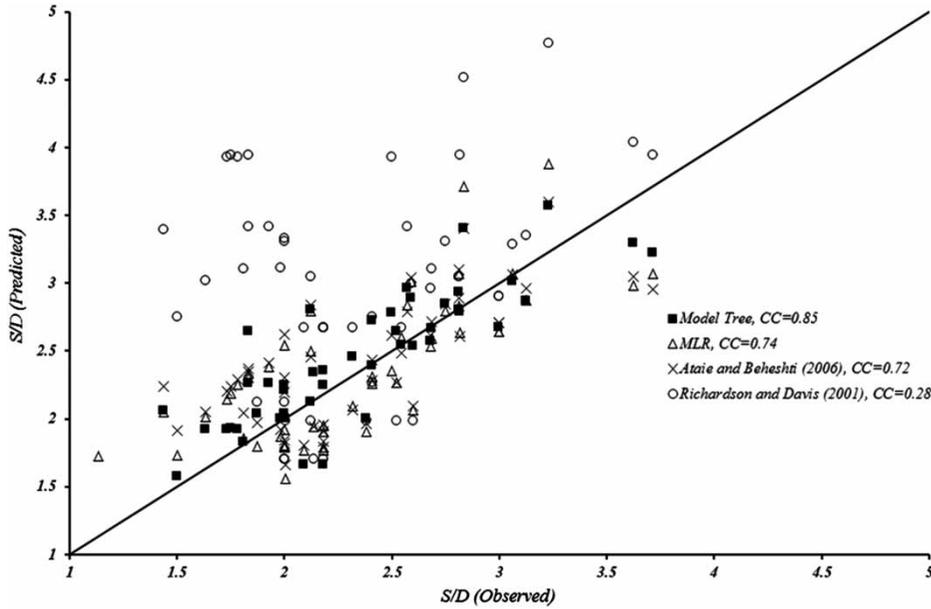
Figure 5 │ Comparison of the measured and predicted normalized scour depths using different approaches.
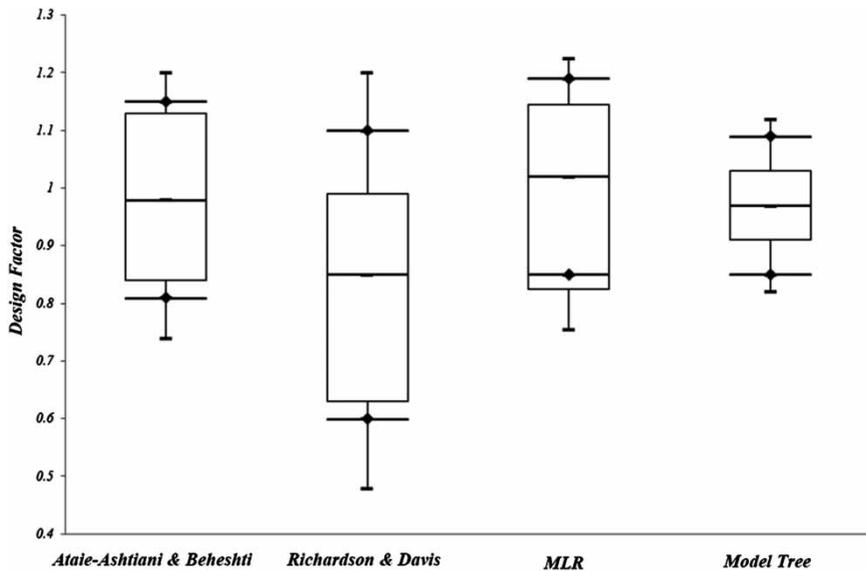

Figure 6 │ Box plots of different methods.

of laboratory data consisting of different pile group arrangements were used for building the model. Then, the results of the model tree were compared with those of empirical methods of Richardson & Davis (2001) and Ataie-Ashtiani & Beheshti (2006). Error measures show that model tree

approach is more accurate than the empirical methods. For example the RMSE of the MT approach is 0.3 while this index is 1.02 when using the Richardson & Davis (2001) and 0.38 when using the Ataie-Ashtiani & Beheshti (2006) approaches. Results of this study show that the

scour depth around pile groups is governed by the Froude number (Fr), gap to diameter ratio (G/D), water depth to pile diameter ratio (h/D) and number of piles normal to the flow (n). According to the given formulas, the S/D increases with increasing Fr, h/D and n, while it decreases with increasing G/D. It was also found that for large G/D values, n becomes less important. In order to address the uncertainty in the developed formulas, a probabilistic approach using design factors was suggested for practical purposes. It was shown that by using the box plot it is possible to use the design factor that is related to acceptable risk level.

ACKNOWLEDGEMENTS

REFERENCES

Amini, A., Melville, B. W., Ali, T. M. & Ghazali, A. H. 2011 Clear water local scour around pile groups in shallow-water flow. *J. Hydraul. Eng.* **138**, 177–185.

Ataie-Ashtiani, B. & Beheshti, A. A. 2006 Experimental investigation of clear-water local scour at pile groups. *J. Hydraul. Eng.* **132**, 1100–1104.

Bateni, S. M. & Jeng, D. S. 2007 Estimation of pile group scour using adaptive neuro-fuzzy approach. *Ocean Eng.* **34**, 1344–1354.

Bayram, A. & Larson, M. 2000 Analysis of scour around a group of vertical piles in the field. *J. Waterw. Port Coast. Ocean Eng.* **126**, 215–220.

Bhattacharya, B., Price, R. K. & Solomatine, D. P. 2007 Machine learning approach to modeling sediment transport. *J. Hydraul. Eng.* **133**, 440–450.

Bonakdar, L. & Etemad-Shahidi, A. 2011 Predicting wave run-up on rubble-mound structures using M5 model tree. *Ocean Eng.* **38**, 111–118.

Breusers, H. N. C. 1972 *Local Scour Near Offshore Structures*. Delft Hydraulics Laboratory, The Netherlands, Publication No. 105.

Breusers, H. N. C., Nicollet, G. & Shen, H. W. 1977 Local scour around cylindrical piers. *J. Hydraul. Res.* **15**, 211–252.

Breusers, N. H. C. & Raudkivi, A. J. 1991 *IAHR Hydraulic Structures Design Manual: Scouring, Vol. 2*. Balkema, Rotterdam, The Netherlands.

Coleman, S. E. 2005 Clear-water local scour at complex piers. *J. Hydraul. Eng.* **131**, 330–334.

Etemad-Shahidi, A. & Bali, M. 2011 Stability of rubble-mound breakwaters using H_{50} wave height parameter. *Coast. Eng.* **59**, 38–45.

Etemad-Shahidi, A. & Bonakdar, L. 2009 Design of rubble-mound breakwaters using M5' machine learning method. *Appl. Ocean Res.* **31**, 197–201.

Etemad-Shahidi, A. & Ghaemi, N. 2011 Model tree approach for prediction of pile groups scour due to waves. *Ocean Eng.* **38**, 1522–1527.

Etemad-Shahidi, A. & Mahjoobi, J. 2009 Comparison between M5' model tree and neural networks for prediction of significant wave height in Lake Superior. *Ocean Eng.* **36**, 1175–1181.

Ettema, R., Melville, B. W. & Barkdoll, B. 1998 Scale effect in pier-scour experiments. *J. Hydraul. Eng.* **124**, 639–642.

Ghazanfari-Hashemi, S., Etemad-Shahidi, A., Kazeminezhad, M. H. & Mansoori, A. R. 2010 Prediction of pile groups scour in waves using support vector machines and ANN. *J. Hydroinform.* **13**, 609–620.

Hancu, S. 1971 Sur le calcul des affouillements locaux dams la zone des piles des ponts. In: *Proceedings of the 14th IAHR Congress*, Paris, France, **Vol. 3**. International Association for Hydraulic Research, Delft, The Netherlands, pp. 299–313.

Hannah, C. R. 1978 Scour at Pile Groups. Research Rep. No. 28-3, Civil Engineering Dept., Univ. of Canterbury, Christchurch, New Zealand.

Kambekar, A. R. & Deo, M. C. 2003 Estimation of pile group using neural networks. *Appl. Ocean Res.* **25**, 225–234.

Kazeminezhad, M. H., Etemad-Shahidi, A. & Yeganeh-Bakhtiary, A. 2010 An alternative approach for investigation of wave-induced scour around pipelines. *J. Hydroinform.* **12**, 51–65.

Khosronejad, A., Montazer, G. A. & Ghodsian, M. 2003 Estimation of scour properties around vertical pile using ANNs. *Int. J. Sed. Res.* **18**, 290–300.

Laursen, E. M. & Toch, A. 1956 *Scour Around Bridge Piers and Abutments*. Bulletin No. 4, Iowa Highway Research Board, Ames, Iowa.

Liriano, S. L. & Day, R. A. 2001 Prediction of scour depth at culvert outlets using neural networks. *J. Hydroinform.* **3**, 231–238.

Melville, B. W. 1997 Pier and abutment scour: Integrated approach. *J. Hydraul. Eng.* **123**, 125–136.

Melville, B. W. & Chiew, Y. M. 1999 Time scale for local scour depth at bridge piers. *J. Hydraul. Eng.* **125**, 59–65.

Melville, B. W. & Coleman, S. E. 2000 *Bridge Scour*. Water Resources, Colorado.

Melville, B. W. & Sutherland, A. J. 1988 Design method for local scour at bridge piers. *J. Hydraul. Eng.* **114**, 1210–1226.

Quinlan, J. R. 1992 Learning with continuous classes. In: *Proceedings of AI'92* (J. Adams & H. Sterling, eds.). World Scientific, Singapore, pp. 343–348.

Richardson, E. V. & Davis, S. R. 2001 Evaluating Scour at Bridges. Hydraulic Engineering Circular No. 18 (HEC-18), Rep. No. FHWA NHI 01-001, Federal Highway Administration, Washington, DC.

Salim, M. & Jones, J. S. 1998 Scour Around Exposed Pile Foundations. *Compilation of Conf. Scour Papers (1991–1998)*. ASCE, Reston, VA.

Shen, H. W. 1971 *Scour Near Piers. River Mechanics*, Vol. II. USGS, Ft. Collins, Colorado.

Sheppard, D. M. & Glasser, T. 2004 Sediment scour at piers with complex geometries. *Proc. 2nd Int. Conf. on Scour and Erosion*, World Scientific, Singapore.

Sheppard, D. M., Odeh, M. & Glasser, T. 2004 Large scale clear water local pier scour experiments. *J. Hydraul. Eng.* **114**, 1210–1226.

Sumer, B. M. & Fredsoe, J. 1998 Wave scour around a group of vertical piles. *J. Waterw. Port Coast. Ocean Eng.* **124**, 248–256.

Sumer, B. M., Bundgaard, K. & Fredsoe, J. 2005 Global and local scour at pile groups. In: *Proceedings of the 15th International Offshore and Polar Engineering Conference*. Seoul, pp. 577–583.

Wang, Y. & Witten, I. H. 1997 Induction of model trees for predicting continuous lasses. In: *Proceedings of the Poster Papers of the European Conference on Machine Learning*. University of Economics, Faculty of Informatics and Statistics, Prague.

Zounemat-Kermani, M. 2012 Hourly predictive Levenberg-Marquardt ANN and multi linear regression models for prediction of dew point temperature. *Meteorol. Atmos. Phys.* **1**, 1–12.

Zounemat-Kermani, M., Beheshti, A. A., Ataie-Ashtiani, B. & Sabbagh-Yazdi, S. R. 2009 Estimation of current-induced scour depth around pile groups using neural network, adaptive neuro-fuzzy inference system. *Appl. Soft Comp.* **9**, 746–755.