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An evaluation of expansion equations for fluidized solid–liquid systems

Omer Akgiray and Elif Soyer

ABSTRACT

Numerous equations can be found in the literature for the prediction of bed expansion during liquid–solid fluidization. There is no general agreement regarding which equation is the most accurate. In this work, fluidization experiments have been carried out with plastic and glass spheres of 11 different sizes, varying from 1.11 mm to 6.01 mm. Water was used as the fluidizing medium in all the experiments. Widely used correlations are compared and evaluated using the data collected in this work and certain long-standing data from the literature. A new equation with a simple and meaningful form is developed for the prediction of bed expansion in liquid fluidized beds of spheres. The proposed correlation is based on an extension of the fixed-bed friction factor concept to fluidized beds. The new equation has a number of advantages such as a wider operating range, improved accuracy, simple and continuous form, and a fundamental basis.

Key words | bed expansion, filter backwash, fluidization, liquid–solid systems, porosity

Omer Akgiray (corresponding author)
Marmara University, Faculty of Engineering,
Department of Environmental Engineering,
Goztepe, 34722, Istanbul,
Turkey
Phone/Fax: (+90) 216-3481369
E-mail: *akgiray@eng.marmara.edu.tr*

Elif Soyer
Istanbul Technical University, Faculty of Civil Engineering,
Department of Environmental Engineering,
Maslak 34469, Istanbul,
Turkey
E-mail: *esoyer@ins.itu.edu.tr*

INTRODUCTION

Liquid–solid fluidization has a number of applications in engineering. The expansion of granular filter media during backwashing is an important example (AWWA 1999; Akkoyunlu 2003). Another area of application that is of growing interest is fluidized-bed reactors used in wastewater treatment. It is important to have an understanding of fluidization principles and an ability to predict bed expansion as a function of liquid velocity to design such systems properly.

This paper focuses on single-medium beds consisting of uniform (i.e. single size) spheres and introduces a new equation for the prediction of the velocity-voidage relationship during fluidization. Extension of such a model to handle non-spherical particles is discussed by Dharmarajah & Cleasby (1986) and Soyer & Akgiray (2005). If an accurate model for the expansion of a uniform bed is available, the expansion of a non-uniform bed can also be predicted. A bed with a size gradation is considered to consist of several layers of approximately uniform size according to the sieve analysis data, and the expansion of each layer is separately calculated. The unexpanded depth of each layer is proportional to the volume fraction of that layer. The total expansion is calculated by adding the expansions of all the layers (Fair *et al.*, 1971). This calculation method can also be used to predict the total expansion of a multimedia bed.

Numerous equations have been proposed to predict the expansion of liquid fluidized beds of spherical particles. Reviews and listings of such equations can be found in Couderc (1985), Hartman *et al*. (1989), Di Felice (1995) and Epstein (2003). Surprisingly, there is no general agreement regarding which equation is the most accurate. The major goal of this work was to obtain new and independent data to test and to evaluate the existing correlations. A new, simple equation has emerged during the course of this work.

A number of the more popular correlations employ the following empirical relation (Lewis & Bowerman, 1952;

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Richardson & Zaki, 1954):

$$\frac{V}{V_i} = \varepsilon^n \quad (1)$$

Here V is the superficial liquid velocity, V_i is the intercept velocity, and ϵ is the mean porosity of the fluidized bed. The mean porosity is defined by the relation $L(1-\epsilon) = L_0(1-\epsilon_0)$, where $L =$ depth of the fluidized bed, $L_0 =$ depth of the fixed-bed, and $\epsilon_0 =$ fixed-bed porosity. The most widely used correlation based on Equation 1 has been that published by Richardson & Zaki (1954). Richardson and Zaki set $V_i = V_{t\infty}$ for sedimentation and $\log V_i = \log V_{t\infty} - d/D$ for fluidization, where $V_{t\infty}$ is the terminal settling velocity of a bed particle in an infinitely large container, d is the particle diameter, and D is the diameter of the fluidization column. The index n is calculated as shown in Table 1 (Richardson, 1971). More recently, Khan & Richardson (1990) published another set of equations that supersede the Richardson-Zaki correlation. These two correlations as well as the other widely used correlations for spherical particles are listed in Table 1.

The following notation is used in Table 1 and in the following paragraphs. $Re_{t\infty} = V_{t\infty}\rho d/\mu$ is the Reynolds number based on the terminal settling velocity of an isolated particle in an infinitely large container; $Re_t = V_t\rho d/\mu$

Table 1 | Velocity-voidage relations for particulate, liquid fluidized beds of spheres

Authors	**Correlation**
Richardson & Zaki (1954)	$\frac{V}{V_i} = \varepsilon^n$ where $n = 4.65 + 20\frac{d}{D}$ for $Re_{t\infty} < 0.2$
Richardson (1971)	$n = \left(4.4 + 18\frac{d}{D}\right)Re_{t\infty}^{-0.03}$ for $0.2 < Re_{t\infty} < 1$
	$n = \left(4.4 + 18\frac{d}{D}\right)Re_{t\infty}^{-0.1}$ for $1 < Re_{t\infty} < 200$
	$n = 4.4\, Re_{t\infty}^{-0.1}$ for $200 < Re_{t\infty} < 500$
	$n = 2.4$ for $Re_{t\infty} > 500$
Wen & Yu (1966)	$\varepsilon^{4.7}Ga = (18\,Re + 2.7\,Re^{1.687})$
Riba & Couderc (1977)	$dGa/Re^e = (af(\varepsilon)^b)^c$ where a = 1.21, b = 1.28 if $\epsilon < 0.85$,
	a = 0.77, b = 2.70 if $\epsilon > 0.85$; c = 1, d = 1/18, e = 1 if Ga < 18.
	c = 1.4, d = 1/13.9, e = 1.4 if 18 < Ga < 10^5; c = 2, d = 3, e = 2 if Ga > 10^5
	$f(\epsilon) = (1-1.21(1-\epsilon)^{2/3})^{-1}$
Garside & Al-Dibouni (1977)	$\varepsilon^{5.14} + 0.048\varepsilon^{2.28}Re_t^{\varepsilon+0.2} = Re(1 + 0.06\,Re_t^{\varepsilon+0.2})/Re_t$ if $\epsilon \leq 0.85$
	$\varepsilon^{5.14} + 0.06\varepsilon^{3.65}Re_t^{\varepsilon+0.2} = Re(1 + 0.06\,Re_t^{\varepsilon+0.2})/Re_t$ if $\epsilon > 0.85$
	$Re_{t\infty}/Re_t = 1 + 2.35(d/D)$
Gibilaro *et al.* (1986)	$\varepsilon^{4.8}Ga = Re^2((17.3/Re)^{\alpha} + 0.336^{\alpha})^{1/\alpha}$
	$\alpha = 2.55 - 2.1[\tanh(20\varepsilon - 8)^{1/3}]^3$
Khan & Richardson (1990)	$\frac{V}{V_t} = \varepsilon^n$
	$V_t/V_{t\infty} = 1 - 1.15(d/D)^{0.6}$
	$\frac{4.8-n}{n-2.4} = 0.043Ga^{0.57}\left[1 - 1.24\left(\frac{d}{D}\right)^{1.27}\right]$

is the Reynolds number based on the terminal settling velocity of an isolated particle in a finite cylindrical container of diameter D; $Re = V\rho d/\mu$ is the Reynolds number based on the superficial liquid velocity V in fluidization; $\mu =$ viscosity of the liquid; $\rho =$ density of the liquid; $Ga = d^3\rho(\rho_p - \rho)g/\mu^2$ is the Galileo number; and $\rho_p =$ density of the particles.

Ideally, an equation relating velocity and voidage would have the following features: (i) it should be based on a plausible physical model (such as the capillary-tube model for fixed beds); (ii) it should be applicable to both spherical and non-spherical particles; (iii) it should be applicable to both fixed-beds and fluidized-beds all the way up to the limit $\epsilon \to 1$ (i.e. single isolated particle settling in a liquid); and (iv) it should account for container wall effects correctly and accurately. These goals are very hard to achieve with a single equation. While fixed-bed equations applicable to both spherical and non-spherical particles exist, prediction methods for settling velocities of non-spherical particles are notoriously unreliable (Kelly & Spottiswood, 1982). Furthermore, most engineering applications involve beds of particles at low expansions. (Here 'low expansion' means porosities less than about 0.90, and covers a range from minimum fluidization point up to 150–500% expansion for typical granular materials. Porosities observed in filter backwash are normally well below 0.90, and therefore remain safely within the range of porosities considered.) A contention of this paper is that, for engineering design purposes, the extension of fixed-bed models to fluidized beds may provide the most suitable method of correlation. This approach is explained next.

One of the most commonly used head loss equations for fixed-beds is the Blake-Kozeny Equation (Fair & Hatch, 1933):

$$\frac{h}{L_0} = \frac{k\mu}{\rho g}\frac{(1-\varepsilon_0)^2}{\varepsilon_0^3}\left(\frac{6}{\psi d_{eq}}\right)^2 V \tag{2}$$

in which $h =$ loss of head in the bed, L_0 = depth of the fixed-bed, $V =$ velocity based on the empty cross-section of the bed, $\epsilon_0 =$ fixed-bed porosity, d_{eq} = equivalent diameter defined as the diameter of the sphere with the same volume as a bed particle, $\psi =$ sphericity defined as the surface area of the equivalent volume sphere divided by the actual surface area of the particle. The proportionality constant k is known as the Kozeny's constant and a widely quoted value for k is 5 (Coulson & Richardson, 1968).

Fair & Hatch (1933) assumed that the Blake-Kozeny equation is applicable to fluidized beds as well. Replacing L_0 and ϵ_0 by L and ϵ, respectively, they have equated Equation 2 to the fluidized bed head loss given by:

$$\frac{h}{L} = \frac{(\rho_p - \rho)(1-\varepsilon)}{\rho} \tag{3}$$

The result was the following relationship between velocity and porosity:

$$\frac{\varepsilon^3}{1-\varepsilon} = \frac{k\mu}{g(\rho_p - \rho)}\left(\frac{6}{\psi d_{eq}}\right)^2 V \tag{4}$$

It is noteworthy that Fair and Hatch reported different values of k for fixed ($k = 5$) and fluidized ($k = 4$) beds. Leva (1959) also considered the extension of the Blake-Kozeny equation to particulately fluidized beds and stated that '…an analysis of initial dense-phase fluidization by way of the laws of fixed beds may be logical…' He used the constant $k = 200/36$ for both fixed and fluidized beds.

It is well known that the Blake-Kozeny equation is not valid at high fluid velocities, and the Ergun equation can be used for a much wider range of Reynolds numbers:

$$\frac{h}{L_0} = \frac{k_1\mu}{\rho g}\frac{(1-\varepsilon_0)^2}{\varepsilon_0^3}\left(\frac{6}{\psi d_{eq}}\right)^2 V + \frac{k_2}{g}\frac{(1-\varepsilon_0)}{\varepsilon_0^3}\left(\frac{6}{\psi d_{eq}}\right)V^2 \tag{5}$$

Based on fixed-bed pressure drop data, Ergun (1952) reported the values $k_1 = 150/36 = 4.17$ and $k_2 = 1.75/6 = 0.29$. Akgiray & Saatçi (2001) considered the application of the Ergun equation to fluidized beds. In this approach, the head losses calculated by Equations 3 and 5 are set equal to each other. (This same approach was used by Wen & Yu (1966) to calculate minimum fluidization velocity. They did not, however, consider the application of the Ergun equation to fully fluidized beds.) Akgiray & Saatçi (2001) presented an exact explicit solution of the resulting equation giving the fluidized-bed porosity as a function of superficial velocity. This exact solution has recently been adopted by Crittenden *et al.* (2005) for the prediction of filter-bed expansion.

Figure 1 displays the fluidization and sedimentation data published by Wilhelm & Kwauk (1948), Loeffler (1953), Wen & Yu (1966), and Hartman *et al*. (1989). (See Equations 6 and 7 below for the definitions of ϕ and Re_1 used in this figure.) The fluidization data in Figure 1 were analysed by Akgiray *et al*. (2004) to determine the optimal values of k_1 and k_2 for expanded beds. The following were concluded: The Ergun equation cannot fit the fluidization data very accurately in the entire range of Reynolds numbers for which experimental data are available. Furthermore, the values of k_1 and k_2 depend on the range of data used in their determination. Considering the fluidization data in the range of Reynolds numbers of interest in filter backwashing (see Figure 1, points in the range $-2.0 < \log \mathrm{Re}_1 < 2.0$), the following values were obtained: $k_1 = 3.52$ and $k_2 = 0.27$ ($r^2 > 0.996$ with 474 measurements). Equation 9 (with $k_1 = 3.52$ and $k_2 = 0.27$) gives an excellent agreement with the fluidization data in the range $-2.0 < \log \mathrm{Re}_1 < 2.0$, provided that the porosity is below about 0.90. It can be shown that these two conditions are satisfied for silica sand, anthracite coal, granular activated carbon, garnet and ilmenite, with sizes from 0.2 mm to 3.0 mm, and for expansions up to 100%. (The expansion will typically be much less than 100% during filter backwashing.) Figure 1 displays the curves predicted by the Fair-Hatch Equation (i.e. Ergun with $k = k_1 = 4.17$ and $k_2 = 0$) and the modified Ergun Equation (with $k_1 = 3.52$ and $k_2 = 0.27$).

The equation proposed by Fair and Hatch can be found in many widely used chemical engineering texts such as Foust *et al*. (1960), Geankoplis (1993) and McCabe *et al*. (2001). These authors derive Equation 4 by neglecting the nonlinear term in the Ergun equation, and therefore they take $k = k_1 = 150/36 = 4.17$, i.e. the value obtained by Ergun (1952) for fixed beds.

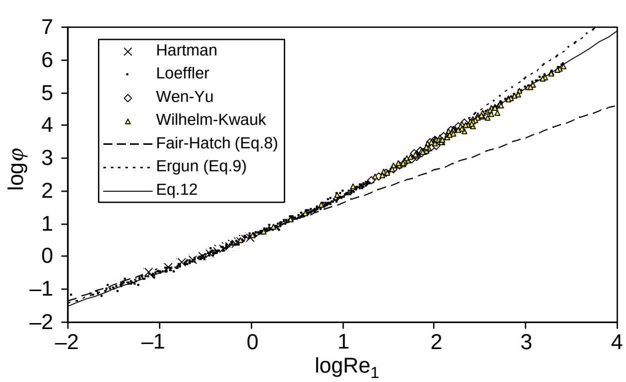

Figure 1 | Spherical particle data collected from the literature (Wilhelm & Kwauk, 1948; Loeffler, 1953; Wen & Yu, 1966; Hartman *et al.*, 1989).

The following can be concluded from Figure 1 (Akgiray *et al.*, 2004): Neglecting the second term in the Ergun equation leads to a very poor agreement with the fluidization data at high Reynolds numbers. Furthermore, for small Reynolds numbers, the use of the fixed-bed value of $k_1 = 4.17$ gives friction factors larger than those experimentally observed. While the Ergun equation is applicable over a wider range of Reynolds numbers than the Fair-Hatch equation, it cannot accurately fit the fluidization data over the entire range of Reynolds numbers for which experimental data are available in the literature. An alternative empirical equation was therefore developed by Akgiray *et al*. (2004). This method is applicable over a wider range of conditions and is more accurate than the Ergun equation. To explain this method, as well as the new proposed equation, certain notation and terms are first defined.

Richardson & Meikle (1961) analysed their own data and the data by Richardson & Zaki (1954) and Loeffler & Ruth (1959) to plot a curve of a dimensionless group ϕ *versus* the modified Reynolds number Re_1:

$$\varphi = f \times \mathrm{Re}_1^2 = \frac{\varepsilon^3}{(1-\varepsilon)^2} \frac{\psi^3 d_{eq}^3 \rho(\rho_p - \rho) g}{216\mu^2} \qquad (6)$$

$$\mathrm{Re}_1 = \frac{\psi d_{eq} \rho V}{6\mu(1-\varepsilon)} \qquad (7)$$

Note that $f = \varphi/\mathrm{Re}_1^2$ is a friction factor. It has been noted that the Fair-Hatch (Equation 8) and Ergun Equations (Equation 9) are also correlations relating ϕ and Re_1 (Akgiray & Saatçi, 2001):

$$\varphi = k\,\mathrm{Re}_1 \qquad (8)$$

$$\varphi = k_1\,\mathrm{Re}_1 + k_2\,\mathrm{Re}_1^2 \qquad (9)$$

Dharmarajah & Cleasby (1986) used the same method of correlation and analysed the data for fluidized spheres published by Wilhelm & Kwauk (1948) and Loeffler (1953) to develop the following equations by carrying out a regression analysis:

For $Re_1 < 0.2$:

$$\varphi = 3.01\, Re_1 \quad (10a)$$

For $Re_1 > 0.2$:

$$\log\varphi = 0.56543 + 1.09348\log Re_1 + 0.17979(\log Re_1)^2 - 0.00392(\log Re_1)^4 \quad (10b)$$

While Equation 10 has a good overall accuracy (see Tables 2 and 3), it has a number of shortcomings: First, the correlation is quite cumbersome and lacks a theoretical basis. Second, it contains a discontinuity at $Re_1 = 0.2$. Furthermore, two different porosity values are possible in a vicinity of the discontinuity (see Akgiray *et al.*, 2004). This leads to an ambiguity in the application of the correlation. To remedy the latter shortcoming, Akgiray *et al.* (2004) proposed the following single, continuous equation applicable to spheres:

$$\log\varphi = 0.565013 + 1.157034\log Re_1 + 0.12866(\log Re_1)^2 + 0.02195(\log Re_1)^3 - 0.008(\log Re_1)^4 \quad (11)$$

This equation has the following advantages over Equations 10a and 10b: (i) it is simpler to use (with Equation 10a and 10b, it is not clear in advance which equation is to be used when Re_1 is not known); and (ii) it is continuous (Equation 10 has a discontinuity at $Re_1 = 0.2$ and this is not physically reasonable). Equations 10 and 11 have been found to be more accurate than any of the other existing correlations for spheres (Akgiray *et al.*, 2004). Unfortunately, Equations 10 and 11 have both been obtained by 'brute force' (i.e. *via* regression analysis employing bulky expressions with large numbers of adjustable parameters), and they lack a theoretical basis. Therefore, a more elegant and theoretically meaningful solution to this problem is called for. This is achieved in the present work and explained in the next section.

Table 2 | Percentage errors in the predicted values of porosity for the literature data (based on 600 measurements in the range $-1.96 < \log Re_1 < 3.42$; see Figure 1)

Authors	Mean deviation in porosity (%)
Richardson & Zaki (1954)	5.18
Wen & Yu (1966)	4.68
Riba & Couderc (1977)	9.18
Garside & Al-Dibouni (1977)	3.26
Dharmarajah & Cleasby (1986)	2.52
Gibilaro *et al.* (1986)	6.98
Khan & Richardson (1990)	3.56
Akgiray *et al.* (2004) (Equation 11)	2.52
Equation 12 in this work	2.39

Table 3 | Percentage errors in the predicted values of porosity for the data obtained in this work (based on 332 measurements in the range $0.76 < \log Re_1 < 3.54$; see Figure 2)

Authors	Mean deviation in porosity (%)
Richardson & Zaki (1954)	7.22
Wen & Yu (1966)	5.69
Garside & Al-Dibouni (1977)	1.70
Dharmarajah & Cleasby (1986)	2.03
Gibilaro *et al.* (1986)	9.94
Khan & Richardson (1990)	4.42
Akgiray *et al.* (2004) (Equation 11)	2.51
Equation 12 in this work	2.04

THE NEW EQUATION

The analytical solution of the Ergun Equation (Equation 9) for porosity (Akgiray & Saatçi, 2001) is very convenient. Unfortunately, Equation 9 deviates from the trend of the data for $\log Re_1 > 2$ (see Figure 1). This deviation stems from the fact that the power 2 in the inertial term of the Ergun equation is too large for fluidization. It is noteworthy that, even for fixed beds, smaller values such as 1.90 (Carman, 1937) and 1.83 (Tallmadge, 1970) were also proposed. It is therefore plausible that a good fit to the fluidization data can be achieved by choosing a power value different from 2.0 for the inertial term. The following form is adopted in this work:

$$\varphi = k_1\, Re_1 + k_2\, Re_1^{P} \quad (12)$$

Note that this equation contains a viscous energy loss term (the linear term) and an inertial loss term (the nonlinear term). It has therefore a much more meaningful form than the curve-fitting correlations of Dharmarajah & Cleasby (1986) and Akgiray *et al*. (2004) (i.e. Equations 10 and 11, respectively).

A nonlinear regression analysis was carried out to determine the best values of k_1, k_2 and P. Based on the data collected from the literature (Wilhelm & Kwauk, 1948; Loeffler, 1953; Wen & Yu, 1966; Hartman *et al.*, 1989), the following values are obtained: $k_1 = 3.137$, $k_2 = 0.673$ and $P = 1.766$ ($r^2 = 0.99897$, based on 600 data points). The resulting curve is shown in Figure 1 together with the literature data on which it is based. Equation 12 has the following advantages:

- Compared with Equations 10 and 11, it is considerably simpler in form.
- It is continuous (the discontinuity in Equation 10 is not physically reasonable).
- The proposed equation is very accurate (see Tables 2 and 3).
- The new equation has a theoretically meaningful form. It consists of a viscous loss term and an inertial loss term, just like the fixed-bed equations by Forchheimer (1930), Carman (1937) and Ergun (1952). Equations 10 and 11, on the other hand, are based on curve fitting with 'brute force'. Although Equation 12 is much simpler in form and has a smaller number of adjustable parameters, it gives equivalent or better accuracy than Equations 10 and 11. This can be attributed to the fact that the form of Equation 12 has some theoretical basis, whereas Equations 10 and 11 are completely empirical.
- It has been shown that Equations 8–12 are applicable to spheres only (Akgiray *et al.*, 2004). However, a correlation of ϕ *versus* Re_1 can be extended to handle non-spherical media (Dharmarajah & Cleasby, 1986; Soyer & Akgiray, 2005), whereas most other existing correlations are strictly restricted to spheres.
- When contrasted with the Ergun equation, Equation 12 has one disadvantage: Just like Equations 10 and 11, it must be solved iteratively to calculate either the porosity or the velocity when the other one is given. On the other hand, the required iterations can be carried out easily with the aid of a computer (see the Appendix).

WALL EFFECTS

While the effect of container walls is normally negligible in full-scale operations, it may be significant in laboratory experiments. Wall effects, therefore, should be given consideration when: (1) data obtained with small columns are used to derive a correlation intended for application to industrial-scale equipment; or (2) a correlation directly applicable to large columns only (such as Equations 8–12 of this paper) is to be applied to a column with a small diameter.

Loeffler (1953) carried out fluidization experiments using the same particles in two different-sized columns. Dharmarajah & Cleasby (1986) analysed Loeffler's data and concluded that wall effects depend on both $\mathrm{Re}_{t\infty}$ and d/D. They have presented the following curve-fitting relation between the actual backwash velocity V and a hypothetical backwash velocity V_0 which is free of wall effects:

$$\frac{V_0}{V} = 10^{\alpha(d/D)} \tag{13a}$$

$$\alpha = 5.18/\mathrm{Re}_{t\infty}^{0.585} \tag{13b}$$

Here V_0 is the backwash velocity required to attain the same porosity in an infinitely large container as velocity V would produce in a container of diameter D. This equation represents Loeffler's data well in the range $\mathrm{Re}_{t\infty} = 1.79$ to 2769. In this work, Equations 8–12 are applied in conjunction with Equation 13. The Richardson-Zaki, Garside-al-Dibouni and Khan-Richardson correlations have their own wall-effect correction terms, whereas the Wen-Yu, Riba-Couderc and Gibilaro correlations do not take wall effects into account. These three correlations were therefore evaluated without any wall-effect corrections. It should be mentioned that the ratio V_0/V remained in the range 1.01–1.03 in all the experiments carried out in this work (i.e. for the data of Figure 2 and Table 3), whereas it varied between 1.003 and 1.77 in the data collected from the literature (Figure 1 and Table 2).

EXPERIMENTAL

Fluidization experiments have been carried out with glass balls of eight different sizes and plastic balls of three

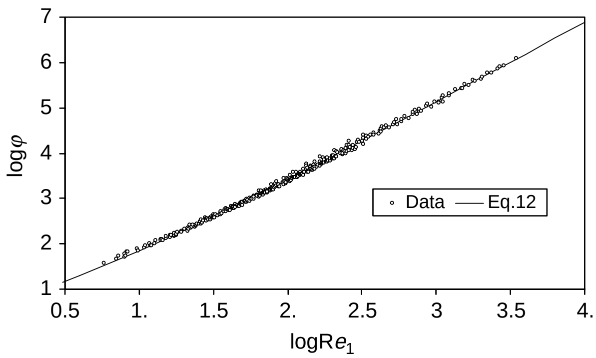

Figure 2 | Spherical particle data collected in this work and the new equation.

different sizes. The properties of the media are listed in Table 4. Densities were measured by a water-displacement technique. Particle diameters have been measured by counting and weighing 200 balls from each medium. The diameter values so determined were verified using a micrometer. A 152 cm high column with an internal diameter of 50.5 mm was employed as the fluidization section. The column was preceded by a 20 cm long equalizing section filled with gravel with an approximate size of 1 cm. A wire mesh screen was placed at the end of the 152 cm fluidization section to prevent the loss of media. Another 20 cm long section (placed above the screen) followed the fluidization section. Water was the fluidizing medium in all the experiments. Bed height, water flow rate and temperature were recorded during the experiments. Porosities were calculated from bed weight, bed height and particle density values. The data collected in this work are displayed collectively in Figure 2. An example of a typical run is shown in Figure 3.

Table 4 | Properties of the media used in the fluidization experiments

Particle type	ρ_p (gr cm^{-3})	d (mm)
Glass balls	2.519	1.11
Glass balls	2.494	1.19
Glass balls	2.529	2.03
Glass balls	2.494	3.18
Glass balls	2.532	2.99
Glass balls	2.499	4.03
Glass balls	2.529	4.98
Glass balls	2.527	6.01
Plastic balls	1.171	1.97
Plastic balls	1.193	2.48
Plastic balls	1.180	2.87

COMPARISON OF THE EQUATIONS

Table 2 displays the mean values of the percentage error ($100|\epsilon_{predicted} - \epsilon_{exp}|/\epsilon_{exp}$) for the correlations considered here. The values in this table are based on the 600 measurements collected from the literature. For each of these correlations, the error values are approximately the same throughout the entire range of Reynolds numbers ($0.01 < Re_1 < 2600$) spanned by the data, except the Riba-Couderc equation, which is highly inaccurate in the range $Re_1 < 2$.

Table 3 shows the agreement of the equations with the data collected in this work. In examining these results, it should be remembered that a small error in porosity can lead to a large error in predicted bed expansion. This can be seen by considering the curves in Figure 4. These curves were obtained by using the relation $L/L_0 = (1-\epsilon_0)/(1-\epsilon)$ and assuming $\epsilon_0 = 0.4$. It is seen that even a 5% error in porosity can lead to unacceptably high errors in predicted bed height. The error in predicted bed expansion is magnified at larger porosities. The point here is, even a 'slight' improvement in the accuracy of predicted porosity may lead to a non-negligible improvement in the accuracy of the predicted bed height.

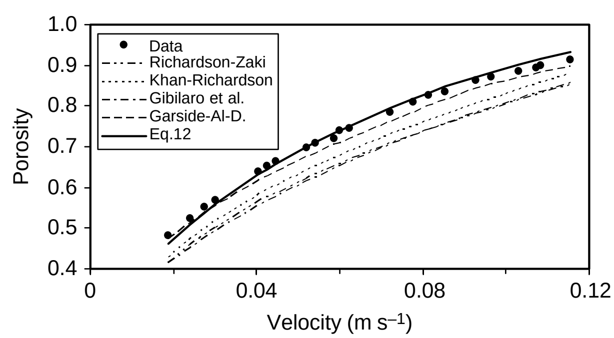

Figure 3 | Fluidization data for 1.19 mm glass balls and the model lines for various equations.

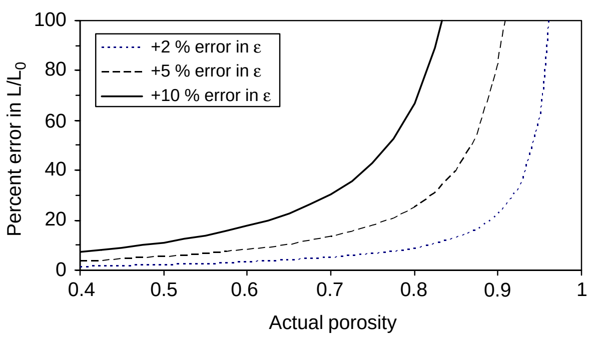

Figure 4 | Percentage error in predicted bed height as a function of error in porosity ($\epsilon_0 = 0.40$).

When the data collected from the literature and the data obtained in this work are considered together, the mean error values in predicted porosity are as follows: Equation 12: 2.26%; Dharmarajah-Cleasby: 2.35%; Equation 11: 2.52%; Garside-Al-Dibouni: 2.70%; Khan-Richardson: 3.87%. The remaining correlations lead to errors larger than 5%.

These results (Tables 2 and 3 considered together) show that, among the correlations restricted to spherical particles, the correlation published by Garside & Al-Dibouni (1977) is the most accurate. It is also seen that, in addition to being extendable to non-spherical particles, the equation proposed in this paper shows the smallest overall mean deviation from the experimental measurements. The data referred to in Tables 2 and 3 consist of $600 + 332 = 932$ measurements in the range $-1.96 < \log \mathrm{Re}_1 < 3.54$ and $0.37 < \epsilon < 0.90$.

Dharmarajah & Cleasby (1986) did not present a comparison of their correlation with other published correlations. It is seen from Tables 2 and 3 that the equation proposed here (Equation 12) and Equations 10 and 11 are all very accurate. (Considering Tables 2 and 3 together, the new equation is slightly more accurate, although the difference in accuracy is not great.) The new equation, however, has a considerably simpler form and has the other advantages mentioned earlier.

SUMMARY AND CONCLUSIONS

The problem of predicting the velocity-mean voidage relationship in particulate fluidization is studied. In accordance with the majority (if not all) of practical engineering applications of fluidization, attention is restricted to porosities less than 0.90.

Long-standing fluidization data from the literature as well as new data obtained in this work are used to evaluate existing popular correlations. It has been found that the classical Richardson-Zaki correlation is much less accurate than some of the newer equations and should now be retired. Although the Khan-Richardson correlation represents a marked improvement over the Richardson-Zaki method, the somewhat older Garside-Al-Dibouni correlation is more accurate. All of these correlations are strictly restricted to beds of spheres.

An alternative simple equation for the prediction of the velocity-voidage relationship is presented. The correlation method is based on the use of a friction factor concept similar to that used for fixed-beds. The proposed equation can be employed to predict the expansion of spheres up to a porosity value of 0.90. The coefficients in the correlation have been determined by a nonlinear regression analysis of long-standing published data for spheres. When applied to the new data collected in this work, the new equation has been found to be very accurate. It is found in this work that, even for spheres, correlations of φ *versus* Re_1 can give better accuracy than the other popular correlations. An important advantage of this type of correlation is that it can be extended to handle non-spherical media (Dharmarajah & Cleasby, 1986; Soyer & Akgiray, 2005).

There are two important motivations for the new correlation. First, although some of the existing correlations seem to be accurate enough (giving mean errors in porosity from 3.5% to 10%), further improvement in accuracy will not be superfluous. This is because relatively small errors in porosity may lead to large errors in fluidized bed height, which is frequently the quantity of practical interest. Second, the only practically useful correlations that can be applied to non-spherical media are those presented by Dharmarajah & Cleasby (1986) and Soyer & Akgiray (2005). These investigators presented quite cumbersome curve-fitting relations between the dimensionless group φ (introduced by Richardson & Meikle, 1961) *versus* the modified Reynolds number Re_1 (defined by Blake, 1922). The equation proposed here is considerably simpler and more elegant than these two correlations. It also remedies a number of shortcomings of these equations.

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APPENDIX

Table 5 shows one possible way to solve Equation 12 using the Solver package within MS Excel. Here A, B and C are column labels, whereas 1, 2, …etc. are row labels in Excel. Starting with an empty worksheet, the table shown below should be typed starting at the top leftmost cell (at the address A1). Next, from the Tools menu, the Add-Ins command should be executed, and the Solver Add-in should be selected. To find the porosity for a given velocity, the velocity value should be typed into B5, and an estimate of the porosity should be entered into cell B6 (a value between about 0.3 and 0.9). Next, the Solver is executed with the goal of making the residual zero. In the Solver interface, the 'Set Target Cell' parameter will be B9, whereas the 'By Changing Cells' parameter should be B6. The 'Equal to Value of' selection should be made, and 0 (number zero) should be specified as the value. This will make sure that the residual error is as close to zero as possible. When the residual is zero, Equation 12 is satisfied. This table format (with the formulas shown) can also be used for manual iterations without using the Solver. One would then try different porosity values by typing different values into cell B6 until the residual in cell B9 is small enough. This table format can also be used to find the velocity for a given porosity by modifying the steps described above in an obvious way.

Table 5 | Numerical solution of Equation 12 using ms Excel

	A	B	C
1	Size	0.00119	M
2	Particle density	2494	$kg\,m^{-3}$
3	Fluid density	996.5	$kg\,m^{-3}$
4	Viscosity	0.000854	$kg\,m^{-1}\,s^{-1}$
5	Velocity	0.0355	$m\,s^{-1}$
6	Porosity	0.595	
7	Phi	= B6^3*B1^3*B3*(B2-B3)*9.81/216/(1-B6)^2/B4^2	
8	Re1	= B1*B3*B5/6/B4/(1-B6)	
9	Residual error	= B7-3.137*B8-0.673*B8^1.766	