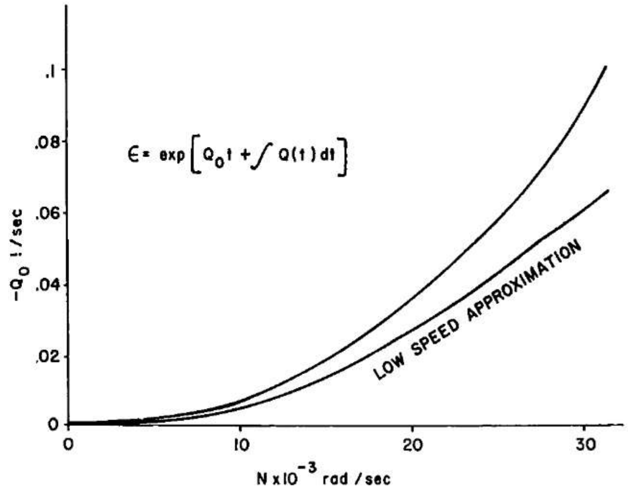

Fig. 6 Asymptotic stability characteristics of stable motion

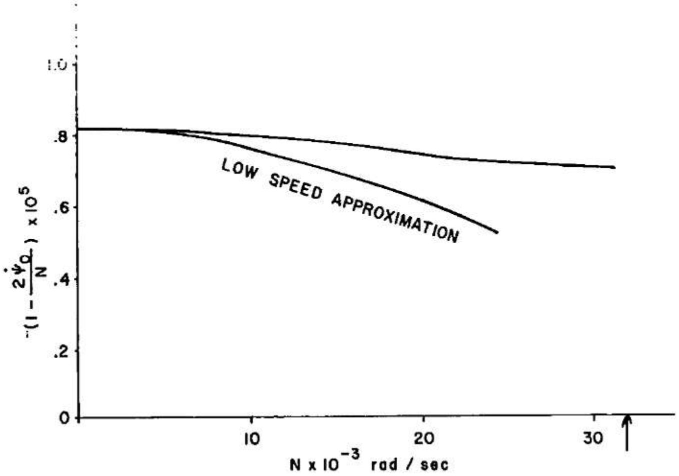

Fig. 7 Mean whirl frequency

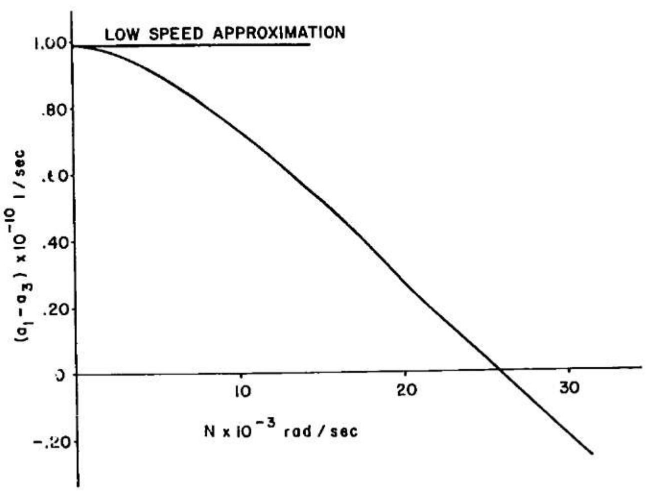

Fig. 8 Damping constant of small oscillations

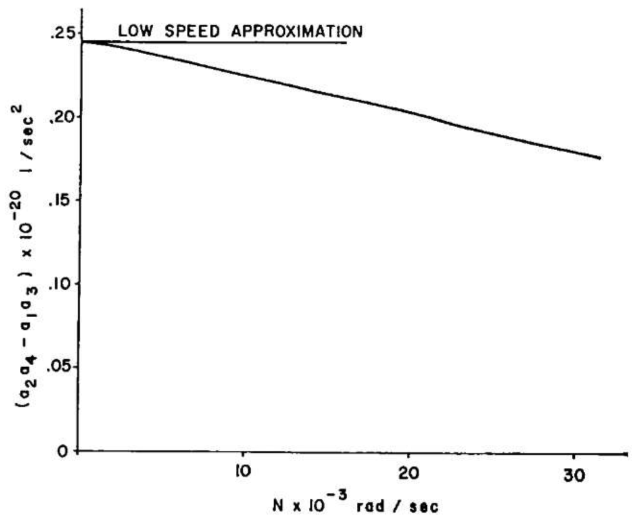

Fig. 9 Spring constant of small oscillations

and the bearing shall run in air at 500 deg F at 5 psia

$$p_0 = 5 \text{ lb/in.}^2 \qquad \mu = 6 \times 10^{-7} \frac{\text{lb/sec}}{\text{ft}^2}$$

As it has been seen, the two quantities Q_0 and Λ_0^* completely determine the essential characteristics of the motion. With a knowledge of the behavior of Q_0 and Λ_0^* the stability criteria given by equations (33), (34), (35) can be investigated for any operating speed N and, what is equivalent, the nature of the motion can be determined. To aid in the solution of equations (28), (29) for Q_0 and Λ_0^* Figs. 2, 3, 4, and 5 are recommended. These simplify the computations somewhat when a trial-and-error approach is used. The results of these computations are shown in Figs. 6, 7, 8, and 9 where the behavior of Q_0, $\left(1 - \frac{2\dot{\psi}_0}{N}\right)$, $a_1 - a_3$, and $a_2a_4 - a_1a_3$ with operating speed N is presented.

Fig. 7 shows the whirl frequency to be extremely close to one-half spin frequency throughout the whole stable operating speed range. This seems to be in agreement with experimental evidence which shows the whirl frequency to be linearly dependent on the operating speed.

The stability criteria are seen to be violated in Fig. 8 at a speed of 26, 750 rad/sec. When the quantity $(a_1 - a_3)$ becomes negative this, in effect, gives a negative damping constant for the small oscillations. Judging from the slope of $(a_1 - a_2)$ with N one would expect the critical speed to be very sharp and well defined.

The quantity $a_2a_4 - a_1a_3$ shown in Fig. 9 plays the role of a spring constant for the small oscillations. It maintains itself at a large positive value dropping only slightly with increasing speed from the low-speed approximation.

DISCUSSION

V. Castelli[2]

The contribution presented by this paper is certainly valuable in view of the frequent application of gas bearings to space-vehicle systems where conditions of no-load prevail during inertial flight.

However, a few questions should be raised in connection with the presented treatment. First, a matter of consistency of approximations: in the treatment of Reynolds equation a rotating coordinate system was adopted and the precessing rate was assumed to be constant so that for orbits of nearly constant amplitude the time derivative of pressure could be neglected. Later in the development, the force components are expressed in terms of a series in $\Lambda^* - \Lambda_0^*$ which is a term measuring the deviations of the orbiting angular velocity from a constant value. Aside from the fact that this seems to have only "quasi-static" value, is this a consistent approximation?

The second question is also raised on a matter of consistency of approximations. The contribution of the unbalanced frictional force is included in this analysis in an effort to improve the ap-

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proximation to the real situation. It appears, though, that the expression for a component of the frictional force can be reduced to

$$F_{3T} = \frac{c}{D} \Lambda \frac{\pi}{3} LDP_0 \frac{\epsilon}{C}$$

and for one of the pressure force components

$$(F_1\epsilon)_T = \frac{\Lambda^*}{1 + \Lambda^{*2}} \frac{\pi}{3} \frac{3}{2} LDP_0 \frac{\epsilon}{C} 0 \tag{43}$$

The frictional force is still of order c/D with respect to the normal pressure forces and must be neglected in agreement with assumptions made in deriving the Reynolds equation used in this paper. It should be pointed out, however, that the inclusion of the frictional force will probably not entail any appreciable error in the analysis.

If the validity of this theory is proved it will be very valuable for guidance component designers since, although it does not produce usable closed form solutions, the complexity of the calculations it requires is modest enough to be worth performing.

C. H. T. Pan[3]

The author has attempted to introduce the influence of fluid film shear to study the dynamical stability of a rigid rotor in unloaded self-acting plain cylindrical journal bearings.

In previous studies (e.g. reference [1 and 2]),[4] bearing force due to film shear has been neglected on the basis that, relative to the force due to film pressure, it is of the order $\Lambda(C/R)$ which is usually much less than $^1/_{100}$. In this paper, the author has pointed out that, during half-frequency whirl of a rotor in unloaded bearings, the bearing force due to film pressure actually degenerates to a value of similar order and therefore has proposed to include the effects of film shear. The actual stabilizing effect provided by film shear, according to the numerical example, is perhaps only of academic interest. Here, the threshold speed of instability is raised above 20,000 rad/sec by using a superprecise bearing which has a 50 microin. radial clearance in 9-in. length!

The low speed stability criterion as given by equation (42) may be derived alternately by an argument considering static equilibrium. That is, suppose the bearing parameters, L/D, Λ, and C/R, are given, then one can find a whirl frequency ψ_0 such that tangential components of bearing forces due to film shear and film pressure are in exact equilibrium. The system is stable provided the radial fluid film force is larger than the centrifugal force so that the eccentricity tends to reduce. This criterion assigns an upper limit to the rotor mass but has no reference to the rotor speed whatsoever, whence ψ_0 differs from one half of rotor speed only by a term of the order of C/R. Incidentally, the criterion favors large L/D provided rotor mass remains constant; the contrary is true if rotor mass per unit length remains constant.

Since static equilibrium does not assure dynamic stability, the author has performed further analysis and given the additional conditions as equations (33) through (35). In performing the dynamic analysis, the author employed a scheme for the calculation of time dependent film pressure, which has been independently developed by the discusser in reference [3]. In this scheme, time dependence is included in the parameters $\epsilon(t)$ and $\psi(t)$ and the time dependent film pressure is expressed as an expansion of time derivatives of all orders of these two parameters. In practice, this expansion has to be truncated, thus the author included only the effect of $\dot{\epsilon}$. Experimental evidence seems to indicate that the whirl orbits of the considered rotor-bearing system are nearly circular and one might infer that higher order time derivatives of ϵ should be negligible. On the other hand, it is not apparent whether one can legitimately neglect the effect of $\ddot{\psi}$ and $\ddot{\epsilon}$ relative to the rotor inertia. In fact, since the static equilibrium condition, or the low speed stability criterion according to the author's terminology, actually limits the rotor mass to a very small value, an order of magnitude analysis on the equations of motion would readily show that the effects of $\ddot{\psi}$ and $\ddot{\epsilon}$ on film pressure should not be neglected. For this reason, this discusser hesitates to endorse the conclusions reached by the author via his dynamic stability analysis.

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[4] Numbers in brackets designate References at end of paper.

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