

Discussion

View Factor Algebra for Two Arbitrarily Sized Nonopposing Parallel Rectangles

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Evaluation of the radiative heat transfer between two surfaces requires the use of a view factor, F_{1-2} , to give the fraction of radiation emitted from surface 1 that directly strikes surface 2. This discussion gives an equation to calculate the view factor between two arbitrarily sized nonopposing parallel rectangles in terms of view factors for opposing rectangles of the same size. This was originally presented by Hamilton and Morgan (1952) but their equation (referenced by Siegel and Howell, 1981, and Chapman, 1987) is missing one term. A closed-form solution derived by Hsu (1967) for the general case of two arbitrary parallel rectangles is also referenced as the recommended method for calculating the view factor. Hsu's approach is later generalized by Yuen (1980) to incorporate general polygons that do not have to be parallel to each other.

This discussion derives the expression for parallel rectangles that are not directly opposed. Using the notation of Fig. 1, this corresponds to F_{1-9}' . View factors for the geometries shown in Figs. 2 and 3 are used as building blocks for F_{1-9}' . The final result will be written entirely in terms of view factors for parallel opposing rectangular plates of the same size as shown in Fig. 4. This as well as basic view factor relations used in the derivation are given by Incropera and DeWitt (1990). The view factor $F_{a-b'}$ for the geometry shown in Fig. 2 can be found using the notation of Hamilton and Morgan as follows:

Let:

$$\begin{aligned}
 G_{i-j} &\equiv A_i F_{i-j} & G_{i-(ij)'} &\equiv A_i F_{i-(ij)'} \\
 G_{ij-i'} &\equiv (A_i + A_j) F_{ij-i'} & G_{i^2} &\equiv G_{i-i'}
 \end{aligned}
 \tag{1}$$

Then:

$$\begin{aligned}
 G_{(ab)^2} &= G_{ab-(ab)'} = G_{a-(ab)'} + G_{b-(ab)'} \\
 &= G_{a^2} + G_{a-b'} + G_{b-a'} + G_{b^2}
 \end{aligned}
 \tag{2}$$

Using a result from Siegel and Howell (1981), $G_{a-b'} = G_{b-a'}$ gives:

$$G_{a-b'} = 1/2 (G_{(ab)^2} - G_{a^2} - G_{b^2})
 \tag{3}$$

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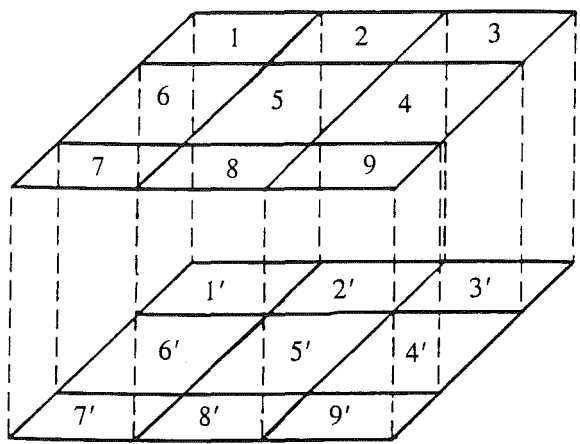


Fig. 1 Geometry for nonopposing parallel rectangles

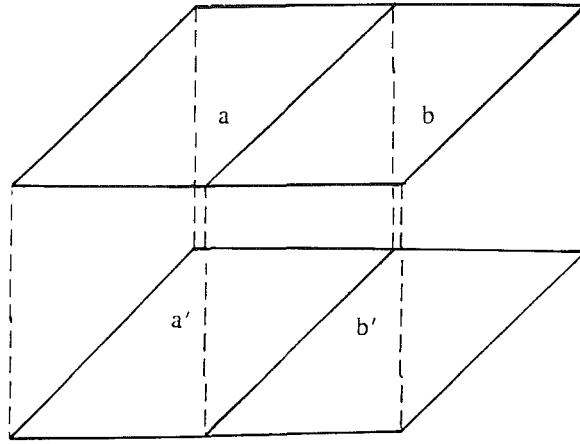


Fig. 2 Nonopposing parallel rectangles aligned on one edge

Similarly for Fig. 3:

$$\begin{aligned}
 G_{ce-(df)'} &= G_{c-(df)'} + G_{e-(df)'} = 1/2 (G_{(cdef)^2} - G_{(ce)^2} \\
 &\quad - G_{(df)^2}) = G_{c-d'} + G_{c-f'} + G_{e-d'} + G_{e-f'} \\
 &= 1/2 (G_{(cdef)^2} - G_{(ce)^2} - G_{(df)^2})
 \end{aligned}
 \tag{4}$$

It can be shown that $G_{c-f'} = G_{e-d'}$ by writing them in integral form, interchanging the order of integration, and renaming the dummy variables of integration. This and rewriting $G_{c-d'}$ and $G_{e-f'}$ gives:

$$\begin{aligned}
 G_{c-f'} &= 1/4 [G_{(cdef)^2} + G_{c^2} + G_{d^2} + G_{e^2} \\
 &\quad + G_{f^2} - (G_{(cd)^2} + G_{(ce)^2} + G_{(df)^2} + G_{(ef)^2})]
 \end{aligned}
 \tag{5}$$

Returning to Fig. 1, write G_{1-9}' as:

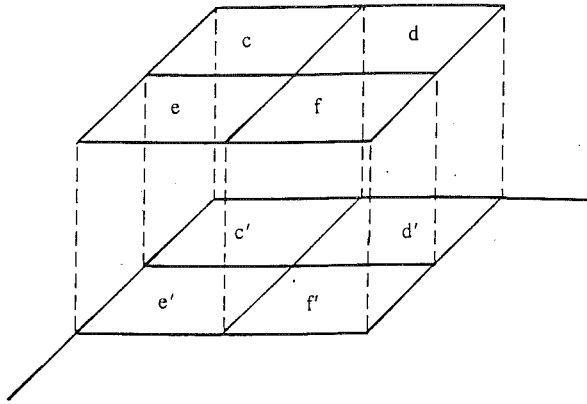


Fig. 3 Nonopposing parallel rectangles aligned on one corner

$$G_{1-9'} = G_{1-(4589)'} - G_{1-4'} - G_{1-5'} - G_{1-8'} \quad (6)$$

$G_{1-4'}$ and $G_{1-8'}$ can be rewritten:

$$G_{1-4'} = G_{1-(45)'} - G_{1-5'} \text{ and } G_{1-8'} = G_{1-(58)'} - G_{1-5'} \quad (7)$$

Use Eq. (5) to expand $G_{1-(4589)'}$, $G_{1-(45)'}$, $G_{1-5'}$, and $G_{1-(58)'}$ and combine Eqs. (6) and (7) to arrive at:

$$G_{1-9'} = 1/4 [G_{(123456789)^2} - (G_{(125678)^2} + G_{(234589)^2} + G_{(456789)^2} + G_{(123456)^2} + (G_{(1256)^2} + G_{(2345)^2} + G_{(4589)^2} + G_{(5678)^2} + G_{(456)^2} + G_{(258)^2}) - (G_{(25)^2} + G_{(45)^2} + G_{(58)^2} + G_{(56)^2} + G_{5^2})] \quad (8)$$

This equation is the same as that presented by Hamilton and Morgan except for the term $G_{5-5'}$, which was inadvertently omitted in the original paper. Equation (8) is sensitive to the values that are used for each term so it is recommended that graphs should not be used. Hsu's result is more general than

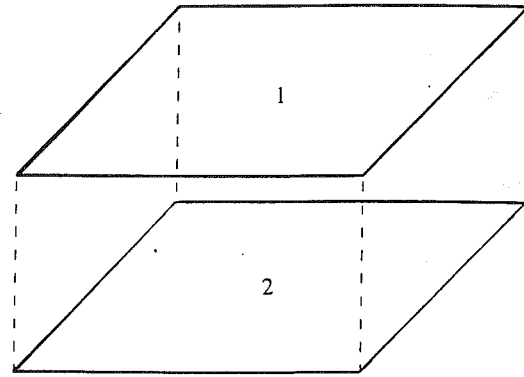


Fig. 4 Parallel opposing rectangles of the same size

Eq. (8) because it can also be used for rectangles that are directly opposed. Since the programming effort is similar for either method, Hsu's equation is recommended.

References

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