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Discussion

Natural Convection From L-Shaped Corners With Adiabatic and Cold Isothermal Horizontal Walls[1]

A. Bejan.[2] In this note I would like to show a way in which the two main conclusions of Angirasa and Mahajan could be anticipated qualitatively. The following argument is based on an earlier paper, which was apparently overlooked.

The second configuration studied by Angirasa and Mahajan, namely the cold horizontal wall joined with a hot vertical wall, was investigated earlier by Kimura dn Bejan (1985a). The corresponding natural convection flow in a corner filled with fluid saturated porous medium was documented in this very journal (Kimura and Bejan, 1985b).

Kimura and Bejan's (1985a) study contained a scaling theory of the high Rayleigh number regime, finite difference simulations that validated the theory, isothermal versus constant flux walls, and an asymptotic series analysis of the flow and heat transfer in the limit $\mathrm{Ra} \rightarrow 0$. The corner region model is reproduced here in Fig. 1. The scaling theory predicted that in $\mathrm{Pr} \geq 1$ fluids the total heat transfer rate q' [W/m] from the vertical wall to the horizontal wall is of the order of

$$\mathrm{Nu} \sim \left(\frac{L}{H}\right)^{3/7} \mathrm{Ra}_H^{1/7} \tag{1}$$

where $\mathrm{Nu} = q'/k(T_H - T_C)$ and $\mathrm{Ra}_H = g\beta(T_H - T_C)H^3/\alpha\nu$. The finite-difference calculations showed that Eq. (1) is correct (as expected, within a factor of order 1, see Fig. 7 of Kimura and Bejan, 1985a).

One high Ra_H implication of Eq. (1) is that in the corner of Fig. 1 the heat transfer rate is *negligible* when compared with the heat transfer rate that occurs when the same wall (H, T_H) is immersed in a cold fluid reservoir maintained at T_c. The heat transfer rate in this second case can be estimated based on scale analysis (e.g., Bejan, 1984),

$$\mathrm{Nu} \sim \mathrm{Ra}_H^{1/4} \tag{2}$$

For example, when $\mathrm{Ra}_H \sim 10^8$ and $L/H \sim 1$, Eq. (1) yields $\mathrm{Nu} \sim 10$ and Eq. (2) yields $\mathrm{Nu} \sim 10^2$. The gap between the two Nu estimates widens as Ra_H increases. In conclusion, the natural convection thermal resistance between a vertical wall and a cold reservoir is much smaller than the resistance between the same wall and a cold horizontal wall attached as in Fig. 1.

Equations (1) and (2) are relevant as a review of the existing work, and because they can be used to anticipate Angirasa and Mahajan's conclusion regarding the vertical wall perpendicular to a cold isothermal wall. Their numerical results for $10^5 < \mathrm{Ra}_H < 10^9$ and $0.7 < \mathrm{Pr} < 7$ were correlated by the power law

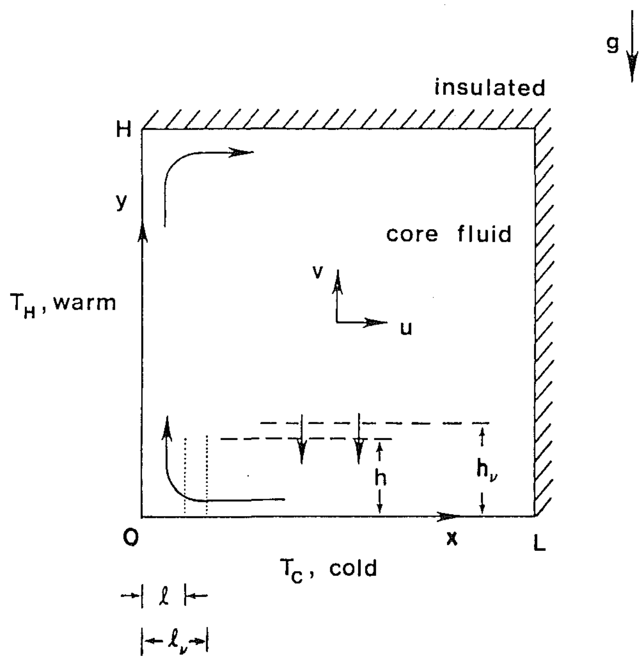

Fig. 1 Natural convection and boundary layer structure in the corner region bounded by a hot vertical wall and a cold horizontal wall (Kimura and Bejan, 1985a)

$$\mathrm{Nu} = 0.833\ \mathrm{Ra}_H^{0.221} \tag{3}$$

which, numerically, turns out to be very close to the classical result for the vertical wall immersed in a cold isothermal reservoir, $\mathrm{Nu} = 0.671\ \mathrm{Ra}_H^{1/4}$ if $\mathrm{Pr} > 1$ (e.g., Bejan, 1984, p. 129).

Why then does Eq. (3) look a lot like Eq. (2), and not like Eq. (1)? The answer is that in Angirasa and Majahan's numerical work the horizontal wall was set isothermal and cold *in addition* to specifying that the fluid pool is isothermal and cold. In other words, the authors' heat transfer rate could sink simultaneously into two heat sinks, the cold reservoir and the cold horizontal wall. The shape of Eq. (3) shows that the heat transfer prefers to sink into the cold reservoir, which must mean that the path of least resistance is from the vertical wall to the cold reservoir. As pointed out above, this finding should be expected based on a comparison of Eqs. (1) and (2), coupled with the observation that Ra_H is high.

All this can be summarized by saying that when the vertical wall is bathed (with high Ra_H) by a reservoir that is maintained cold, the presence of an isothermal cold wall at $y = 0$ has a negligible effect on the total heat released by the vertical wall. If this effect is negligible when the horizontal wall is thermally active (cold), then it must certainly be negligible when the horizontal attachment is inactive (adiabatic). In this way we

[1] By D. Angirasa and R. L. Mahajan, published in the February 1993 issue of the ASME JOURNAL OF HEAT TRANSFER, Vol. 115, pp. 149–157.

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recover Angirasa and Mahajan's conclusion to their first problem, in which the vertical wall was attached to an adiabatic wall.

References

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Kimura, S., and Bejan, A., 1985a, "Natural Convection in a Differentially Heated Corner Region," *The Physics of Fluids*, Vol. 28, pp. 2980–2989.

Kimura, S., and Bejan, A., 1985b, "Natural Convection in a Stably Heated Corner Filled With Porous Medium," ASME JOURNAL OF HEAT TRANSFER, Vol. 107, pp. 293–298.

Authors' Closure

We thank Professor Bejan for his thoughtful comments on our work. We agree with him that it is possible to obtain qualitative understanding of the effects studied from scaling theory. The objective of our paper was to provide quantitative results for an important configuration and describe the physics underlying these observed effects. His comment on heat transfer following the path of least resistance in a corner is, in general, valid for all heat transfer processes in different configurations as well.