Role of Particle-Hole Interaction in the Four-Particle-Four-Hole States in $^{16}$O

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Role of the particle-hole interaction in the four-particle-four-hole states in $^{16}$O is investigated with relation to the alpha-cluster-like spatial correlations by using the vertically-truncated-subspace shell model which was proposed by Kamimura, Matsuse and one of the present authors (K.T.). It is concluded that the coupling between the two kinds of mode (spatially correlated four-particle mode and four-hole mode) is nearly independent of the spatial localization of the four particles outside the $^{16}$O core and the particle-hole interaction does not strongly mix the states of the two "rotors" (the ground band of $^{20}$Ne and that of $^{12}$C).

§ 1. Introduction

Various approaches have been tried in order to understand particle-hole states in the sd-shell nuclei. Particularly, the phenomenological analysis with the use of the weak-coupling model has suggested that the so-called "mysterious" $0^+ (6.05\,\text{MeV})$ state in $^{16}$O is a weakly coupled state of strongly correlated four particles outside the $^{16}$O core and four holes in the core. A series of the low-lying excited ($T=0$) states in $^{16}$O can, therefore, be understood as states analogous to the ground bands of $^{20}$Ne and $^{12}$C. It appears that the shell model calculation considering four particles outside the inert $^{12}$C core supports the above viewpoint.

On the other hand, the alpha-cluster model has been successfully applied to this region of nuclei and the importance of the alpha-cluster-like four-body correlations has been well recognized. Recently, Suzuki and Ikeda have applied the orthogonality-condition model to study the excitation mechanism of $^{16}$O on the viewpoint of the alpha-cluster model, and shown that the structure of the $T=0$ excited states in $^{16}$O is qualitatively understood by using a simple coupling scheme of $^{12}$C + $\alpha$.

One of the important aspects of the alpha-cluster-like four-body correlations is the spatial correlations, or the spatial localization, of four particles outside the core. Kamimura, Matsuse and one of the present authors (K.T.) have proposed the vertically-truncated-subspace shell model which is useful for treating the effects of the spatial correlations.

In the present article, we investigate a role of the particle-hole interaction between the spatially-correlated-four-particle mode outside the $^{16}$O core and four-hole mode inside the core using the vertically-truncated-subspace shell model.
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We, furthermore, discuss the usefulness of the weak-coupling model. In § 2, a formalism to handle the four-particle-four-hole states in \(^{16}\)O is presented.

In § 3, the calculated results are shown and discussed in connection with the weak-coupling model. In § 5, concluding remarks are given.

\section*{2. Formalism for four-particle-four-hole states in \(^{16}\)O}

The total Hamiltonian describing the particle-hole states in \(^{16}\)O is expressed as

\[ H = H_p + H_h + V_{ph}, \]

where \(H_p\) refers to the particle states unoccupied in the \(^{16}\)O core, \(H_h\) the hole states and \(V_{ph}\) is the particle-hole interaction.

First, we construct spatially correlated four-particle modes outside the \(^{16}\)O core and four-hole modes in the core in the same manner as in the paper of Kamimura, Matsuse and one of the authors (K.T.),\(^{7}\) (hereafter referred to as I). The four-particle mode with angular momentum \(LM\), spin \(S=0\) and isospin \(T=0\) is

\[ A_p^\dagger(LM, S=T=0) = \frac{1}{\sqrt{4!}} \int \psi_{pLM}(r_1 r_2 r_3 r_4) X_{S=T=0}(1234) \]

\[ \times \phi^\dagger(x_1) \phi^\dagger(x_2) \phi^\dagger(x_3) \phi^\dagger(x_4) (\prod_{i=1}^{4} dx_i), \]

\[ \psi_{pLM}(r_1 r_2 r_3 r_4) = \int \langle r_1 r_2 r_3 r_4 | Q_{p} | r_1' r_2' r_3' r_4' \rangle \phi_{pLM}(r_1' r_2' r_3' r_4') (\prod_{i=1}^{4} dr_i'), \]

where the following form of \(\phi_{pLM}\) is assumed in the vertically-truncated-subspace shell model proposed in I:

\[ \phi_{pLM}(r_1 r_2 r_3 r_4) = \int \exp \left\{ -\frac{\alpha^2}{2} \sum_{i=1}^{4} (r_i - R)^2 \right\} \psi_{pLM}(R) Y_{LM}(\Omega_R) dR. \]

Here the suffix \(p\) denotes a set of quantum numbers characterizing the mode except \(L, M, S\) and \(T\). The operator \(\phi^\dagger(x)\) is the field operator of nucleon and \(x\) stands for all variables describing one nucleon; \(x=(r, \sigma, \tau)\), and \(\phi_{pLM}\) is a generator-coordinate weight function.

Similarly we define the four-hole mode

\[ B_h^\dagger(LM, S=T=0) = \frac{1}{\sqrt{4!}} \int \psi_{hLM}(r_1 r_2 r_3 r_4) X_{S=T=0}(1234) \]

\[ \times \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) (\prod_{i=1}^{4} dx_i), \]

\[ \psi_{hLM}(r_1 r_2 r_3 r_4) = \int \langle r_1 r_2 r_3 r_4 | Q_h | r_1' r_2' r_3' r_4' \rangle \psi_{hLM}(r_1' r_2' r_3' r_4') (\prod_{i=1}^{4} dr_i'), \]

where
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\[ \psi_{hLM}(r_1, r_2, r_3, r_4) = \int \exp \left\{ -\frac{\alpha^2}{2} \sum_{i=1}^{4} (r_i - R)^2 \right\} \Phi_{hL}(R) Y_{LM}(\Omega_4) dR, \]  

(2·5)

and \( \Phi_{hL} \) is a generator-coordinate weight function for the hole states. The operator \( Q_{sp} \) (or \( Q_{sh} \)) appeared in Eq. (2·2b) (or Eq. (2·4b)) is the projection operator to the four-particle (or four-hole) states and the explicit form of its matrix element is shown in I.

The modes \( A^+_p(LM, S=T=0) \) and \( B^+_h(LM, S=T=0) \) are determined by diagonalizing the Hamiltonians \( H_p \) and \( H_h \) within the subspaces \{ \( A^+_p(LM, S=T=0) \) \} and \{ \( B^+_h(LM, S=T=0) \) \}, where \( |0\rangle \) denotes the \( ^{16}\text{O} \) core being assumed as the closed shell of \( 0s^{-4} \) and \( 0p^{-4} \) orbit.

The wave function \( \psi_{pLM}(r_1, r_2, r_3, r_4) \) of the outer four particles is composed of many shell-model-single-particle states, keeping persistently the \([4]\)-symmetry. It depends on the “size” parameter \( \alpha^2 \) how many states are involved in it, and is determined by the generator-coordinate weight function \( \Phi_{pL}(R) \). On the other hand, the possible number of independent states involved in the wave function \( \psi_{hLM}(r_1, r_2, r_3, r_4) \) of the four holes is three \((0s)^{-4} \), \((0s)^{-2}(0p)^{-2}[4] \), \((0p)^{-2}[4] \) for \( L=0 \), two \((0s)^{-2}(0p)^{-2}[4] \), \((0p)^{-4}[4] \) for \( L=2 \) and one \((0p)^{-4}[4] \) for \( L=4 \). Therefore, the parameter \( \alpha^2 \) in Eq. (2·5) has no physical meaning and the resultant four-hole mode never depends on the choice of the value \( \alpha^2 \).

Using the above-mentioned four-particle and four-hole modes, we can compose the basis vectors of the four-particle-four-hole states under consideration in \( ^{16}\text{O} \) as

\[ |\rho_L, hL'; IK\rangle = [A^+_p(LM, S=T=0) \cdot B^+_h(L'M', S'=T'=0)]_{IK}|0\rangle. \]  

(2·6)

Diagonalizing the Hamiltonian (2·1) within the space spanned by the basis vectors (2·6), we obtain the four-particle-four-hole eigenstates in \( ^{16}\text{O} \).

Now, consider the weak-coupling model in which the states consist of weakly coupled ones of the ground bands of \( ^{20}\text{Ne} \) and \( ^{12}\text{C} \). The states in the ground band of \( ^{20}\text{Ne} \) are written as

\[ |^{20}\text{Ne}, LM\rangle = A^+_{LM}|0\rangle, \quad L = 0, 2, 4, \ldots, \]  

(2·7)

where \( A^+_{LM} \) is the four-particle eigenmode with the lowest energy among various modes \( A^+_p(LM, S=T=0) \) for each angular momentum \( LM \). We assume the ground band of \( ^{12}\text{C} \) to be \((0p)^{-4}[4]\) \((04)-states and write the corresponding four-hole mode as \( B^+_{LM} \). Then the states in the ground band of \( ^{12}\text{C} \) are represented as

\[ |^{12}\text{C}, LM\rangle = B^+_{LM}|0\rangle, \quad L = 0, 2, 4. \]  

(2·8)

The basis vectors in the weak-coupling model are, therefore, written as

\[ |L, L'; IK\rangle = [A^+_{LM} \cdot B^+_{LM'}]_{IK}|0\rangle. \]  

(2·9)

In order to see whether the picture of the weak-coupling model is good or not, we should diagonalize the Hamiltonian (2·1) within the space spanned by the basis vectors (2·9).
§ 3. Calculated results and discussion

3.1 Single-particle Hamiltonian and residual interaction

As the shell-model-single-particle (hole) Hamiltonian involved in \( H_p(H_h) \), we assume the harmonic oscillator Hamiltonian with the oscillator constant \( \beta^2 = m\omega_0/h = 0.362 \text{ fm}^{-2} \) corresponding to \( \hbar \omega_0 = 15.0 \text{ MeV} \). Now, it should be noticed that one-body-spin-orbit potential has never any contribution because of the property of the spin and isospin wave function \( X_{S=T=0}(1234) \).

For the residual interactions (the particle-particle interaction in \( H_p \), the hole-hole interaction in \( H_h \) and the particle-hole interaction \( V_{ph} \)), a Gaussian interaction is assumed:

\[
V(r) = -(V^{13}P_{13} + V^{21}P_{21} + V^{31}P_{31} + V^{33}P_{33}) \exp(-\mu r^2),
\]

where \( P_{ST+1,S'T+1} \) is the projection operator to the state of spin \( S \) and isospin \( T \) and \( V^{ST+1,S'T+1} \) stands for the strength of the interaction in the state with the quantum numbers \( T \) and \( S \). Because of the wave function \( X_{S=T=0}(1234) \), the residual interactions become

\[
V_{pp \text{ or } hh}(r) = -V_0^{(p) \text{ or } (h)} \exp(-\mu r^2),
\]

\[
V_{ph}(r) = -V_0^{(ph)} \exp(-\mu r^2),
\]

where

\[
V_0^{(p) \text{ or } (h)} = \frac{1}{2} (V^{13} + V^{33}),
\]

\[
V_0^{(ph)} = \frac{1}{16} (V^{13} + 3V^{21} + 3V^{31} + 9V^{33}).
\]

We treat the interaction strengths \( V_0^{(p) \text{ or } (h)} \) and \( V_0^{(ph)} \) as two independent parameters. We assume the same value of the range parameter

\[
\lambda = \sqrt{\frac{1}{2} \frac{\beta^2}{\mu}} = 0.7
\]

as in I, which was used in the shell-model calculations of \( sd \)-shell nuclei.\(^8\)

3.2 Free excitation energy of the first excited four-particle-four-hole state in \( ^{16}O \)

Let us now define a quantity \( \Delta \) which corresponds to the excitation energy of the first excited four-particle-four-hole state in \( ^{16}O \) when the particle-hole interaction is neglected:

\[
-\Delta = E_0(^{20}Ne) + E_0(^{12}C) - 2E_0(^{16}O),
\]

where, for instance, \( E_0(^{16}O) \) denotes the ground-state energy of \( ^{16}O \). The empirical value of \( \Delta \) can be estimated from the binding energies of \( ^{20}Ne, ^{12}C \) and \( ^{16}O \) and it is \( \sim 1.3 \text{ MeV} \).\(^8\)

\(^8\) Since the Coulomb interaction is neglected throughout this paper, the Coulomb correction is included in this value, while the experimental value without the correction is 2.43 MeV.
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If we want to reproduce the binding energy and the levels in the groundband of $^{20}\text{Ne}$ within the framework of the sd-shell model, we need a rather strong effective interaction, i.e., $V_0^{(p)} \approx 50 \text{ MeV}$. If the same value is used for $V_0^{(n)}$, the binding energy of $^{12}\text{C}$ becomes too large and the quantity $\Delta$ becomes negative ($\approx -17 \text{ MeV}$).

It was, however, shown in I that, taking the spatial correlations into account, we can reproduce both the binding energy and the levels of $^{20}\text{Ne}$ by using a considerably weak interaction, i.e., $V_0^{(p)} \approx 35 \text{ MeV}$. Using the same strength for $V_0^{(n)}$, we get a result $\Delta \approx 1.04 \text{ MeV}$, which shows a good agreement with corresponding empirical value $\approx 1.3 \text{ MeV}$. Therefore, it is very reasonable to take account of the spatial correlations in the four particles outside the $^{16}\text{O}$ core in order to understand the structure of $^{20}\text{Ne}$, $^{12}\text{C}$ and the four-particle-four-hole states in $^{16}\text{O}$ from a unified viewpoint.

3.3 Effect of the spatial correlations on the coupling between four-particle mode and four-hole mode

For the purpose of studying the effect of the spatial correlations on the coupling between four-particle mode and four-hole mode, we diagonalized the Hamiltonian (2.1) within the space spanned by the basis vectors

$$|pL, L'=0; LM\rangle = [A_p^+ (LM, S=T=0) \cdot B_{L'=0}^{M'=0}] |0\rangle,$$  

(3.6)

where the angular momentum of the four-hole mode is fixed to zero for the sake of simplicity. Therefore, the four-hole state in the basis vectors (3.6) is $(0p)^{-4}[4]L=0$-state, as mentioned in the previous section. Calculations were carried out for various values of the “size” parameter $\alpha^2$ of the four particles outside the core. The strength of the particle-hole interaction was taken to be $V_{ph}^{(p)} = 2.165 \text{ MeV}$ so as to reproduce nearly the excitation energy 6.05 MeV of the first $0^+$ state at the minimum point $\alpha^2 = 1.558^2$.

The calculated results are shown in Fig. 1. In this figure, the solid lines (which indicate the excitation energies of the four-particle-four-hole eigenstates including the particle-hole interaction) are almost parallel to the dashed lines (not including the particle-hole interaction). This means that the coupling between the four-particle mode and the four-hole mode is nearly independent of the parameter $\alpha^2$ which is a measure of the spatial localization of the four particles outside the core. This is more apparent seen in Fig. 2, where the expectation values of $V_{ph}$ with respect to the calculated four-particle-four-hole eigenstates are shown for the cases of $L=0$ and $L=2$ for example. These expectation values are almost independent of both $\alpha^2$ and $L$. From these facts, it is concluded that the coupling between the two kinds of mode does not depend on the degree of spatial localization of the outer four particles.

*) In Ref. 8, Akiyama, Arima and Sebe used an interaction with $V_{ph}^{(p)} = 52.5 \text{ MeV}$ and obtained good agreements with experiments of $^{20}\text{Ne}$. 
One may ask that, if a single-particle potential with finite depth is used instead of the harmonic oscillator potential, a situation different from the above-mentioned may occur. To answer this question, we performed a similar calculation using the finite-well potential used in a previous paper. However, the calculated results hardly change the above-stated conclusion.

It was consequently clarified that the particle-hole interaction is nearly diagonal in the sense that the level structure of the four-particle-four-hole states hardly change whether the particle-hole interaction is taken into account or not. The condition under which the weak-coupling model works well is that the nondiagonal matrix elements of the mode-mode interaction are much smaller than the difference between the diagonal energies. This condition is satisfied by the mode-mode interaction under consideration.

3.4 Weak coupling between the two "rotors"

We diagonalized the Hamiltonian (2.1) within the space spanned by the basis vectors (2.9) in order to investigate the role of the particle-hole interaction in the coupling of the two "rotors" i.e., the ground "rotational" bands of $^{20}$Ne and $^{12}$C. The two "rotors" $A_{1/2}^M$ and $B_{1/2}^N$ are determined by the Hamiltonian $H_p$ and $H_h$ respectively, and their energy levels are shown in Fig. 3. The structure of $A_{1/2}^M$ was investigated in detail in I. Energy levels of the ground band of $^{20}$Ne are well reproduced by our model but those of $^{12}$C are not because of the severe restriction of the model space (only $(0p)^{-1}[4](04)$-states). Therefore, instead of the energy eigenvalues of the "rotor" $B_{1/2}^N$, we used the experimental
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Fig. 3. The energy levels of the two "rotors" are compared with the experimental ones of the ground band of $^{12}$C and that of $^{20}$Ne.

Fig. 4. Dependence of the energies of the coupled-two-rotor eigenstates on the strength $V_0^{(ph)}$.

values of excitation energies of $^{12}$C.

The dependence of the energies of the coupled-two-rotor eigenstates on the strength $V_0^{(ph)}$ is shown in Fig. 4. It is seen from this figure that the energy splittings induced by the particle-hole interaction are small compared with the increase of the absolute values of the energies. This means that the particle-hole interaction does not strongly mix the coupled states of the two "rotors". Consequently, levels analogous to the ground bands of $^{20}$Ne and $^{12}$C appear starting from the first $0^+$ (6.05 MeV) states. In Fig. 5, we show the energy levels of $^{16}$O calculated with $V_0^{(ph)} = 2.165$ MeV. The mixing coefficients involved in these coupled-two-rotor eigenstates are represented in Table I.

Fig. 5. Level structures of four-particle-four-hole states with $T=0$ in $^{16}$O calculated with $V_0^{(ph)} = 0.0$ MeV and $V_0^{(ph)} = 2.165$ MeV, are compared with the experimental one. The symbols $(L \times L')$ on the left-hand side denote the angular momenta of the two "rotors", where $L$ belongs to $^{20}$Ne "rotor" and $L'$ to $^{12}$C.
Table I. Mixing coefficients in the coupled-two-rotor eigenstates calculated with $V^{(6)}_{\text{os}} = 2.165$ MeV. The values in the second column are the excitation energies of these states. The symbol $(L, L')$ denotes the angular momenta of the two "rotors", where $L$ belongs to $^{20}\text{Ne} "rotor"$ and $L'$ to $^{12}\text{C}$. The states higher than the first $8^+$ state are omitted.

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§ 4. Concluding remarks

We have investigated the role of the particle-hole interaction in the four-particle-four-hole states in $^{16}O$ with relation to the alpha-cluster-like spatial correlations. We are interested in the following two points. One is the dependence of the coupling between the four-particle mode and the four-hole mode on the "size" parameter $\alpha^2$ of the four particles outside the $^{16}O$ core. The other is the role of the particle-hole interaction in the coupling of the two "rotors", i.e., the ground band of $^{20}Ne$ and that of $^{12}C$. The results of the present calculations are summarized as follows:

(1) The coupling between the two modes is nearly independent of the spatial localization of the four particles outside the core in contrast with the particle-particle interaction. In the other words, the particle-hole interaction does not change the value of the "size" parameter $\alpha^2$ which minimizes the energy of the four-particle mode outside the core. Then we can safely use the same value of the parameter $\alpha^2$ in treating $^{20}Ne$ and $^{16}O$.

(2) The particle-hole interaction does not strongly mix the coupled-two-rotor states. Consequently, the bands which closely resemble the ground bands of $^{20}Ne$ and $^{12}C$ appear starting from the first excited $0^+$ state in $^{16}O$.

These results support that the nondiagonal matrix elements of the mode-mode coupling are small and the picture of the weak-coupling model is well realized in our model space. This conclusion seems to be almost independent of the detailed properties of the particle-hole interaction.

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References

5) Prog. Theor. Phys. Suppl. No. 52 (1972), "Alpha-Like Four-Body Correlations and Mole-
   cular Aspects in Nuclei".