present writer have independently taken up Bolotin's suggestion and constructed both continuos and discrete systems which utilize jet reaction forces to obtain nonconservative loadings on elastic systems. For a two-degree-of-freedom model of the tangentially loaded column, Walter measured a critical load of 0.5 lb which compared favorably with a predicted critical load of 0.52 lb. The observed instability, of the expected "flutter" type, was repeatedly initiated without any more disturbance than that due to opening the valve on a compressed gas bottle. No attempt will be made to publish the experimental results of Walter and the present writer because it is believed that Herrmann and his group have priority.

Author's Closure

The author would like to thank Professors Augusti, Levinson, and Roco and Herrmann for their valuable comments.

Although the experiment differs from Beck's problem, it must be realized that the location of the point of intersection results in identical boundary conditions. The main difference between the two systems is due to the fact that values for the second mode frequency for varying values of \( k \) differ. This was pointed out by Professors Huang, Nachbar, and Nemat-Nasser of the University of California, San Diego. By distorting the tip of the column only, an effort was made to stay in the fundamental mode. The mode of failure from the experiment appeared to be in the second mode. This confirms that for values of \( k \) larger than 1.5, such failure must be expected.

**Diffraction of Horizontal Shear Waves by a Parabolic Cylinder and Dynamic Stress Concentrations**

B. Rulf, 2 I was surprised to find in the December, 1966, issue of the *Journal of Applied Mechanics* a long paper devoted to the problem of diffraction of a plane harmonic scalar wave by a parabolic cylinder. This problem has been thoroughly investigated by many authors; some of the most important of them [1, 2, 3] have not been quoted in the foregoing paper. Even though the problem has previously been solved for the diffraction of electromagnetic waves, the authors themselves have pointed out that diffraction of scalar electromagnetic waves is mathematically completely analogous to diffraction of SH-waves in elasto-dynamics. Thus, I was unable to find in their paper new results which have been hitherto unknown.

For example, the singularity of the solution of the half-plane diffraction problem at \( r = 0 \) has been discussed at length in the literature [4]. The authors' finding that the stresses at the tip of a line crack or a rigid rib behave like \( (kr)^{-1/2} \) is not unexpected. This paper does not even summarize the state-of-the-art in this problem. The authors mention only the series solution which was first derived by Epstein in 1914. This solution is useless for the high-frequency short-wavelength limit, since it converges extremely slowly in that region. Apparently because of this, the authors give merely the geometrical-optics result, which is the lowest-order approximation of the solution, and cannot account for the nonzero field in the shadow region. A different series representation, which converges rapidly at high frequencies, can be obtained from the Epstein series by means of a Watson transform [1].

More generally, the exact solution can be given by a contour integral, from which we can obtain both series representations by residue evaluations. For the case of plane-wave diffraction, this integral is given by [3]

\[
w = \frac{1}{2\sqrt{2\pi}} \int \frac{e^{i \frac{1}{2} \pi} \cos \theta/2}{\sin \theta/2} \frac{d\psi}{\partial \xi} D_{\nu}(-\eta) + A_{1}(\nu) D_{\nu}(-\eta) D_{\nu}(-\ell \xi) \quad (1)
\]

where

\[
A_{1}(\nu) = \frac{D_{\nu}(-\eta \nu)}{D_{\nu}(\eta \nu)}
\]

and

\[
A_{2}(\nu) = \frac{D_{\nu}(-\nu \eta)}{D_{\nu}(\nu \eta)}
\]

The coefficients \( A_{1}(\nu) \) or \( A_{2}(\nu) \) are to be used for the boundary conditions \( \psi(\xi, \eta) = 0 \) or \( \partial \psi(\xi, \eta)/\partial \eta = 0 \), respectively. (The notation here is the same as in the paper I am commenting on.) It is easy to verify that the residues of the integral (1) at the points where \( \sin \theta/2 = 0 \) yield exactly the Epstein series. If, however, the path of integration is deformed around the poles of \( A_{1}(\nu) \) which are located at the points where \( D_{\nu}(\eta \nu) = 0 \) or \( D_{\nu}(\nu \eta)/\partial \eta = 0 \), respectively, an alternative solution is obtained.

The authors' finding that the stresses at the tip of a parabolic cylinder have not been quoted in the foregoing paper. Even though the technique has been discussed in references [1, 2, 3] and given a unified interpretation by Keller [3], the solution (1) is valid for angles of incidence in the range \( 0 \leq \theta < \pi \) and does not display the divergence difficulty at \( \theta = \pi/2 \) mentioned by the authors.

Some works in elastodynamics may not be very familiar with the existing literature on diffraction theory, since such results appear mainly in periodicals dealing with electromagnetic waves or applied mathematics. Therefore, it may be useful to publish occasionally review papers by acknowledged authorities on this subject in periodicals like the *Journal of Applied Mechanics*. However, the paper under discussion is, in my opinion, neither such a review nor does it bring any essentially new research results.

**References**

4. Reference [3], Chapter 9; this work contains many additional references.

**Authors' Closure**

The authors welcome a critical discussion of their work. Unfortunately, all of Mr. Rulf's criticisms are either inappropriate or in error. His main objections are: (a) The problem has been thoroughly investigated by others in the field of electromagnetics and we failed to review all of the literature related to the subject. (b) We present no new results. (c) The \( (kr)^{-1/2} \) singular behavior of stresses at a sharp edge is not unexpected. (d) Our high-frequency asymptotic results contain only the lowest-order approximation. Below is our answer.

First, we point out that, while some electromagnetic wave problems are mathematically analogous to elastic SH-wave problems, the information sought is often quite different. Furthermore, studies of scalar wave propagation problems cast in terms of elastic SH-waves are important for gaining physical insight into phenomena connected with the more complicated, vector-scalar, elastic wave boundary-value problems which have no counterpart in electromagnetics.
The title, introduction, and extensive amount of numerical results of our paper clearly show that we were interested in applying the solutions of diffraction of scalar, harmonic SH-waves for calculating dynamic stress concentrations near the base of a parabolic cylinder and at the tip of a crack and rigid ribbon. This being the objective, we adequately summarized the literature on dynamic stress concentrations as pertained to our problem. The mathematical solutions of the wave equation in parabolic coordinates were provided by Epstein in 1914, Lamb in 1907, and in polar coordinates by Sommerfeld in 1896, and their works were properly referenced in the paper. The recent works mentioned by the discusser, while of interest in electromagnetics for diffraction of high-frequency, short waves, do not add any new information that was needed in our investigation and thus they were not listed. Moreover, none of the previous “thorough investigations” contains numerical results analogous to ours. For the parabolic cylinder with nonzero focal length, we believe our results are new, especially since there are no available tables of Weber functions with argument \((1±i)x\) \((x\ \text{real})\) and negative \(v\). The results of our paper clearly show that we were interested in applying the solutions of the wave equation in parabolic coordinates were provided by Epstein in 1914, Lamb in 1907, and in polar coordinates by Sommerfeld in 1896, and their works were properly referenced in the paper. The recent works mentioned by the discusser, while of interest in electromagnetics for diffraction of high-frequency, short waves, do not add any new information that was needed in our investigation and thus they were not listed. Moreover, none of the previous “thorough investigations” contains numerical results analogous to ours. For the parabolic cylinder with nonzero focal length, we believe our results are new, especially since there are no available tables of Weber functions with argument \((1±i)x\) \((x\ \text{real})\) and negative \(v\). The final factor in the integrand of equation (1) should be \(D_0(\lambda \xi)\), the order of the Weber functions in equations (2) and (3) should be \(-v-1\), and the sign of \(A_k(\xi)\) should be negative.

It was not our aim to present in detail the complete, high-frequency, asymptotic solution to the problem as done in the references cited by the discusser. First, in many applications of dynamical stress analysis, the wavelengths of disturbances are comparable to, or longer than, the characteristic dimensions of the structure. Often, the low-frequency, long-wavelength limit is more important since this is the case of static loading. We devoted considerable attention to analysis and interpretation of the static limits in the solutions. This, together with our discussion on the kinetic boundary conditions, cannot be found in the previous “thorough investigations.” Furthermore, we found that the series solution yielded with sufficient accuracy the high-frequency limits given in the paper. Contrary to the statements in the discussion, “Apparently because of this, the authors . . . cannot account for the nonzero field in the shadow region,” the high-frequency limit in the shadow region along a parabolic cylinder is indeed zero.

On the matter of the singular behavior of stress at the root of a crack and rigid ribbon, we are not concerned with what people expect or do not expect. We established rigorously, by considering the zero focal-length limiting cases of a rigid or cavity parabolic cylinder, that one of the stress components remains finite while the other became infinite of the order \((kr)^{-1}\). This singular behavior has been discussed before in dynamic elasticity, but not by considering the limit from a smooth object to one with an edge. Moreover, workers in stress analysis are not satisfied just with an order-of-magnitude answer; they also need to know the coefficients of the singular terms. It would have been useful if the discusser had contributed a better method than the one we used for calculating the stress intensity factors, instead of mentioning irrelevant references.

Finally, we advise the discusser to confine his criticism to the future to the contents of the paper. We tolerate his mock surprise, sarcasm, provincial viewpoint, and his authoritative-sounding, but incorrect comments, but we object strongly to his sounding remarks on the field of elastodynamics and the journal in which we chose to publish our research work.

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Hydrodynamic Gyroscope

R. A. Milroy.

In December, 1959, I completed an electrical circuit by passing current through ball bearings just as Mr. Then did in building his hydrodynamic gyroscope. Obviously, Mr. Then was as unaware of the motoring effect generated by passing current through ball bearings as I was. Fortunately, I was working on a smaller mechanism where the effect was more pronounced; however, there are two ways I can read Mr. Then’s statement, “The whole assembly is remarkably free of friction about the vertical and horizontal axes, spinning completely around.” Mine did spin around at nearly 1000 rpm with nothing driving it but the electrical current passing through the ball bearings.

A ball-bearing motor can be easily constructed by placing two bearings on a conductive shaft and passing current into the outer race of one, through the balls to the inner race, down the shaft to the inner race of the other bearing, through the balls, and out of the outer race. The motor requires practically no voltage but rather high current and will run in either direction on a-c or d-c current, Fig. 1.

It took me a long time to find where anyone used this effect. A recent article indicated that railroad cars can be accelerated 1 m/sec by passing current through their wheels. Whether Mr. Then’s results were adversely affected or not I do not know, but the effect can be significant.

Author’s Closure

Mr. R. A. Milroy’s discussion presents a striking experimental fact. Needless to say, I was unaware of such a motor effect. If there was a tendency for the gyroscope to rotate due to the foregoing effect, it is very likely that it would be masked by the gyroscopic reaction moment. It is possible that this effect accounted for some of the friction that was attributed to air friction.


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Lubrication of a Porous Bearing—Reynolds’ Solution

W. T. ROULEAU.

This paper is a valuable contribution to the theory of hydrodynamically lubricated porous bearings because it considers the limiting case of an infinitely long full bearing, which has not heretofore been discussed. The companion paper


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