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\[ e^+e^- \rightarrow \text{Hadrons Total Cross Sections and the Parton-Antiparton Interactions} \]

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The rise of the factor \( R = \sigma(e^+e^- \rightarrow \text{hadrons})/(4\pi\alpha^2/3s) \) for energies available in recent experiments is explained within the framework of the parton field theory. Assuming that the electromagnetic current couples minimally to the parton fields, we find that contributions to \( \sigma(e^+e^- \rightarrow \text{hadrons}) \), due to the parton-antiparton interactions, dominate over the so-called light-cone contribution, at least in the energy region available at present. We show within the pole approximation for partons that the factor \( R \) is bounded by 5 times ordinary quark-parton predictions for \( R \). Experimental data on \( R \) is well reproduced by a suitable choice of the parton-antiparton interaction term. A comment is directed toward a possible connection of the absence of isolated free partons to the "sercitinous" scaling in the process \( e^+e^- \rightarrow \text{hadrons} \).

§ 1. Introduction and summary

Recent progress in \( e^+e^- \) colliding beam machines made it possible to perform \( e^+e^- \rightarrow \text{hadrons} \) experiments at higher energies. Experimental data on the factor \( R = \sigma(e^+e^- \rightarrow \text{hadrons})/(4\pi\alpha^2/3s) \) obtained at several laboratories exhibit the trend of an impressive rise as the centre-of-mass total energy grows from 2 to 5 GeV. This experimental trend suggests anomalous behaviour of the imaginary part of the photon propagator when the one-photon approximation in the process \( e^+e^- \rightarrow \text{hadrons} \) is assumed to be valid. In particular it is recognized that most of the quark-parton type models with simple light-cone dominance fail to explain the present experimental results. There would be many different viewpoints to understand the failure of the quark-parton predictions in the process: (1) We are not yet in the asymptotic region where the light-cone dominance is valid; (2) some unknown leptonic interaction shows up in the present energy region; (3) unknown particles including charmed constituents of hadrons are produced; and so on. We are rather close to the viewpoint (1) in the present paper: The contribution of the parton-antiparton interaction as shown in Fig. 1(b) to the cross section \( \sigma(e^+e^- \rightarrow \text{hadrons}) \) may dominate over the light-cone contribution of Fig. 1(a) at least in the present experimental energy region. We shall prove in this paper that the above possibility is in fact the

* Preliminary results of the present paper in the case of the scalar partons were already reported by T. Muta.
Our assumptions to start with are the existence of the bare constituents (partons) of hadrons and the general framework of quantum field theory. Our main results are Eqs. (2.18) and (3.19) with which we can accommodate the present experimental data satisfactorily if a suitable model is chosen from the quark-parton type models. These equations then set unitarity bounds for $R$. The results are quite general except for the pole approximation for the partons made in the course of the calculation.

Since in our formulation the contribution due to the parton-antiparton interaction dominates over the light-cone contribution even at reasonably high energies, an approach to the scaling limit may be very slow and is from above in conformity with the result obtained in asymptotically free gauge field theories. This slow approach to the scaling limit (if it exists) in the time-like region may be called a "serotinous" scaling. We shall show that this serotinous scaling can be clearly explained if the absence of the isolated free partons is assumed.

In §2 our formulation is illustrated in the case of scalar partons. The formulation is then applied to a more realistic case of spinor partons and the results obtained are compared with experimental data in §3. A comment on the possible connection of the absence of isolated free partons with the serotinous scaling in the process $e^+e^\rightarrow$-hadrons is given in §4. Some technical problems related to the parton pole approximation are discussed in Appendices A and B.

§2. Scalar partons

We first consider the simplest case of scalar partons in order to explain our formulation of the parton-field theory in the process $e^+e^\rightarrow$-hadrons. A more realistic case of spinor partons will be dealt with in the next section. For simplicity we consider only one kind of the charged scalar parton $\phi$ with charge $eQ$. Generalization to include several kinds of partons is straightforward. As was mentioned in §1 we only assume the existence of bare partons and the general framework of quantum field theory. The electromagnetic current $j_\mu(x)$ is then expressed by the bare parton field $\phi(x)$ in the following form:

$$j_\mu(x) = -iQ\phi^+(x)\bar{\phi}(x) - eQ^2\phi^+(x)\phi(x)A_\mu(x),\quad (2.1)$$

where $A_\mu(x)$ is the electromagnetic field. The $e^+e^\rightarrow$-hadrons total cross section
The process $e^+e^-\rightarrow \text{hadrons}$ in the one-photon approximation.

\[ \sigma = (16\pi^2\alpha^2/s) \text{Im} \pi(s), \quad s=k^2, \quad (2.2) \]

\[ (k_\mu k_- - k^2 g_\mu) \pi(k^2) = i \int dxe^{ik_\mu x} \langle 0| T^* [j_\mu(x)j_\nu(0)] |0\rangle = \pi_{\mu\nu}(k), \quad (2.3) \]

where, though $j_\nu$ is hermitian, we preserve the symbol $\dagger$ on $j_\nu(0)$ in order to stress for later convenience that there is a hermitian conjugate of $j_\nu(0)$. The relevant amplitude in the above calculation is shown in Fig. 2. We insert Eq. (2.1) in Eq. (2.3) to express $\pi_{\mu\nu}(k)$ by vacuum expectation values of the parton fields. To do this we must take care of the ill-defined operator product at the same space-time point in Eq. (2.1). We use the following prescription:

\[ j_\mu(x) = iQ \lim_{s_1, s_2 \to 2} \left( \frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2} \right) \phi^\dagger(x_1) \phi(x_2); \quad (x_1-x_2)^2<0, \quad (2.4) \]

where we have neglected the gauge term linear in $A_\mu(x)$ since it is irrelevant to the calculation of $\text{Im} \pi(s)$. The product $\phi^\dagger(x_1)\phi(x_2)$ in Eq. (2.4) must be understood as a normal product when it is transformed into the interaction representation. We rewrite Eq. (2.4) in the following form:

\[ j_\mu(x) = \left[ Q/(2\pi)^2 \right] \int dp_1 dp_2 (p_1 - p_2) \mu e^{-i(p_1 + p_2) \cdot x} \times \int dx_1 dx_2 e^{i(p_1 + p_2 + k) \cdot x_2} \phi^\dagger(x_1) \phi(x_2). \quad (2.5) \]

Substituting Eq. (2.5) into Eq. (2.3) we obtain

\[ \pi_{\mu\nu}(k) = \left[ iQ^2/(2\pi)^2 \right] \int dp_1 dp_2 dq_1 dq_2 (p_1 - p_2) \mu (q_1 - q_2) \delta(p_1 + p_2 - k) \times \int dx_1 dx_2 dx_3 dx_4 \exp i(p_1 \cdot x_1 + p_2 \cdot x_2 - q_1 \cdot x_3 - q_2 \cdot x_4) \times \langle 0| T^* [\phi^\dagger(x_1) \phi(x_2) \phi^\dagger(y_2) \phi(y_1)] |0\rangle. \quad (2.6) \]

The above technique is similar to that of Landshoff, Polkinghorne and Short\(^6\) and Ezawa\(^7\) which is developed in deep inelastic electron scattering. The

\(^*)\) Our metric is $g_{\mu\nu} = (+---)$.  

\(^6\) Landshoff, Polkinghorne and Short.  

\(^7\) Ezawa.  

\(\text{Fig. 2.}\) The process $e^+e^-\rightarrow \text{hadrons}$ in the one-photon approximation.
vacuum expectation value appearing in Eq. (2.6) is nothing but the 4-point Green's function which we divide into two parts; the disconnected and connected parts.

\[
\langle 0 | T^* \left[ \phi^I(x_1) \phi^J(x_2) \phi^I(y_3) \phi^J(y_4) \right] | 0 \rangle = \langle 0 | T \left[ \phi^I(y_1) \phi^I(x_1) \right] | 0 \rangle \langle 0 | T \left[ \phi^I(x_2) \phi^I(y_2) \right] | 0 \rangle + \langle 0 | T^* \left[ \phi^I(x_1) \phi^I(x_2) \phi^I(y_3) \phi^I(y_4) \right] | 0 \rangle \text{c},
\]

(2.7)

where \( \langle 0 | T [...] | 0 \rangle \) denotes the connected part. It should be noted here that \( \phi^I(x_1) \) and \( \phi^J(x_2) \) (or \( \phi^I(y_1) \) and \( \phi^J(y_2) \)) are not contracted because the product of these operators is of the normal product form. The Fourier transform of the connected part of Eq. (2.7) is equal to the nonamputated virtual parton-antiparton scattering amplitude. Denoting the amputated virtual parton-antiparton scattering amplitude by \( T \) we find

\[
\int dx_1 dx_2 dy_3 dy_4 e^{-i p_1 \cdot x_1 - i p_2 \cdot x_2 - i q_1 \cdot y_3 - i q_2 \cdot y_4} \langle 0 | T^* \left[ \phi^I(x_1) \phi^I(x_2) \phi^I(y_3) \phi^I(y_4) \right] | 0 \rangle \text{c}
\]

= \( A_s'(p^2) A_s'(p'^2) \) \( (2\pi)^4 i \delta^4(p_1 + p_2 - q_1 - q_2) T A_s'(q^2_1) A_s'(q'^2_2) \),

(2.8)

where \( A_s'(p^2) \) is a full propagator of partons defined by

\[
i A_s'(p^2) = \int dx e^{i s \cdot x} \langle 0 | T \left[ \phi(x) \phi(0) \right] | 0 \rangle.
\]

Hence we get

\[
\pi_{\mu\nu}(k) = - \left[ i Q^2 / (2\pi)^4 \right] \int dp (2p - k)_{\mu} (2p - k)_{\nu} A_s'(p^2) A_s'(k - p)^2
\]

\[
- \left[ Q^2 / (2\pi)^4 \right] \int dp dq (2p - k)_{\mu} (2q - k)_{\nu} A_s'(p^2) A_s'(k - p)^2
\]

\[
\times T A_s'(q^2) A_s'(k - q)^2.
\]

(2.9)

Here the virtual parton-antiparton invariant amplitude \( T \) is a function of invariants \( s = k^2 \) and \( t = (p - q)^2 \) and the virtual parton masses \( p^2, (k - p)^2, q^2 \) and \( (k - q)^2 \). For later convenience we denote the first term of the right-hand side of Eq. (2.9) by \( \pi_{\mu\nu}^{(0)}(k) \) and the second term by \( \pi_{\mu\nu}^{(1)}(k) \). These two terms \( \pi_{\mu\nu}^{(0)}(k) \) and \( \pi_{\mu\nu}^{(1)}(k) \) correspond to Fig. 1(a) and Fig. 1(b) respectively.

We shall estimate the absorptive part of these two terms in the following. For this purpose we use the Lehmann spectral representation for the parton propagator \( A_s'(p^2) \):

\[
A_s'(p^2) = \int_{-\infty}^{\infty} \frac{d\lambda p(\lambda)}{\lambda - p^2} \quad \text{with} \quad \int_{-\infty}^{\infty} d\lambda p(\lambda) = 1.
\]

(2.10)

We insert Eq. (2.10) in the expression for \( \pi_{\mu\nu}^{(0)}(k) \), Eq. (2.9), and take its absorptive part by using the Cutkosky rule. If we define \( \pi_i(k^2) = (k^2 k^2 / k^2 - g^{**}) \times \pi_{\mu\nu}^{(0)}(k) / 3k^2 \) where \( i = 0, 1 \), we find that \( \pi(s) = \pi_0(s) + \pi_1(s) \) and \( R = R_0 + R_1 \) with \( R_i = 12\pi \text{Im} \pi_i(s) \). We finally obtain...
It is easy to see that $R_0 \sim q^2/4$ as $s \to \infty$. Thus we establish that $R_0$ corresponds to the usual light-cone term. It should be noted here that the asymptotic result $R_0 = q^2/4$ is obtained irrespectively of whether or not the Lehmann spectral function $\rho(\lambda)$ has a discrete spectrum corresponding to an isolated free parton. What we have used to get $R_0 = q^2/4$ is the normalization condition (2.10) which is derived solely from the canonical commutation relation for the bare parton fields. The existence of the light-cone term $R_0 \sim q^2/4$ does not conflict with the absence of the isolated free partons.

We next estimate $R_1$. To do this we deal with the absorptive part of $\pi^{(0)}_{\mu}(k)$. We substitute Eq. (2.10) into the expression for $\pi^{(0)}_{\mu}(k)$, Eq. (2.9), and get

$$R_1 = \left[8q^2/(2\pi)^3\right] \text{Im} \int d\lambda_1 d\lambda_2 d\mu_1 d\mu_2 \rho(\lambda_1) \rho(\lambda_2) \rho(\mu_1) \rho(\mu_2) \int dp dq \times \left[ p \cdot q - (p \cdot k) (q \cdot k)/k^2 \right] T/(p^2 - \lambda_1 + i\epsilon) ((k - p)^2 - \lambda_2 + i\epsilon) \times (q^2 - \mu_1 + i\epsilon) ((k - q)^2 - \mu_2 + i\epsilon).$$

(2.12)

Up to this point we have made no approximation. It would be of interest if one could succeed in finding a theory in which the isolated free parton does not appear though the bare parton field is present in Lagrangian. A possible consequence on the serotinous scaling, when such a theory exists, will be discussed in 4. In the present section we remain in a rather provisional stage: We assume that the parton propagator $\Delta(p^2)$ is dominated by a pole at a certain effective parton mass $m$, $\Delta(p^2) \sim Z/(p^2 - m^2)$ with $Z \sim 1$ where $Z$ is a wave-function renormalization constant of the parton field due to its strong interaction. Hence we obtain $\rho(\lambda) \sim \delta(\lambda - m^2)$. As mentioned before, the virtual parton-antiparton invariant amplitude $T$ is a function of the virtual parton masses $p^2, (k - p)^2, q^2, (k - q)^2$ in addition to $s$ and $t$. If we assume that $T$ decreases sufficiently rapidly as the virtual parton masses become large, we may safely approximate the $p$ and $q$ integrations in Eq. (2.12) by parton poles at $m^2$ alone. Thus we get the following approximate result for $R_1$:

$$R_1 = (Q^2/\pi^2 k^2) \int d p d q \left[ (p \cdot k) (q \cdot k)/k^2 - p \cdot q \right] \delta^+(p^2 - m^2) \delta^+((k - p)^2 - m^2) \times \delta^+(q^2 - m^2) \delta^+((k - q)^2 - m^2) \text{Im} T(s, t),$$

(2.13)

where $\delta^+(p^2 - m^2) = \theta(p_0) \delta(p^2 - m^2)$ and replacement $(p^2 - m^2)^{-1} \to 2\pi i \delta^+(p^2 - m^2)$ was done under the integral (2.12) (see Appendix B). Note that the negative

* This assumption is similar to that of the parton field theory in Ref. 6).
sign comes about when we take the imaginary part (see Appendix A). In the centre-of-mass system where \( k_z = (\sqrt{s}, 0, 0, 0) \), we can perform the integrations in Eq. (2.13) to get the following result:

\[
R_1 = \frac{Q^2}{16\pi} \left( 1 - 4m^2/s \right)^{1/2} \int_{-1}^{1} dz z \text{Im} T(s, z),
\]

(2.14)

where \( z = \cos \theta \) with \( \theta \) being the centre-of-mass parton-antiparton scattering angle and \( T(s, z) = T(s, t) \), the on-shell parton-antiparton invariant amplitude. We use the partial-wave expansion of the invariant amplitude \( T \),

\[
T(s, z) = \frac{16\pi}{\sqrt{1 - 4m^2/s}} \sum_{l=0}^{\infty} (2l + 1) P_l(z) a_l(s)
\]

(2.15)

to derive the following relation:

\[
R_1 = \frac{Q^2}{16\pi} \left( 1 - 4m^2/s \right)^{1/2} \text{Im} a_l(s),
\]

(2.16)

where \( a_l(s) = (\eta_l e^{i\delta_l} - 1)/2i \) with \( \eta_l \) and \( \delta_l \) being the elasticity and the phase shift in the \( l \)-th partial wave respectively. The unitarity condition for the parton-antiparton scattering amplitude, \( \text{Im} a_l(s) \leq 1 \), sets an upper bound on \( R_1 \) such that

\[
R_1 \leq \frac{Q^2}{16\pi} \left( 1 - 4m^2/s \right)^{1/2}.
\]

(2.17)

Thus \( R_1 \) can reach at most 4 times the light-cone value \( R_s = Q^2/4 \). By making the same approximation as above in estimating \( R_s \), i.e., \( \rho(\lambda) \sim \delta(\lambda - m^2) \), we find

\[
R = R_s + R_1 = \frac{Q^2}{4} \left( 1 - 4m^2/s \right)^{1/2} \left[ 1 + 4 \text{Im} a_l(s) \right].
\]

(2.18)

The upper bound for \( R \) is then 5\( R_s \):

\[
R \leq 5 \left( \frac{Q^2}{4} \right) \left( 1 - 4m^2/s \right)^{1/2}.
\]

(2.19)

If we have several kinds of partons, the argument given here is not altered. We simply replace \( Q^2 \) by the sum of the charge squared of each kind of partons, assuming the same mass for all charged partons. The value of \( Q^2 \) depends on the choice of a specific model and so we may test validity of various models through Eq. (2.19). Once we choose a proper model, we can estimate \( \text{Im} a_l(s) \) from experimental data on \( R \) through Eq. (2.18). The \( l=1 \) parton-antiparton total cross section is

\[
\sigma_1 = \left[ 12\pi/(s-4m^2) \right] \left[ R/Q^2 \left( 1 - 4m^2/s \right)^{1/2} - 1 \right].
\]

(2.20)

The effective parton mass \( m \) is an arbitrary parameter in our formalism. The precocious scaling in deep inelastic electron scattering suggests that it should be about 1 GeV.\(^{[19]}\) It is also concluded in deep inelastic electron scattering that spins of partons are mainly 1/2. In order to make our model more realistic we shall consider the case of spinor partons in the next section.
§ 3. Spinor partons

We apply our formulation developed in § 2 to the case of spin 1/2 partons. As before we consider a single type of partons $\phi$ with charge $eQ$. Generalization to include several types of partons is straightforward. The electromagnetic current in the present case is given by $j_{\mu}(x) = Q\bar{\phi}(x)\gamma_{\mu}\phi(x)$. It must be redefined in a manner similar to Eq. (2·4). By using the same technique as before we can derive the following relation:

$$\pi_{\mu}(k) = \frac{iQ^2}{(2\pi)^d} \int dp dpdq dq_2 \delta(p_1 + p_2 - k) \times \gamma_{\mu}^{\alpha_1\alpha_2^- \beta_1 \beta_2} G_{\alpha_1 \beta_1 \alpha_2 \beta_2}(p_1 p_2 q_1 q_2) , \quad (3·1)$$

where Green's function $G$ is given by

$$G_{\alpha_1 \beta_1 \alpha_2 \beta_2} = \int dx dx dy dy \phi_{\alpha_1}(x) \gamma_{\mu}^{\alpha_1 \beta_1^*} \phi_{\alpha_2}(y) \psi_{\beta_2} \psi_{\beta_1}(y)\psi_{\beta_1}(y) \rangle \langle 0 \rangle . \quad (3·2)$$

Green's function given by Eq. (3·2) is related to the virtual parton-antiparton scattering amplitude $T$ in the following manner:

$$G_{\alpha_1 \beta_1 \alpha_2 \beta_2} = -(2\pi)^d \delta^d(p_1 - q_1) \delta^d(p_2 - q_2) S_{P}^{\alpha_1 \alpha_2 \mu_1 \mu_2} (p_{12}^{\alpha_1 \alpha_2 \beta_1 \beta_2}) + S_{P}^{\alpha_1 \alpha_2} (p_{12}^{\alpha_1 \alpha_2 \beta_1 \beta_2}) \delta^d(p_1 + p_2 - q_1 - q_2) \times T_{\alpha_1 \alpha_2 \beta_1 \beta_2} (p_{12}^{\alpha_1 \alpha_2 \beta_1 \beta_2}) S_{P}^{\alpha_1 \alpha_2 \mu_1 \mu_2} (p_{12}^{\alpha_1 \alpha_2 \beta_1 \beta_2}) , \quad (3·3)$$

where $S_{P}^{\alpha_1 \alpha_2} (p)$ is a full propagator of partons,

$$iS_{P}^{\alpha_1 \alpha_2} (p) = \int dx e^{ipx} \langle 0 \mid T \left[ \bar{\phi}_{\alpha_2}(x) \phi_{\alpha_1}(0) \right] \rangle 0 \rangle ,$$

and $S_{P}^{\alpha_1 \alpha_2 \mu_1 \mu_2} (p)$ is a charge conjugated propagator; $S_{P}^{\alpha_1 \alpha_2 \mu_1 \mu_2} (p) = -S_{P}^{\alpha_2 \alpha_1 \mu_2 \mu_1} (-p)$. We replace $G$ in Eq. (3·1) by Eq. (3·3). We then obtain for $\pi_{\mu}(k) = \pi_{\mu}^{(0)}(k) + \pi_{\mu}^{(0)}(k)$,

$$\pi_{\mu}^{(0)}(k) = -i [Q^2 / (2\pi)^d] \int dp Tr \left[ S_{P}^{\mu \nu} (-p) \gamma_{\nu} S_{P}^{\alpha_1 \alpha_2 \mu_1 \mu_2} (k - p) \gamma_{\nu} \right] , \quad (3·4)$$

$$\pi_{\mu}^{(0)}(k) = -[Q^2 / (2\pi)^d] \int dp dq \left[ S_{P}^{\mu \nu} (-p) \gamma_{\nu} S_{P}^{\alpha_1 \alpha_2 \mu_1 \mu_2} (k - p) \right]_{\alpha_1 \alpha_2} \times T_{\alpha_1 \alpha_2 \beta_1 \beta_2} (k - q) \gamma_{\nu} S_{P}^{\alpha_1 \alpha_2 \mu_1 \mu_2} (q) \gamma_{\nu} \gamma_{\nu} \gamma_{\nu} . \quad (3·5)$$

We employ the Lehmann spectral representation for $S_{P}^{\alpha_1 \alpha_2} (p)$,

$$S_{P}^{\alpha_1 \alpha_2} (p) = \int_0^\infty d\lambda \rho_{\alpha_2} (\lambda) \gamma^\mu \rho_{\alpha_1} (\lambda) \frac{p^\mu - \lambda}{p^2 - \lambda} \quad \text{with} \quad \int_0^\infty d\lambda \rho_{\alpha_2} (\lambda) = 1 \quad (3·6)$$

in order to reexpress $\pi_{\mu}^{(0)}(k)$ and $\pi_{\mu}^{(0)}(k)$ in terms of the spectral functions $\rho_{\alpha_2} (\lambda)$ and $\rho_{\alpha_1} (\lambda)$. In the same way as in § 2 we derive the following expression for $R_{\phi}$.
\[ R_s = Q^2 \int_0^\infty d\lambda \int_0^\infty d\mu \, \theta(\sqrt{s} - \sqrt{\lambda} - \sqrt{\mu}) \left( 1 - (\sqrt{\lambda} + \sqrt{\mu})/s \right)^2 \left( 1 - (\sqrt{\lambda} - \sqrt{\mu})/s \right)^2 \times \left[ (1 - (\lambda + \mu)/s) \rho_1(\lambda) \rho_2(\mu) + 4\rho_1(\lambda) \rho_1(\mu)/s \right]. \quad (3.7) \]

Here again we see that \( R_0 \) corresponds to the light-cone term. In fact \( R_s \rightarrow Q^2 \) as \( s \rightarrow \infty \). It should also be noted that the existence of the pole in \( S_\gamma'(p) \) corresponding to the isolated free parton is not necessarily required to guarantee the light-cone limit \( R_s = Q^2 \). Next we substitute Eq. (3.6) into Eq. (3.5) and find out the expression for \( R_1 \).

\[ R_1 = - \left[ 2Q^2/(2\pi)^2 \right] \text{Im} \int d\lambda d\lambda_1 d\mu_1 d\mu_2 \]
\[ \times \int dp dq \tau \left( p^2 - \lambda_1 + i\epsilon \right) \left( (k - p)^2 - \lambda_2 + i\epsilon \right) \left( q^2 - \mu_1 + i\epsilon \right) \left( (k - q)^2 - \mu_2 + i\epsilon \right), \quad (3.8) \]

where \( \tau = (k^* k^*/k^2 - g^\mu\nu) \tau_{\mu\nu}/k^2 \) with

\[ \tau_{\mu\nu} = \left[ (\rho_1(\lambda_1) - \gamma \cdot p \rho_1(\lambda_1)) \gamma_{\nu}(\rho_1(\lambda_2) + \gamma \cdot (k - p) \rho_2(\lambda_2)) \right]_{\alpha_1 \alpha_2 \beta_1 \beta_2} T_{\alpha_1 \alpha_2 \beta_1 \beta_2} \times \left[ (\rho_1(\mu_2) + \gamma (k - q) \rho_2(\mu_2)) \gamma_{\nu}(\rho_1(\mu_1) - \gamma \cdot q \rho_2(\mu_1)) \right]_{\alpha_1 \alpha_2} \quad (3.9) \]

Now we make the same assumptions as before: The parton propagator is dominated by a pole at an effective parton mass \( m \), i.e., \( \rho_1(\lambda) \sim m\delta(\lambda - m^2) \) and \( \rho_2(\lambda) \sim \delta(\lambda - m^2) \), and \( T_{\alpha_1 \alpha_2 \beta_1 \beta_2} \) decreases sufficiently rapidly as the virtual parton masses become large, so that the \( p \) and \( q \) integrations in Eq. (3.8) may be approximated by poles at \( m^2 \). Thus we have (the overall negative sign comes about on taking the imaginary part (see Appendices A and B))

\[ R_1 = \left[ 2Q^2/(2\pi)^2 \right] \int dp dq \delta^+(p^2 - m^2) \delta^+((k - p)^2 - m^2) \]
\[ \times \delta^+(q^2 - m^2) \delta^+((k - q)^2 - m^2) \text{Im} \tau, \quad (3.10) \]

where

\[ \tau = - \left[ (2m)^2/k^2 \right] \sum_{l_1, l_2, r_1, r_2} \overline{v}_{l_1}(p) \gamma_{\nu} u_{l_2}(k - p) \overline{u}_{r_1}(k - q) \gamma^\nu v_{l_1}(q) \phi_{l_2 l_1 r_1 r_2} \quad (3.11) \]

with \( \phi_{l_2 l_1 r_1 r_2} \) the helicity amplitude defined by

\[ \phi_{l_2 l_1 r_1 r_2} = v_{l_1}^\dagger(p) \overline{u}_{l_2}^\dagger(k - p) T_{\alpha_1 \alpha_2 \beta_1 \beta_2} \overline{v}_{r_1}(q) u_{r_2}(k - q). \]

Note that we have employed here the following relations:

\[ (\gamma \cdot p - m)/2m = \sum_\lambda v_\lambda(p) \overline{v}_\lambda(p); \quad (\gamma \cdot p + m)/2m = \sum_\lambda u_\lambda(p) \overline{u}_\lambda(p). \]

In the \( e^+e^- \) centre-of-mass system the integrations in Eq. (3.10) may be easily performed to give

\[ R_1 = (Q^2/32\pi) (1 - 4m^2/s) \int_0^1 dx \text{Im} \tau(s, z), \quad (3.12) \]
where $\tau$ is given by Eq. (3.11) and $z = \cos \theta$ with $\theta$ being the parton-antiparton scattering angle. We are now left with the work to express $\tau$ in terms of $J=1$ parton-antiparton partial-wave amplitudes. Apparently the kinematics needed here is exactly parallel to that for the nucleon-antinucleon system. We closely follow the kinematics given by Goldberger, Grisaru, McDowell and Wong in the case of nucleon-nucleon scattering. We set

$$\phi_1 = \phi_{+++}, \quad \phi_2 = \phi_{+--}, \quad \phi_3 = \phi_{+-+}, \quad \phi_4 = \phi_{++-}, \quad \phi_5 = \phi_{+++}.$$  

(3.13)

The helicity amplitudes (3.13) exhaust all independent amplitudes if $P, T$ and $C$ are conserved in parton-antiparton scattering. By carefully reducing Eq. (3.11) we obtain

$$\tau = 2 \left[ (\phi_1 + \phi_3) / s \right] \rho_{00}(\theta) + (1 / m^2) \left[ \phi_0 d_{11}^1(\theta) + \phi_0 d_{1-1}^1(\theta) \right] + (4 \sqrt{2 / m \sqrt{s}}) \phi_0 d_{15}^1(\theta),$$  

(3.14)

where $d_{\rho}^{j}(\theta)$ is the usual rotation matrix. We insert in Eq. (3.14) the following partial-wave expansion of the helicity amplitudes $\phi_{\lambda, \lambda'; \rho, \rho'}$:

$$\phi_{\lambda, \lambda'; \rho, \rho'}(s, z) = (4 \pi / m^2 \sqrt{1 - 4m^2 / s}) \sum_{J} (2J + 1) d_{\rho}^{j}(\theta) \phi_{J, \lambda, \lambda', \rho, \rho'}^{J}(s),$$  

(3.15)

where $\rho = \rho_1 - \rho_2$ and $\lambda = \lambda_1 - \lambda_2$, and perform the integration in Eq. (3.12). We find

$$R_1 = 4Q^4 \sqrt{1 - 4m^2 / s} \left[ (2m^2 / s) \text{Im}(\phi_1^1 + \phi_3^1) + \text{Im}(\phi_3^1 + \phi_4^1) + (4 \sqrt{2 / m \sqrt{s}}) \text{Im} \phi_5^1 \right],$$  

(3.16)

where $\phi_1^1 = \phi_{2+++}^2(s), \phi_2^1 = \phi_{2--}^2(s)$, etc. It is easy to observe that combinations $\phi_1^1 + \phi_3^1, \phi_2^1 + \phi_4^1$ and $\phi_5^1$ are transition amplitudes between parton-antiparton states with $J^{PC}=1^{--}$ which is the quantum number of the photon. Thus only the $^1S_1$ and $^3D_1$ parton-antiparton state contribute to the expression for $R_1$. Making use of the nuclear bar phase shift, we may rewrite the expression for $R_1$.

$$R_1 = 4Q^4 \sqrt{1 - 4m^2 / s} (1 / 3) \left[ (2 + m / \sqrt{s}) \text{Im} a_- + (1 - 2m / \sqrt{s}) \text{Im} a_+ \right.$$  

$$+ \sqrt{2} (1 + m / \sqrt{s}) (1 - 2m / \sqrt{s}) \text{Im} b \right],$$  

(3.17)

where $a_-$ and $a_+$ are $^3S_1$ and $^3D_1$ parton-antiparton scattering amplitudes respectively which can be expressed in terms of the phase shifts $\delta_\pm$, coupling parameter $\varepsilon$ and elasticities $\eta_\pm$ as follows:

$$a_\mp = \left[ \eta_\pm \exp(2i\varepsilon) \cos 2\varepsilon - 1 \right] / 2i,$$

and $b$ is a transition amplitude between the $^3S_1$ and $^3D_1$ states and is given by

$$b = \sqrt{\eta_+ \eta_-} \sin 2\varepsilon \exp \{ i(\delta_+ - \delta_-) \}.$$

Equation (3.17) for $R_1$ is bounded by the unitarity condition for parton-antiparton scattering. The upper bound for Eq. (3.17) is attained when $\delta_\pm = \pi / 2$ and $\varepsilon = 0$.\n
Hence
\[ R_1 \leq 4Q^2 \sqrt{1 - 4m^2/s} (1/3) [(1 + m/\sqrt{s})^2 (1 + \eta_-) + (1 - 2m/\sqrt{s})^2 (1 + \eta_+)/2]. \]

Taking into account that \( \eta_+ \leq 1 \), we finally get
\[ R_1 \leq 4Q^2 \sqrt{1 - 4m^2/s} (1 + 2m^2/s). \] (3.18)

This result is almost the same as that in the case of the scalar partons, Eq. (2.17). Calculating \( R_2 \) given by Eq. (3.7) in the same approximation as described below Eq. (3.9) and adding the result for \( R_1 \), Eq. (3.17), we obtain
\[ R = Q^2 \sqrt{1 - 4m^2/s} [1 + 2m^2/s + (4/3) (2(1 + m/\sqrt{s}) \Im a_- + (1 - 2m/\sqrt{s}) \Im a_+ + \sqrt{2}(1 + m/\sqrt{s}) (1 - 2m/\sqrt{s}) \Im b)]. \] (3.19)

Hence
\[ R \leq 5Q^2 \sqrt{1 - 4m^2/s} (1 + 2m^2/s). \] (3.20)

As mentioned before, \( Q^2 \) must be regarded as a sum of the charges squared of each kind of partons when we have several kinds of partons. It is well known that \( Q^2 = 2/3 \) for the Gell-Mann-Zweig quark model,\(^{15}\) \( Q^2 = 1 \) for the Maki-Hara quartet model,\(^{16}\) \( Q^2 = 2 \) for the coloured quark model,\(^{10}\) \( Q^2 = 10/3 \) for the coloured quartet model,\(^{16}\) and \( Q^2 = 4 \) for the Han-Nambu three-triplet model.\(^{17}\) As is seen in Fig. 3 the present experimental data\(^{11}\) already exceed the value \( R = 5 \) at around \( s = 17 \text{ GeV}^2 \). Hence referring to Eq. (3.20) we may conclude that the Gell-Mann-Zweig quark model and the Maki-Hara quartet model are in trouble. The coloured quark model together with the other two models gives the upper bound well above the present experimental data. In Fig. 3 we drew the upper bound for the coloured quark model. As a possible candidate to describe the

![Fig. 3. The bound of \( R \) and the phenomenological form for \( R \) in the coloured quark model are compared with the experimental data.](https://academic.oup.com/jpa/article-abstract/3/1/200/1860865/1.2018.06.0865)
experimental data we choose the coloured quark model and try to make a phenomenological fit to the experimental data. To do this we assume that 
\[ \varepsilon \sim 0; \quad \text{Im} \, a_+ \sim \text{Im} \, a_- (= \text{Im} \, a). \]
This assumption simplifies our expression for \( R \) to result in
\[ R = Q^2 \sqrt{1 - 4m^2/s} (1 + 2m^2/s) (1 + 4 \text{Im} \, a), \tag{3.21} \]
with \( Q^2 = 2 \). Given the data on \( R \) we can estimate \( \text{Im} \, a \) by Eq. (3.21), if the effective mass \( m \) of the partons is taken to be 1 GeV which is consistent with the precocious scaling in deep inelastic electron scattering.\(^{10}\) The value of \( \text{Im} \, a \) as a function of \( s \) is presented in Fig. 4. We notice that the absorptive part of the \( J^{PQ} = 1^{--} \) parton-antiparton scattering amplitude, \( \text{Im} \, a \), rises sharply near the threshold and reaches the value 0.5 (complete absorption \( \eta = 0 \)) at around \( s = 25 \text{ GeV}^2 \). The following phenomenological form for \( \text{Im} \, a \)
\[ \text{Im} \, a = 0.9 (1 - 4/s)^{1/4} \tag{3.22} \]
nicely reproduces the experimental data between \( 4 \leq s \leq 25 \text{ GeV}^2 \). The curve "Best Fit" in Fig. 3 was drawn by Eq. (3.21) with Eq. (3.22). It should be stressed that Eq. (3.21) is purely phenomenological and does not necessarily mean that \( \text{Im} \, a \to 0.9 \) as \( s \to \infty \). It is more plausible to suspect that \( \text{Im} \, a \) starts decreasing at a certain energy much higher than \( \sqrt{s} = 5 \text{ GeV} \) and vanishes asymptotically to disclose the light-cone term \( R_o \).

\section*{4. A comment on the absence of the free quarks}

In the last two sections we have been assuming that the parton propagator has a pole corresponding to the isolated free parton. The assumption was made partly because this made it possible to estimate the contribution \( R \) explicitly. Unfortunately the isolated free partons have not been observed so far. In this sense our formulation developed here may be regarded as a provisional one for the future theory. On the other hand, the Bjorken scaling in deep inelastic electron scattering suggests the existence of the asymptotically free constituents in the nucleons. These point-like objects would correspond to the bare partons. Under these circumstances it is very interesting if one could construct a theory in which the isolated free partons do not appear though the bare parton fields
are present. In the present paper, however, we do not enter into discussion of the possibility of this kind of the theory, but we would rather like to examine consequences of such a theory if it exists. Since there is no isolated free parton in this theory, the parton propagator $S_{p'}(p)$ has no pole at all and consists only of continuum spectra. The spectrum in Eq. (3·6) starts at $\lambda = s_0$ which would correspond to the threshold of a continuum composed of hadrons and a triplet gluon. In such a case the threshold $s_0$ must be very high because the mass of the gluon is supposed to be very large $\sim 10$ GeV in conformity with the possible breakdown of the Bjorken scaling for large $q^2$.

Thus $s_0$ is of order 100 GeV$^2$. Let us now go back to Eqs. (3·7) and (3·8). In the present case integrations in $\lambda, \mu, \lambda_1, \lambda_2, \mu_1$ and $\mu_2$ in Eqs. (3·7) and (3·8) start at $s_0$ instead of 0. As is easily seen in Eq. (3·7), $R_0$ is non-vanishing only when $s > 4s_0$, while the spectrum of $R_1$ in $s$ may extend down to the low $s$ region since low-mass hadronic states can contribute to the absorptive part of $\tau$ in Eq. (3·8). Hence for $s \leq 4s_0 \sim 400$ GeV$^2$ only the parton-antiparton interaction term $R_1$ contributes to $R$ and the light-cone term $R_0$ would be significant only at $s \gg 400$ GeV$^2$. Thus in our formulation of the process $e^+e^-\rightarrow$ hadrons the absence of the isolated free partons implies the serotinous scaling in this process.$^{19,8}$ The argument given here does not contradict the precocious scaling in the electron-nucleon scattering process since the light-cone term may become dominant already at relatively low $q^2(= -s)$ in the space-like region.

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Appendix A

In this Appendix we shall show that the negative sign comes about when we take the imaginary part in Eq. (2·12) to get the approximate form (2·13). We start with the expression (2·3) for $\pi_\mu(k)$. Denoting twice the absorptive part of $\pi_\mu(k)$ by $w_\mu(k)$ we see that

$$w_\mu(k) = \sum_n (2\pi)^4 \delta^4(p_n - k) \langle 0|j_\mu(0)|n\rangle \langle n|j_\mu^+(0)|0\rangle. \tag{A·1}$$

We replace the electromagnetic current $j_\mu(x)$ by the product of two-parton fields as in Eq. (2·4). One may easily show that the amplitude $\langle n|j_\mu^+(0)|0\rangle$ is

$^8$ John Kogut proposed a similar idea that the parton-parton interactions become literally strong for $s \geq 10$ GeV$^2$. The idea, however, is different from ours in that the interactions vanish for $s \leq 10$ GeV$^2$ in conformity with the precocious scaling. Our opinion is that the situation concerning the Bjorken scaling in the space-like region may be different from that in the time-like region.
composed of the connected and disconnected parts in the two-parton system, i.e.,
diagrammatically
\[
< n | j_i^\nu(0) | 0 > = T_n \quad (A \cdot 2)
\]
where \( T_n \) is the off-shell transition amplitude between the two-parton state and the final state \( | n \rangle \). The connected part of \( < n | j_i^\nu(0) | 0 > \), i.e., the second term of the right-hand side of Eq. (A \cdot 2), has the following form:
\[
< n | j_i^\nu(0) | 0 > = -iQ (2\pi)^{-4} \int dq (2q - p_n) \Delta f' (q^2) \Delta f' ((p_n - q)^2) T_n \quad (A \cdot 3)
\]
Substituting Eq. (A \cdot 2) into Eq. (A \cdot 1) we find diagrammatically
\[
\omega_{\mu\nu}(k) = T_n + \text{off-diagonal terms.} \quad (A \cdot 4)
\]
The first term of the right-hand side of Eq. (A \cdot 4) corresponds to \( 2|\omega_{\mu\nu}^{(0)}(k)| \) and hence the rest of the terms in Eq. (A \cdot 4) must be equal to \( 2|\omega_{\mu\nu}^{(0)}(k)| \) which we denote by \( \omega_{\mu\nu}^{(0)}(k) \). In the approximation* of neglecting off-diagonal terms in Eq. (A \cdot 4) we may identify the second term of r.h.s. of Eq. (A \cdot 4) with \( \omega_{\mu\nu}^{(0)}(k) \). We have
\[
\omega_{\mu\nu}^{(0)}(k) = Q^2 (2\pi)^{-3} \int dp dq (2p - k) (2q - k) \Delta f' (p^2) \Delta f' ((k - p)^2) \times \Delta f' (q^2) \Delta f' ((k - q)^2) \sum_n (2\pi)^4 \delta^4(p_n - k) T_n^* T_n \quad (A \cdot 5)
\]
The unitarity condition for the virtual parton-antiparton scattering amplitude \( T \) reads
\[
\sum_n (2\pi)^4 \delta^4(p_n - k) T_n^* T_n = 2 \text{ Im } T \quad (A \cdot 6)
\]
Hence we finally get
\[
\omega_{\mu\nu}^{(0)}(k) = 2Q^2 (2\pi)^{-2} \int dp dq (2p - k) (2q - k) \Delta f' (p^2) \times \Delta f' ((k - p)^2) \Delta f' (q^2) \Delta f' ((k - q)^2) \text{ Im } T \quad (A \cdot 7)
\]
Making the on-shell approximation \( \Delta f' (p^2) \sim 2\pi i \delta^+(p^2 - m^2) \), which will be discussed in Appendix B, and noting that \( R_1 = 2\pi (k^* k^*/k^2 - g^{\mu\nu}) \omega_{\mu\nu}^{(0)}(k)/k^2 \) we obtain Eq. (2 \cdot 13) through Eq. (A \cdot 7).

* Here the off-diagonal terms are neglected since they cancel out when the on-shell approximation is applied. Note that we are only interested in the overall sign appearing in the on-shell approximation given by Eq. (B \cdot 9).
The rule obtained here is that after making the on-shell approximation in Eq. (2.12) one must put the overall negative sign to get the correct result (2.13).

**Appendix B**

Here we shall discuss the validity of the on-shell approximation introduced below Eq. (2.12). The relevant amplitude which we shall discuss is

\[
A(k^2) = \int dp dq B(p, q, k) / (p^2 - m^2 - i\epsilon) \left( (k-p)^2 - m^2 - i\epsilon \right) \times (q^2 - m^2 + i\epsilon) \left( (k-q)^2 - m^2 + i\epsilon \right),
\]

where

\[
B(p, q, k) = \left[ (p \cdot k) (q \cdot k) / k^2 - p \cdot q \right] \text{Im} \mathcal{T}.
\]

The amplitude \(A(k^2)\) is proportional to \((k^4 \mathcal{K}/k^2 - q^2)\mathcal{W}_{\mu\nu}(k) / k^2\) in the pole approximation for the propagators, \(A^\prime(p) \sim 1 / (p^2 - m^2 + i\epsilon)\).

We shall show that the factors \((p^2 - m^2 - i\epsilon)^{-1} (k-p)^2 - m^2 - i\epsilon)^{-1} (q^2 - m^2 + i\epsilon)^{-1} \times ((k-q)^2 - m^2 + i\epsilon)^{-1}\) in the integrand of Eq. (B.1) can be replaced in the approximate sense by \((2\pi i)^2 \delta^+(p^2 - m^2) \delta^+(k-p)^2 - m^2) \delta^+(q^2 - m^2) \delta^+(k-q)^2 - m^2)\) when \(B(p, q, k)\) satisfies a condition which will be given later.

We first perform the \(p_\nu\) and \(q_\nu\)-integrations in Eq. (B.1). For this purpose we introduce the centre-of-mass system for the parton-antiparton system, i.e., \(k_\mu = (\sqrt{s}, 0, 0, 0)\). In this frame \(B(p, q, k)\) is a function of \(p_0, q_0, p^2, q^2, p \cdot q\) and \(s\) alone. In order to perform the \(q_0\)-integration we examine the analytic structure in \(q_0\) of the integrand \(B(p, q, k) / (q^2 - m^2 + i\epsilon) (k-p)^2 - m^2 - i\epsilon)\). Except for possible singularities of \(B(p, q, k)\) we have only poles at \(q_0 = \pm (E_q - i\epsilon)\) and \(q_0 = \sqrt{s} \pm (E_q - i\epsilon)\) where \(E_q = \sqrt{q^2 + m^2}\). These poles are shown in Fig. 5(a).

The condition we impose on \(B(p, q, k)\) is as follows:

\[B(p, q, k)\] decreases sufficiently rapidly at infinity in the complex \(q_0\)-plane (\(p_\nu\)-plane).

Under the condition we may add a large semicircle to the path of the \(q_0\) integration \((-\infty < q_0 < \infty)\) if we neglect contributions of the possible poles and cuts in \(q_0\) of \(B(p, q, k)\) (this assumption amounts to our "on-shell approximation"'). We close the path of the integration in \(q_0\) in the upper half plane as shown by a solid line in Fig. 5(a) to obtain

\[
A(s) = \int \frac{d^4 p}{(p^2 - m^2 - i\epsilon) ((k-p)^2 - m^2 - i\epsilon)} \times \int d^4 q \cdot 2\pi i \frac{1}{4\sqrt{s} E_q} \left[ -B(q_0 = -E_q) + B(q_0 = \sqrt{s} - E_q) \right],
\]

\[\text{(*) The large } q_0 \text{ in the assumption corresponds to the large virtual mass } \sqrt{q^2} \text{ of the partons.}\]
where we suppressed other arguments than $q_0$ in $B(p, q, k)$. Within our approximation of neglecting all the singularities of $B(p, q, k)$ in $q_0$, the result (B·3) must be the same as the one which would be obtained by closing the path of the $q_0$-integration in the lower half plane as shown by a dotted curve in Fig. 5(a). This condition imposes the following relations on $B(p, q, k)$:

$$B(q_0 = -E_q) = B(q_0 = \sqrt{s} + E_q), \quad B(q_0 = E_q) = B(q_0 = \sqrt{s} - E_q). \quad (B·4)$$

The $p_0$-integration may be performed in the same way as in the case of the $q_0$-integration. Here the distribution of the relevant poles in $p_0$ is exhibited in Fig. 5(b) with $E_p = \sqrt{p^2 + m^2}$. Again we neglect the possible singularities of $B(p, q, k)$ in $p_0$ and add a large semicircle in the upper half plane. We find

$$A(s) = \int d^4p d^4q \frac{2\pi i}{4\sqrt{s}E_p} \frac{2\pi i}{4\sqrt{s}E_q} \left[ \frac{B(p_0 = E_p, q_0 = -E_q)}{(E_q - \sqrt{s}/2 - ie)(E_q - \sqrt{s}/2 + ie)} - \frac{B(p_0 = E_p, q_0 = E_q)}{(E_q + \sqrt{s}/2)(E_q + \sqrt{s}/2)} \right. \quad (B·5)$$

where we used Eq. (B·4). For the $p_0$-dependence of $B(p, q, k)$ we get similar relations to those in Eq. (B·4). Using these relations and taking polar coordinates in $p$ and $q$, we derive

$$A(s) = \frac{(2\pi i)^3}{16s} \int d\Omega_p d\Omega_q \int_m^{\infty} \sqrt{E_p^2 - m^2} dE_p \int_m^{\infty} \sqrt{E_q^2 - m^2} dE_q \left[ \frac{B(p_0 = E_p, q_0 = -E_q)}{(E_q + \sqrt{s}/2)(E_q - \sqrt{s}/2 + ie)} - \frac{B(p_0 = E_p, q_0 = E_q)}{(E_q - \sqrt{s}/2 - ie)(E_q - \sqrt{s}/2 + ie)} - \frac{B(p_0 = -E_p, q_0 = -E_q)}{(E_q + \sqrt{s}/2)(E_q + \sqrt{s}/2)} + \frac{B(p_0 = -E_p, q_0 = E_q)}{(E_q - \sqrt{s}/2 - ie)(E_q + \sqrt{s}/2)} \right], \quad (B·6)$$

where $d\Omega_p$ and $d\Omega_q$ represent integrations in the directions of $p$ and $q$ respectively. Equation (B·6) may be rewritten in the following form:

Fig. 5.  \hspace{1cm} Fig. 6.
\[ A(s) = -\frac{(2\pi i)^2}{16s} \int d\Omega_p d\Omega_q \int_{|E_p| \geq m} \frac{\sqrt{E_p^2 - m^2} dE_p}{E_p - \sqrt{s}/2 + i\epsilon} \times \int_{|E_q| \geq m} \frac{\sqrt{E_q^2 - m^2} dE_q}{E_q - \sqrt{s}/2 - i\epsilon} B(p_0 = E_p, q_0 = E_q). \] (B.7)

Since \( B(p_0 = E_p, q_0 = E_q) \) decreases sufficiently rapidly as \( |E_q| \to \infty \) \( (|E_p| \to \infty) \) in the complex \( E_q \)-plane \( (E_p \)-plane), we can again add a large semicircle to the path of the \( E_q \)-integration \( (E_p \)-integration). It should here be noted that the path of the \( E_q \)-integration \( (E_p \)-integration) has a break \(-m < E_q < m \) \((-m < E_p < m\)).

We shall approximate the path of the \( E_q \)-integration \( (E_p \)-integration) by adding a path \( L_1 \) \( (L_2) \) shown in Fig. 6(a) \( (6(b)) \). Closing the path of the integration as shown in Figs. 6(a) and 6(b) we finally obtain

\[ A(s) = \left[ \frac{(2\pi i)^4}{16s} \right] (s/4 - m^2) \int d\Omega_p d\Omega_q B(p_0 = \sqrt{s}/2, q_0 = \sqrt{s}/2), \] (B.8)

which is equal to

\[ A(s) = \frac{(2\pi i)^4}{16s} \int dp dq B(p, q, k) \delta^+(p^2 - m^2) \delta^+((k - p)^2 - m^2) \times \delta^+(q^2 - m^2) \delta^+((k - q)^2 - m^2). \] (B.9)

Summing up we observe that the on-shell approximation \((B.9)\) is equivalent to the following statements:

1. \( B(p, q, k) \) decreases sufficiently rapidly as \( |q_0| \to \infty \).
2. The contributions of possible singularities of \( B(p, q, k) \) in \( q_0(p_0) \) to \( A(s) \) are negligible.
3. The contribution coming from the integral region \( |E_q| < m \) \( (|E_p| < m\)) is negligible.

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