Renormalization and $\alpha'$-Expansion
of the Dual Resonance Model

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On the basis of slope-parameter ($\alpha'$) expansion of the dual Born amplitudes, we discuss
the renormalization of the dual resonance model in the context of usual quantum field theory.
In lowest non-trivial order of $\alpha'$ and in the one-loop approximation, we show that the
Veneziano model of general intercept is nonrenormalizable, while the model of unit intercept
is renormalizable. We use the dimensional regularization and 't Hooft's general algorithm
in calculating the counter-Lagrangian.

§ 1.

In the limit of small-slope parameter, the dual Born-amplitudes are approximately described by Lagrangian field theory. That is, we can construct a Lagrangian in the power series of $\alpha'$

$$\mathcal{L} = \mathcal{L}^{(0)} + \alpha' \mathcal{L}^{(1)} + (\alpha')^2 \mathcal{L}^{(2)} + \cdots,$$

such that the $S$-matrix elements derived from (1) in the tree approximation reproduce the first few terms of the $\alpha'$-expansion of the dual Born-amplitudes. For the Veneziano model, $\mathcal{L}^{(0)}$ is the $\phi^4$-theory Lagrangian when the coupling constant $\lambda (= g/\sqrt{\alpha'})$ and the ground state mass are fixed in the expansion, or the massless Yang-Mills Lagrangian when the intercept is fixed at one. In either case, $\mathcal{L}^{(0)}$ is renormalizable. If the higher terms $\mathcal{L}^{(1)}, \mathcal{L}^{(2)}, \cdots$ are included, the field theory described by (1) would become a nonrenormalizable one according to the usual criterion, because (1) then contains a coupling parameter $\alpha'$ with positive dimension of length. However, we cannot conclude from this naive argument nonrenormalizability of the dual resonance model, since (1) is a power series in $\alpha'$ and some cancellations between divergences at each order of $\alpha'$ could occur. In fact, in dual loop theory, a regularization scheme very similar to quantum-field theory renormalization is familiar. It is, however, not clear whether all divergences are really absorbed in physical parameters after rescaling external wave functions, except for the case of unit intercept. Only in the case of unit intercept, there is a formal argument for renormalizability.

In this note, we will discuss the renormalization of the dual resonance model from the standpoint of usual quantum field theory on the basis of the expansion (1). It turns out that the Veneziano model of general intercept is nonrenormalizable already at lowest non-trivial order in the expansion. The case of unit...
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intercept is renormalizable at least up to first order of $\alpha'$ in conformity with the dual-theory renormalization.

§ 2.

Even at lowest non-trivial order, the Lagrangian (1) is highly non-linear one. It is then very convenient to apply the dimensional regularization$^6$ and 't Hooft's general algorithm$^7$ to calculating the counter-Lagrangian. We first briefly recapitulate the formulae. Let the Bose fields be $A_i$, where $i$ is any kind of index. We split $A_i$ into two parts, $A_i = \phi_i + B_i$, with $\phi_i$ being the quantum fields and $B_i$ the background fields which satisfy the classical equations of motion. Then, we expand the Lagrangian in the $\phi_i$ and only take the quadratic part $L_2[\phi, x]$ of the $\phi_i$. We assume that the quadratic part can be arranged in the following form:

$$L_2[\phi^1, x] + L_2[\phi^2, x] = \bar{\phi}_i F_{ij}[B, x] \partial^2 \phi_j + 2 \bar{\phi}_i * N_{ij, \rho}[B, x] \partial_{\rho} \phi_j + \bar{\phi}_i * M_{ij, \rho}[B, x] \varphi_j,$$  

(2)

where $\bar{\phi}_i = (\phi_i + i \phi^* i) / \sqrt{2}$ after doubling$^6$ the quantum fields $\phi_i$ to $\phi_i^1$ and $\phi_i^2$. The counter-Lagrangian which cancels all one-loop divergences at dimension four is then given by

$$\Delta L = \text{Tr} \left\{ \frac{1}{4\epsilon} \mathcal{X} \frac{1}{24\epsilon} \mathcal{Q}_{\mu\nu} \mathcal{Q}_{\mu\nu} \right\},$$  

(3)

where

$$\mathcal{X} = \mathcal{M} - \partial_{\mu} \mathcal{N}_{\rho} - \mathcal{N}_{\mu} \mathcal{M},$$  

(4)

$$\mathcal{Q}_{\mu\nu} = \partial_{\mu} \mathcal{N}_{\nu} - \partial_{\nu} \mathcal{N}_{\mu} + \mathcal{M}_{\mu} \mathcal{N}_{\nu} - \mathcal{M}_{\nu} \mathcal{N}_{\mu},$$  

(5)

$$\mathcal{M} = F^{-1} M,$$  

(6)

$$\mathcal{N}_{\mu} = F^{-1} N_{\mu},$$  

(7)

and $\epsilon = 8\pi^2 (n-4)$ with $n$ being the space-time dimension. For $\Delta L$, we can use the equation of motion for $B_i$ as far as we are only concerned with the $S$-matrix elements.

§ 3.

Now let us consider the case of fixed $\lambda$ and the ground state mass. The $\alpha'$-expansion in this case was studied by Frampton and Wali.$^9$ We quote their result

$$L = \frac{1}{2} (\partial_{\rho} \phi)^2 - \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{6} \phi^3 - \lambda^2 \alpha' \left[ \frac{1}{2} (\partial_{\rho} \phi)^2 \phi^2 + \frac{1}{8} m^2 \phi^4 \right] + \frac{9}{5!} \lambda^2 \alpha' \phi^5,$$  

(8)

which is the correct Lagrangian up to second order of $\alpha'$. Here a numerical
constant is absorbed into $\alpha'$. Note that the first order term $\mathcal{L}^{(1)}$ is zero in this case. We calculate the counter-Lagrangian following the prescription explained in § 2. We then find

\begin{align*}
F &= 1 + \lambda \alpha \alpha^* B^2, \\
N &\equiv \lambda \alpha \alpha^* B \partial_\mu B, \\
M &= -m^2 + \lambda B - \frac{1}{2} \lambda \alpha \alpha^* m^2 B^2 + \frac{1}{2} \lambda \alpha \alpha^*(\partial_\mu B)^2 + 2\lambda \alpha \alpha^* B \partial_\mu B.
\end{align*}

(9) (10) (11)

Up to second order of $\alpha'$ and fourth order of $\alpha$, the counter-Lagrangian is given by

\begin{align*}
\Delta \mathcal{L} = \frac{1}{4\epsilon} \left[ -2\lambda m^2 B + 2\lambda \alpha \alpha^* m^2 (\partial_\mu B)^2 + \lambda (1 - \alpha \alpha^* m^2) B^2 \\
-2m^2 \lambda \alpha \alpha^* B^2 - 4\lambda \alpha \alpha^* B (\partial_\mu B)^2 + \lambda \alpha \alpha^* B^2 \right],
\end{align*}

(12)

apart from irrelevant constants and total derivatives. By using the equation of motion, the above equation is simplified:

\begin{align*}
\Delta \mathcal{L} = -\frac{1}{2\epsilon} m^2 \lambda B + \frac{1}{4\epsilon} (1 + \alpha \alpha^* m^2) B^2 + \frac{1}{2\epsilon} \lambda \alpha \alpha^* m^2 (\partial_\mu B)^2.
\end{align*}

(13)

To eliminate the linear term, we translate the field $B$ into $B' = \lambda / 4\epsilon$ as in the usual $\phi^4$-theory. Then the original Lagrangian plus the counter-Lagrangian is given by

\begin{align*}
\mathcal{L}(B') + \Delta \mathcal{L}(B') &= -\frac{1}{2} \left( 1 - \frac{1}{2\epsilon} \lambda \alpha \alpha^* m^2 \right) (\partial_\mu B')^2 \\
&- \frac{1}{2} \left( m^2 + \frac{1}{4\epsilon} \lambda \alpha \alpha^* m^2 \right) (B')^2 + \frac{\lambda}{6} (B')^2 - \frac{1}{2} \lambda \alpha \alpha^* (B')^2 (\partial_\mu B')^2 \\
&- \frac{1}{8} \lambda \alpha \alpha^* m^2 (B')^4 - \frac{1}{32\epsilon} \lambda \alpha \alpha^* (B')^4 + \frac{9}{5!} \lambda \alpha \alpha^* (B')^4.
\end{align*}

Owing to the presence of the term $-\frac{1}{32\epsilon} \lambda \alpha \alpha^* (B')^4$ independent of the mass parameter, (13) cannot recover the original form (8) by renormalization of mass, coupling, slope parameter and field. We conclude that the Veneziano model of general intercept with fixed ground-state mass is not renormalizable.

We can easily convince ourselves that in order to have a renormalizable theory it is necessary to have a contact term $\mathcal{L}^{(1)} \sim \lambda \alpha \phi^4$ which is of first order of $\alpha'$. If considered in terms of the dual Born amplitudes, this implies, for example, that satellite terms such as $\lambda \alpha' \cdot \{\Gamma(1 - \alpha(s)) \Gamma(1 - \alpha(t))\} / \{\Gamma(2 - \alpha(s) - \alpha(t))\}$ should be present in addition to the usual leading term.

Nonrenormalizability of the case of general intercept is not surprising. In

\footnote{Since our calculation is in the one-loop approximation, the result is correct only to this order.}
the usual dual loop theory of general intercept, it is impossible\footnote{3} for daughter trajectories to absorb all the infinities by re-definition of physical parameters, while for the leading trajectory it is possible by renormalization of the intercept. Further, there is an infinite degree of arbitrariness in defining the dual counter terms. For these reasons, it has not been understood how the renormalization of the dual loops works for the general intercept case. As we have just shown, the Veneziano model of general intercept is nonrenormalizable theory in the context of usual quantum field theory

§ 4.

Next we consider the case of fixed unit intercept with Chan-Paton $\text{SU}(2)$ factors. $\mathcal{L}^{(0)}$ is the massless Yang-Mills Lagrangian. In this case $\mathcal{L}^{(1)}$ is not zero. From the calculation of the three-Reggeon vertex and gauge invariance, it is easily seen that $\mathcal{L}^{(0)}$ is $g \text{Tr}(G_{\mu\nu}G_{\sigma\lambda}G_{\lambda\rho})$, where

\begin{align}
(G_{\mu\nu})^{ab} &= f^{abc}G_{\mu\nu}^c, \\
G_{\mu\nu}^c &= \partial_\mu A_\nu^c - \partial_\nu A_\mu^c + g f^{cab}A_\mu^a A_\nu^b.
\end{align}

Thus the starting Lagrangian which is correct to first order of $\alpha'$ is

$$\mathcal{L} = \frac{1}{2} \text{Tr}(G_{\mu\nu}G_{\mu\nu}) + \frac{1}{2} g \alpha' \text{Tr}(G_{\mu\nu}G_{\sigma\lambda}G_{\lambda\rho}).$$

Here again $\alpha'$ is defined up to a numerical constant.

Now we show that the field theory described by (16) is renormalizable up to first order of $\alpha'$. To this order possible counter terms are, from gauge invariance and dimensional consideration, $\text{Tr}(G_{\mu\nu}G_{\mu\nu})$, $\alpha' \text{Tr}(G_{\mu\nu}G_{\sigma\lambda}G_{\lambda\rho})$, $\alpha' \text{Tr}(G_{\mu\nu}[D_\lambda, [D_\lambda, G_{\mu\nu}]]$ and $\alpha' \text{Tr}(G_{\mu\nu}[D_\lambda, [D_\lambda, G_{\mu\nu}]]$, where

\begin{align}
(D_\mu)^{ab} &= \delta^{ab} \partial_\mu - g f^{abc} A_\mu^c \\
&= (1)^{ab} \partial_\mu - g (A_\mu)^{ab}.
\end{align}

We denote these forms by [I], [II], [III] and [IV], respectively. Other possible forms are always equivalent to one of these forms apart from the total derivatives. Since we can use the equations of motion for the counter terms, we can set [IV] $=0$ in the present approximation. [III] is reduced to [II] after the cyclic identity $[D_\mu, G_{\sigma\lambda}] + [D_\nu, G_{\lambda\rho}] + [D_\lambda, G_{\mu\nu}] = 0$ and the equations of motion are used. Thus we can conclude that the counter-Lagrangian to first order of $\alpha'$ is a linear combination of [I] and [II]. This means that (16) is renormalizable up to first order of $\alpha'$.

Next let us calculate the counter-Lagrangian explicitly in the one-loop approximation. The quadratic part $\mathcal{L}_2[\varphi, x]$ derived from (16) has a local gauge invariance\footnote{4} under $\varphi_\mu^a \rightarrow \varphi_\mu^a + g f^{abc} A_\mu^b \varphi^c - (D_\mu A)^a$. For quantization and also arrangement of the quadratic part to the form (2), we add the following gauge breaking term:
After a straightforward calculation we find

\begin{align}
F_{ab} &= \frac{1}{2} \delta_{ab} - \frac{3}{2} g \alpha' G_{ab}, \\
N_{a\alpha \beta} &= -g \left( \frac{1}{2} \delta_{ab} - \frac{3}{2} g \alpha' G_{ab} \right) B_{a \beta} + \frac{3}{2} g \alpha' L_{a \beta}, \\
M_{ab} &= -2gG_{ab} + g \left( \frac{1}{2} \delta_{ab} - \frac{3}{2} g \alpha' G_{ab} \right) (gB_{a \beta}B_{b \alpha} - \partial_{\mu}B_{a \beta}) \\
&\quad + \frac{3}{2} g \alpha' \left\{ [D_{a \alpha}, [D_{b \beta}, G_{ab}]] + [D_{b \beta}, [G_{ab}, D_{a \alpha}]] \right\} \\
&\quad + \frac{3}{2} g \alpha' \left\{ [D_{a \alpha}, G_{b \beta}]B_{a \beta} - [D_{b \beta}, G_{a \alpha}]B_{a \alpha} - [G_{a \alpha}, D_{b \beta}]B_{a \beta} - [G_{b \beta}, D_{a \alpha}]B_{a \alpha} \right\},
\end{align}

\begin{align}
\chi_{ab} &= -2gG_{ab} - \frac{3}{2} g \alpha' G_{b \beta} G_{a \alpha} \\
&\quad - \frac{3}{2} g \alpha' \delta_{ab} \left( G_{b \beta} G_{a \alpha} - \frac{3}{2} g \alpha' [D_{b \beta}, L_{a \alpha}] \right),
\end{align}

\begin{align}
\mathcal{L}_{ghost} &= -2gG_{ab} - \frac{3}{2} g \alpha' \left\{ [D_{a \alpha}, G_{b \beta}]B_{a \beta} - [D_{b \beta}, G_{a \alpha}]B_{a \alpha} - [G_{a \alpha}, D_{b \beta}]B_{a \beta} - [G_{b \beta}, D_{a \alpha}]B_{a \alpha} \right\},
\end{align}

where

\begin{align}
L_{a\beta,\alpha} &= [G_{b \beta}, D_{a \alpha}] + [G_{b \beta}, D_{a \alpha}] - [D_{a \alpha}, G_{b \beta}]
\end{align}

Here $G_{ab}, D_a$ are given by (14), (15) and (17) with the fields $A_\mu$ being replaced by the background fields $B_\mu$. In spite of complicated appearance of these expressions, the counter-Lagrangian is simply given by

\begin{equation}
\Delta L = -\frac{5}{6\epsilon} g^* \text{Tr} (G_{\mu} G_{\nu} + O(\alpha')),
\end{equation}

apart from the ghost contribution. The first order term of $\alpha'$ is miraculously cancelled out after the cyclic identity and the equations of motion are used. Similar vanishing of counter-Lagrangian occurs also in the pure-gravity calculation.

From the gauge breaking term (18), we see that the Faddeev-Popov ghost Lagrangian is

\begin{equation}
\mathcal{L}_{\text{Faddeev-Popov}} = \phi^* (D_\mu D_\mu - \frac{3}{2} g^* \alpha' G_{\mu} G_{\mu}) \phi + \text{irrelevant terms},
\end{equation}

where the fields $\phi_a$ are the well-known complex scalars with wrong statistics. From (26), the ghost contribution to the counter-Lagrangian is calculated:

\begin{equation}
\Delta L_{\text{ghost}} = -\frac{1}{12\epsilon} g^* \text{Tr} (G_{\mu} G_{\nu}) + O(\alpha'^2).
\end{equation}

The first order term again vanishes. Thus to first order of $\alpha'$ the total counter-Lagrangian is given by
The relations between the bare and renormalized quantities are taken to be

\[ A^B = \left(1 - \frac{11}{3\epsilon} g^B R\right) A^R, \]  

\[ g_B = g_R \left(1 + \frac{11}{3\epsilon} g^B R\right) \]  

and

\[ \alpha'_B = \alpha'_R \left(1 + \frac{22}{3\epsilon} g^B R\right). \]

In comparison with the case \( \alpha' = 0 \), (29) and (30) are unaffected by the presence of the second term in (16) in our lowest order approximation. Especially, \( g_R = 0 \) is an ultra-violet stable fixed point. On a dimensional ground, we may expect that to all orders of \( \alpha' \) this stability is valid because \( \alpha' \) which diverges cannot enter into the relation between \( g_B \) and \( g_R \). This "asymptotic freedom" may be a reason why the dual Born amplitudes are good approximations to the hadronic world, since with a fixed number of external lines the perturbative expansion of dual theory is an expansion in \( g \).

§ 5.

Finally we would like to make two speculative remarks. Our calculations are only in the lowest non-trivial order approximation. To proceed up to higher orders of \( \alpha' \) will be a formidable task. This direction makes us feel that the requirement of renormalizability might uniquely determine higher terms by an iteration from the first order term (16). In order to achieve this, a new technical development seems to be needed.

Very recently, calculations of one-loop counter-Lagrangian in quantized gravity have been performed by several authors. According to their results, Einstein's gravity theory coupled with matter is not renormalizable. From our study, we speculate the following possibility: The Shapiro-Virasoro model is reduced to the Einstein-gravity in an appropriate zero-slope limit. If the Einstein theory is replaced by the Shapiro-Virasoro model, it may be possible that the renormalization of gravity can be achieved in the power series expansion of \( \alpha' \), as we have done for the Veneziano model in lowest order.

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