An Elementary Interpretation of the Average Behavior of Multiple Production Processes at Very High Energies

Masatoshi KOSHIKA

High Energy Physics Laboratory, and
Department of Physics, University of Tokyo, Tokyo

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It is shown that the model of ultra high energy interactions originally based on cosmic ray data can account for virtually all the CERN-ISR and FNAL data, including the angular distributions, the $x$-distributions, the $(\gamma_{\text{max}} - \gamma)$-distributions and the large transverse momentum distributions of single particles as well as the correlations of charged particles to the large transverse momentum $\gamma$-rays.

§ 1. Introduction

The purpose of this paper is to show that a model\(^{1-9}\) of ultra high energy interaction proposed six years ago, which is based on the cosmic ray, was essentially confirmed as well as elaborated by the recently published experimental data of CERN-ISR and FNAL.

The model was based on the results of mass separation analysis of the target fragmentation products produced in the interactions of quasi-monochromatic nucleon beam resulting from the break-ups of ultra high energy heavy nuclei in cosmic radiation.

The model is of two-component type. That is, in inelastic collisions at ultra high energy, the nucleon shakes off its meson cloud and with high probability becomes a specific isobar named Aleph which subsequently decays.

The first component, the shaken-off meson cloud, was called the Pionization\(^*\) component and was characterized by the following:

(a) High multiplicity of $1.75E_0^{1.4}$ or $1.25 \ln E_0$ with $E_0$ the primary laboratory energy of the incoming nucleon, in GeV.
(b) Small $K/\pi$ ratio of $(0.12 \pm 0.04)$.
(c) Average transverse momentum slowly increasing with $s$.
(d) Primary energy insensitive C-M angular distribution approximately proportional to $(1 + 3 \cos^2 \theta) \times d(\cos \theta)$.

The Aleph component had been characterized by:

(a) Small multiplicity independent of the primary energy.

\(^*\) To the knowledge of the author, the concept and the name of “Pionization” was first introduced by G. Cocconi. The (Isobar+Pionization) model was first introduced by Y. Pal and B. Peters.
(b) Large $K/\pi$ ratio of $(0.71 \pm 0.14)$.
(c) Small average transverse momenta of $\pi^\pm$ and $K^\pm$.
(d) Narrow C-M angular distribution in the direction of the collision axis, $<15^\circ$ in C-M system.

From the invariant mass distributions of mass-identified particles as well as from their C-M angular distributions, it was concluded that they were presumably the decay products of the Aleph $\Sigma$ characterized by:

\[
\begin{align*}
\text{Mass; } & (2015 \pm 30) \text{ MeV}, \\
\text{Width; } & <30 \text{ MeV}, \\
\text{Isospin; probably } & 1/2, \\
\text{Strangeness; zero,} & \\
\text{Decay; } & (70\% \text{ to } (N+\phi)) \\
& (30\% \text{ to } (N+\gamma)),
\end{align*}
\]

though it was stressed in an original paper$^3)$ that the Aleph component might consists of a mixture of a few $\Sigma$'s with different masses decaying into $(N+\phi^0)$, $(N+\omega^0)$, etc., as shown in Table I which was reproduced from Table III of Ref. 1.$^*)$

Furthermore, the transition probability $P(N \rightarrow \Sigma)$ of nucleon to Aleph was estimated to be

\[
P(N \rightarrow \Sigma) = \begin{cases} 
<2 \times 10^{-3} \text{ at } \langle E_\sigma \rangle \text{ of } 10 \text{ GeV}, \\
<0.1 \text{ at } \langle E_\sigma \rangle \text{ of } 200 \text{ GeV,} \\
= (0.7 \pm 0.2) \text{ at } \langle E_\sigma \rangle \text{ of } 1700 \text{ GeV,} \\
\text{probably equal to or larger than the above at still larger } E_\sigma.\end{cases}
\]

These were the conclusions obtained from the cosmic ray experiments on ultra high energy interaction. For the sake of discussion in the following sec-

$^*)$ It is interesting to note that some of the vector meson-nucleon resonances of this table are being observed recently though with a very small production cross-sections consistent with our estimation of $(p \rightarrow \Sigma) < 2 \times 10^{-4}$. That is, A. Davidson et al. (Phys. Rev. Letters 32 (1974), 855) reported the 10 $\mu$b cross-section for production of $(\omega\phi)$ resonance at 1820 MeV in $\pi^p$ collisions of 4.5 to 14 GeV/c. This article contains the references reit’d to the subject. The invariant mass distribution obtained by P. J. Davis et al. (Nucl. Phys. B44 (1972), 344) for $(p+\omega)$ from $K^p$ at 12 GeV/c is especially interesting since it does seem to show the peaks at $\approx 1820$ MeV and just above 2000 MeV, although the authors themselves state that there are no narrow structures. The invariant mass distribution of $(p+\phi)$ in the same paper also shows a bump in the region of 2 to 2.6 GeV which may be regarded as the formation of $\Sigma$ (2015) to $(p+\phi)$ with a cross-section of about 2$\mu$b. Both the indications put together it might be that the $\Sigma$ (2015) has channel open for decaying into $(p+\phi)$. D. S. Aures et al. (Phys. Rev. Letters 32 (1974), 1463) analyzed the $\phi$-productions in $\pi^p$ and $K^p$ collisions at 3 to 6 GeV/c. After trying various Regge exchange models without much success, they hinted an alternate model by considering the $s$-channel effect of $\pi^p \rightarrow N^* \rightarrow \phi n$ on the condition that this $N^*$ is required to have unexpectedly large couplings to $(\phi\pi)$, which is exactly the property of our $\Sigma$ (2015±30).
tions, Figs. 1 and 2 were reproduced from Ref. 9).

We shall now look at these conclusions in the light of the recently published experimental results of CERN-ISR and FNAL.

Fig. 1. The angular distributions of mass-identified particles in the target fragmentation region. Cosmic ray data taken from Fig. 12 of Ref. 9).

Table I. Taken from Table III of Ref. 1). Possible mesons to be involved in the forward and the backward peaks in ultra high energy interactions. \( N_+, N_\pm \) and \( N_\pi \) are the expected numbers per decay of \( \pi^+, K^\pm \) and \( \gamma \)-rays, respectively.

<table>
<thead>
<tr>
<th>Candidate Boson (Mass in MeV)</th>
<th>Parent-Baryon Mass</th>
<th>(&lt;N_+&gt;)</th>
<th>(&lt;N_\pm&gt;)</th>
<th>(&lt;N_\pi&gt;)</th>
<th>Probability of decaying into neutrals only</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(1254) )</td>
<td>2192~2316</td>
<td>1.33</td>
<td>&lt;0.04</td>
<td>1.33</td>
<td>0.33</td>
</tr>
<tr>
<td>( B(1220) )</td>
<td>2158~2283</td>
<td>2.67</td>
<td>&lt;0.1</td>
<td>2.67</td>
<td>0</td>
</tr>
<tr>
<td>( A_1(1080) )</td>
<td>2018~2150</td>
<td>2.0</td>
<td>&lt;0.11</td>
<td>2.0</td>
<td>0</td>
</tr>
<tr>
<td>( \gamma'(958) )</td>
<td>1896~2036</td>
<td>1.9±0.11</td>
<td>0</td>
<td>3.65±0.34</td>
<td>0.18</td>
</tr>
<tr>
<td>( o(783) )</td>
<td>1721~1877</td>
<td>1.8</td>
<td>0</td>
<td>2.1</td>
<td>0.10</td>
</tr>
<tr>
<td>( p(758) )</td>
<td>1696~1855</td>
<td>1.33</td>
<td>0</td>
<td>1.33</td>
<td>0</td>
</tr>
<tr>
<td>( \rho(549) )</td>
<td>1523~1716</td>
<td>0.54</td>
<td>0</td>
<td>3.20±0.23</td>
<td>0.73</td>
</tr>
<tr>
<td>( \pi(140) )</td>
<td>1078~1627</td>
<td>0.67</td>
<td>0</td>
<td>0.67</td>
<td>0.33</td>
</tr>
<tr>
<td>( \phi(1019) )</td>
<td>1957~2094</td>
<td>0.24±0.08</td>
<td>0.96</td>
<td>0.24±0.08</td>
<td>0.40</td>
</tr>
<tr>
<td>( f'(1500)^a )</td>
<td>2438~2553</td>
<td>0.38</td>
<td>0.75</td>
<td>1.45</td>
<td>0.44</td>
</tr>
</tbody>
</table>

The table is based on "Data on Particles and Resonant States" by A. H. Rosenfeld et al., Rev. Mod. Phys. Jan. (1967).

\(^a\) For lack of sufficient experimental data, we used the theoretical estimates of S. L. Glashow and R. H. Socolow (Phys. Rev. Letters 15 (1965), 329) which were found in agreement with the existing data.

§ 2. The Pionization component

2-a The charged particle multiplicity of the Pionization component

The average charged multiplicity of this component was given by either \( 1.75 E_\pi^{1/4} \) or \( 1.25 \ln E_\pi E_0 \) in GeV, and when combined with the multiplicity of the Aleph component, \( 2.4 \pm 0.4 \), it reproduces the ISR data successfully. One can indeed look at the points labelled "the other cosmic ray data" in Fig. 40 of the review article by Giacomelli in Batavia Conference. The multiplicity distribution had been noted to be much broader than that of Poisson and it is interesting to see the analysis of NAL multiplicity data by Lach and Mulamud based
on the two-component model which gave 1.3 for the small multiplicity component at 200 GeV while cosmic ray data\(^7\) gave 1.5 at the same average energy.

2-b \textit{The} \(K/\pi\) \textit{ratio of the Pionization component}

In fact, we were the first\(^1\) to conclude experimentally that the overall \(K/\pi\) ratio of \((0.24 \pm 0.08)\) measured by many authors for the particles produced in cosmic ray high energy interactions is a result of mixing the two components of high and low \(K/\pi\) ratios and that the \(K/\pi\) ratio of the Pionization is \((0.12 \pm 0.04)\) at \(\langle E_0 \rangle = 1700\) GeV\(^1\) and \((0.07 \pm 0.03)\) at \(\langle E_0 \rangle = 200\) GeV.\(^7\)

The results of CERN-ISR refer mostly to the Pionization component as will become clear in what follows. The Bologna-CERN-UCR collaboration\(^9\) gives \(K^+/\pi^+ = 0.09\) at \(\sqrt{s} = 44\) GeV for the angular range of 80 to 350 mrad. The CERN-Holland-Lancaster-Manchester collaboration\(^9\) also gave the overall values \(K^+/\pi^+ = 0.13 \pm 0.01\) and \(K^-/\pi^- = 0.08 \pm 0.01\). However, in this experiment the measured angular range is from 40 to 100 mrad and is expected to contain a considerable fraction of the \(\pi^+\)'s, a small fraction of the \(K^\pm\)'s and negligible fraction of protons of the Aleph component. In fact, one can see in Figs. 2 and 3 of Ref. 9) the increasing trend of \(\pi^+/p\) ratio and \(K^+/p\) ratio particularly at small values of \(X \sim 0.16\) and \(X \sim 0.2\), respectively.

These are the \(X\)-regions where one can expect the \(\pi\) and \(K\) components of Aleph to show up; see the later sections.

2-c \textit{The transverse momentum of the Pionization component}

A slow increase with the primary energy of the average transverse momentum of this component was suggested\(^3,5,4\) for \(E_0\) above 100 GeV.

The results of CERN-Columbia-Rockfeller, Saclay-Strassbourg, and British-

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{The \(\eta (=r_\theta \cos \theta /r_\theta \theta_0)\) distribution. Cosmic ray data taken from Fig. 17 of Ref. 9).}
\end{figure}
Scandinavian Collaborations as compiled by Jacob in Refs. 13) and 15) show the increasing enhancement of large $P_T$ tail for increasing $s$.

This certainly causes the expected slow increase in the average $P_T$, though the data have not been analysed in this manner. The large $K^\pm/\pi^\pm$ ratio of 0.49 observed for particles at $X=0$ and $2<P_T<3.5$ GeV/$c$ by the British-Scandinavian Collaboration is certainly very interesting and we shall come back to this in later sections.

2-d The C-M angular distribution of the Pionization component

The shape of the C-M angular distribution of this component was found insensitive to $s$. This was brought about in the ISR data as shown presently.

However, the approximate angular distribution of $(1+3 \cos^2 \theta) d(\cos \theta)$ turned out not to be accurate enough to describe the ISR data. This is the only essential revision needed to cope with the vast amount of the accurate data of ISR experiments.

Thanks to this accumulation of ISR data, we can now propose the following formula for the description of the Pionization component:

$$E d^3 \sigma = \frac{d^3 \sigma}{\pi d(z') dy} = \eta(y) \cdot \zeta(z), \quad (1)$$

where the invariant cross-section is expressed as a factorized function of transverse momentum $z$ and rapidity $y$. The functions $\eta$ and $\zeta$ are given by

$$\eta(y) = \frac{\cosh y \cdot \cosh \delta}{\cosh^2 y + \sinh^2 \delta}, \quad (2)$$
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\[ \zeta(z) = A \cdot \exp\left(-\sqrt{z^2 + m^2/\alpha}\right), \]

where \( \cosh \) and \( \sinh \) signify the hyperbolic functions and the three parameters \( A, \alpha \) and \( \delta \) are all slowly increasing functions of \( s \). The parameter \( A \) is mostly responsible for the \( s \)-dependence of the multiplicity. The parameter \( \alpha \) fixes the slope of the transverse momentum distribution and is especially immune to the change of \( s \). The meaning of the parameter \( \delta \) can be seen by observing that \( \eta(y) \) can be transformed to

\[ \eta(y) = \frac{1}{2} \left( \frac{1}{\cosh(y-\delta)} + \frac{1}{\cosh(y+\delta)} \right). \]  

It is then clear that the rapidity distribution consists of the two emitting centers placed at \( y = \pm \delta \). However, these emitting centers are different, as will be shown shortly, from the conventional Two-Fire-Balls and we shall call them "MVR"'s which stand for Meson-Vortex-Ring's.

The rapidity distribution of the Pionization component is then given by

\[ \frac{d\sigma}{dy} = \eta(y) \cdot \int_{\delta}^{\gamma_{\text{max}}(y)} \zeta(z) \cdot \pi \cdot d(x^2) = \eta(y) \cdot Z(s), \]

where the upper limit of the integration can be safely extended to \( \infty \) because of the exponential dumping of \( \zeta(z) \).

With the approximation \( y = y' = -\ln(\tan \theta/2) \) which is valid for \( \gamma > 1 \), \( \gamma \) being the \( \gamma \)-factor of the particle under consideration, we can compare this formula with the preliminary results of Pisa-Stony Brook experiment. The comparison is shown in Fig. 3 and the parameter \( \delta \) is determined so as to be 1.2 at \( s = 2800 \text{ GeV}^2 \) and 1.1 at \( s = 900 \text{ GeV}^2 \). Also is shown in the figure the attempted fit by Two-Fire-Balls which is obviously inadequate. Besides the additional particles at both the ends of the distribution, which we shall come back to shortly, there are two uncomfortable points at \( y = \pm 0.3 \). This region was covered by the experiment of CERN-Hamburg-Orsay-Vienna collaboration in which the charged particle angular distribution is measured by Charpak chamber system at \( 36^\circ < \theta < 90^\circ \), i.e., \( -1.3 < y < 0 \). The fit is made on the above approximation \( y = \ln(\tan \theta/2) \).

That is, with this approximation Eq. (4) gives

\[ \frac{d\sigma}{d\Omega} = \frac{1}{2\pi} \cdot Z(s) \cdot \frac{1}{\sin \theta} \cdot \left( \frac{\cosh \delta}{1 + \sin^2 \theta \cdot \sinh^2 \delta} \right). \]  

The fit with \( \delta = 1.195 \) is shown in Fig. 4(a) and with just one single additional parameter it is certainly much better than the widely used form of \( 1/\sin^2 \theta \). The same fit is applied to the data of Barbiellini et al. and is shown in Fig. 4(b) where one observes the smallest angle point at \( 48.9^\circ \) considerably above the curve but this is at the edge of their detector and this increase is not observed in Fig. 4(a), so we do not worry about. In these comparisons the same \( \delta \)
value was used for all the \( s \) values in order just to illustrate the applicability of Eq. (5). In these two experiments the apparent increase at \( \gamma \approx \pm 0.3, \theta \approx -73^\circ \) in Pisa-Stony Brook experiment is not observed and, hence, we forget it.\(^*\)

The more accurate formula for the angular distribution is obtained from Eqs. (1), (2) and (3).

Namely,

\[
\frac{d\sigma}{dQ} = \frac{1}{2} \cdot \frac{1}{\sin \theta} \cdot \int d(z') \cdot \zeta(z') \cdot \frac{z \cdot \sqrt{z^2 + m^2 \cdot \cosh \delta}}{z^2 + \sin^2 \theta \cdot (z^2 \sinh^2 \delta + m^2 \cosh^2 \delta)} \tag{5'}
\]

which reduces to (5) for \( z^2 \gg m^2 \), an approximation admissible from the observed average transverse momentum, at least for pions which constitute the majority in the Pionization component. We now turn our attention to the transverse momentum distribution at 90°. Equation (1) gives

\[
\frac{E d^3\sigma}{d^3p \mid \theta=90^\circ} = \eta(0) \cdot \zeta(z) = \zeta(z).
\tag{6}
\]

The comparison of this formula with experimental data was already made in Ref. 16), and the parameter \( \alpha \) is thus fixed to be 6.0 (GeV/c). We note also that the overall transverse momentum distribution of this

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4}
\caption{The "MVR" fit for the angular distribution.}
\end{figure}

\(^*\) After preparing this manuscript we found that in their own publication of their experiment, these points have been corrected to smaller values in agreement with this conclusion; see G. Bellettini et al., Phys. Letters 45B (1973), 69.
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A component can be given by integrating Eq. (1) with respect to \( y \) and we obtain

\[
\frac{d\sigma}{d\mu} = \pi \cdot A(s) \cdot \left[ 4 \text{Arc tan} \left( \frac{E_{\text{max}} + \sqrt{E_{\text{max}}^2 - \mu^2}}{\mu} \right) - \pi \right] \cdot \exp \left( -\frac{\mu}{\alpha} \right),
\]

(6')

where \( \mu^2 = P_{\perp}^2 + m^2 \) and \( E_{\text{max}} \) is the maximum possible energy of the observed particle kind of mass \( m \) in the “MVR” rest system. The total multiplicity of this component is now given by

\[
N^{(m)}(s) = \frac{1}{s_{\text{inel}}} \cdot \pi \cdot \int d(z^2) \cdot dy \cdot \frac{\zeta(z) \cdot \eta(y)}{\sqrt{z^2 + m^2}}.
\]

(7)

The \( x \)-distribution at given transverse momentum is given by

\[
\frac{E}{\pi \sqrt{s}} \cdot \frac{d^2\sigma}{d(x^2) \cdot dx} = \zeta(x) \cdot \frac{\sqrt{x^2 + m^2} \cdot \sqrt{sx^2/4 + z^2 + m^2} \cdot \cosh \delta}{(sx^2/4) + (z^2 + m^2) \cdot \cosh^2 \delta}.
\]

(8)

which results from Eqs. (1), (2) and (3) by the change of variable

\[
y = \frac{1}{2} \ln \left( \frac{E + P_{11}}{E - P_{11}} \right), \quad x = 2P_{11} / \sqrt{s}, \text{ and hence}
\]

\[
cosh^2 y = \frac{(sx^2/4) + z^2 + m^2}{z^2 + m^2}.
\]

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**Fig. 5.** The “MVR” fit of the \( x \)-distribution and the possible indication of the Aleph. See text. The data were taken from Fig. 1 of Ref. 21.

**Fig. 6.** The “MVR” fit of the \( (y_{\text{max}} - y) \) distribution and the possible indication of the Aleph. See text. The data were taken from Fig. 38 of Ref. 11.
This distribution with the choice of \( \delta = 1.2 \) corresponding to \( s = 2800 \text{ GeV}^2 \), is compared with the results of various ISR experiments as compiled by Bellettini\(^{16}\) and is shown in Fig. 5. One should not expect this distribution to hold at large \( x \) because the overall energy momentum conservation should certainly give a considerable reduction. However, the fit in general seems quite satisfactory except in the region of \( x \sim 0.1 \) in the low \( Z, P_\perp = 0.2 \text{ GeV}/c \), data in which one sees a considerable additional particles accumulated above this "MVR" fitting. We shall come back to this point in the next section. One should note the fact that the experimental data correspond to various \( s \) values while the curve is for \( s = 2800 \text{ GeV}^2 \). Judging from the form of Eq. (8), the author feels that it would be more advisable to work with the variable \( P_{1\parallel} \), or still better \( P_{1\parallel}/\sqrt{z^2 + m^2} \). Equation (8) then becomes

\[
E \cdot \frac{d^3\sigma}{\pi d(z') dP_{1\parallel}} = \zeta(z) \cdot \frac{\sqrt{z^2 + m^2} \cdot \sqrt{P_{1\parallel}^2 + z^2 + m^2} \cdot \cosh \delta}{P_{1\parallel}^2 + (z^2 + m^2) \cdot \cosh^2 \delta},
\]

which, because of the slow variation with \( s \) of \( \alpha \) and \( \delta \), is more appropriate for the purpose of compiling the data. The measured \( (\gamma_{\text{max}} - \gamma) \)-distribution\(^{15}\) at \( Z = 0.4 \text{ GeV}/c \) is also compared with our distribution, Eqs. (1), (2) and (3), with \( \delta = 1.2 \) and is shown in Fig. 6. The \( \pi^\pm \) and \( K^+ \) data seem to follow the curve rather satisfactorily, while the \( K^- \) and \( \bar{P} \) data seem to behave differently at small \( (\gamma_{\text{max}} - \gamma) \) values. The \( p \) data show the opposite, increasing, behavior at small \( (\gamma_{\text{max}} - \gamma) \) values. We come back to this point in the discussion on the Aleph component.

In all these comparisons with the experimental data, the emphasis was given to show the adequacy of the form of our fitting formula by confronting it with the data of individual experiments. The comparison between the data sets of different experiments was not attempted unless they were compiled by somebody close to the experimental groups. The \( \chi^2 \)-test was not attempted. With a pocketable calculating gadget and without the first hand access to the experimental data including the knowledge of possible systematic errors involved, this is just about the limit the author can reach and he sincerely wishes that somebody at CERN close to the experimental groups and to the large computing facilities attempts a detailed fitting of the data with the formula described in this section.

Fig. 7. An artist drawing what happens at the asymptotic energies. "MVR" and Aleph.
Table II. The Aleph component.

<table>
<thead>
<tr>
<th>C-M angular range content, about 80% of particles</th>
<th>Cosmic Ray Data</th>
<th>Revision due to Accelerator Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>0&lt;40 mrad for proton 0&lt;60 mrad for K⁺ 0&lt;160 mrad for π⁻ for s=3200 GeV⁺</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>P_z range contain. 80% and &lt;P_z&gt; in GeV/c</td>
<td>Proton, P_z&lt;0.5 &lt;P_z&gt;=0.3±0.07</td>
<td>None</td>
</tr>
<tr>
<td>K⁺, P_z&lt;0.4 &lt;P_z&gt;=0.2±0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>π⁺, P_z&lt;0.4 &lt;P_z&gt;=0.2±0.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relevant X-range X=2P_{t1}/√s</td>
<td>Proton, ±(0.23~0.70)</td>
<td>None</td>
</tr>
<tr>
<td>K⁺, ±(0.11~0.32)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>π⁺, ±(0.04~0.13)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relevant γ-range</td>
<td></td>
<td>None</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average no. of particles in the relevant kinematical range</td>
<td>0.7±0.1 protons, 0.5±0.1 K⁺, 0.7±0.1 π⁺, 0.7±0.1 γ-rays per C-M hemisphere of inelastic collision</td>
<td>The number of π⁺ and γ-rays are to be reduced down to about 0.1. This is due to the improved fitting of Pionization component. See text.</td>
</tr>
<tr>
<td>Formation probability</td>
<td>&lt;2×10⁻¹, E_0&lt;30 GeV, &lt;0.1, &lt;E_0&gt;=200 GeV, 0.7±0.1, &lt;E_0&gt;=1700 GeV, probably stays high at still higher energies</td>
<td>See text. 0.5±0.1, &lt;E_0&gt;=1700 GeV</td>
</tr>
<tr>
<td>Mass, M, Width, Γ, and Decay</td>
<td>M=2015±30 MeV, Γ&lt;30 MeV, and 70% to (p+π), 30% to (p+γ)</td>
<td>Decay [mostly (p+φ)] (p+ω)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>See Text.</td>
</tr>
</tbody>
</table>
§ 3. The Aleph component

Before we enter into the discussion of this component, we look at Table II which shows the regions of kinematical variables where the Aleph component should be seen. The table was prepared by the use of the cosmic ray data of Ref. 9) (see also Figs. 1 and 2). In the last column is given the revisions brought about by the improvement of the Pionization fitting as described in the preceding section. The first column of Table II gives the C-M angular range which should contain about 80% of the particles belonging to this Aleph component. The values are for the average \( s \) of 3200 GeV\(^2\) corresponding to \( \langle E_0 \rangle = 1700 \) GeV and they are expected to vary as \( s^{-1/2} \). Among the ISR experiments referred to in (11), (13) and (15) the ones which should contain at least a part of the Aleph component are CERN-Holland-Lancaster-Manchester collaboration covering 20 to 200 mrad., Bologna-CERN-UCR collaboration covering 80 to 350 mrad, Pisa-Stony Brook collaboration covering 2° to 90°, and Aachen-CERN-Genoa-Harvard-Turin collaboration covering 0.04<\( t < 4.0 \) GeV\(^2\). The second column of Table II gives the \( P_{\perp} \) range containing about 80% of the particles belonging to the Aleph component. The third and fourth ones give the \( x = 2P_{\perp}/\sqrt{s} \) range and the rapidity \( y = \frac{1}{2} \ln \left( (E + P_{\perp})/(E - P_{\perp}) \right) \) range, respectively, where the Aleph component is expected to be seen. These ranges were converted from the \( \eta = \gamma \beta \cos \theta / \gamma_0 \delta_0 \) range in the fifth one, in terms of which the cosmic ray data were presented\(^9\) (see Fig. 2). The \( \gamma_0 \delta_0 \) here refers to the initial proton in C-M system. The cosmic ray data on the Aleph component were obtained at \( \theta = \pi \) and hence \( |\eta| = \gamma \beta / \gamma_0 \delta_0 \). It is interesting to note the result reported by Cronin et al. on large \( P_{\perp} \) hadrons from FNAL which says the use of the variable \( x = 2P_{\perp}/\sqrt{s} = (m/M) \cdot \gamma \beta / \gamma_0 \delta_0 \) at \( \theta = 90^\circ \) gives a simpler and more sensible presentation of the experimental data. The sixth one gives the average number of particles belonging to the Aleph component per interaction and per C-M hemisphere. The seventh one gives the transition probability of nucleon to Aleph in inelastic collision and the last one deals with the properties of the isobar Aleph as deduced from the cosmic ray data.

We shall now see if the sign of Aleph is in fact visible at the expected region of kinematical variables and not visible at the unexpected regions.

We begin with the data of Pisa-Stony Brook collaboration\(^{15}\) which measured the angular distribution of charged particles, mass-unseparated, and expressed the results in terms of \( y' = - \ln \tan (\theta/2) \) which is a good approximation of the rapidity \( y \) for not too small angles. This experiment covering the angular range of 35 mrad to \( \pi/2 \) radian is expected to contain some fractions of charged particles belonging to the Aleph component (see Table II). In fact we observe the additional particles at the expected region of \( y \). In Fig. 3 these additional particles are shown together with the expected ranges for \( p, K^\pm \), and \( \pi^\pm \) taken from Table II. The effect is more clearly seen in their \( y \)-distributions of dif-
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Different multiplicity groups as presented in Fig. 8(a) of Ref. 13. It is considerably pronounced in the low multiplicity group of $1<n<5$ where the background due to the Pionization is small. Looking now at the mass-separated data, we have already noticed an enhancement of $\pi^\pm$ only at $Z=0.2$ GeV/c and $X\sim 0.1$, as shown in Fig. 5. This is exactly the region where we expect the $\pi^\pm$ of the Aleph component, see Table II, and the effect is not seen at other $Z$ or $X$ regions where we do not expect to find it.

This enhancement is observable when $E_0$ exceeds 300 GeV as can be seen in Figs. 14 and 15 of Ref. 20) and Fig. 14 of Ref. 15). The effect does not seem to be observable at $E_0$ of 200 GeV. The possible sign of increase in $\pi^+/\rho$ at the respective $X$-regions had been noted before in connection with the $K/\pi$ ratio of the Pionization component.

There are some more indications of Aleph in the proton data though diluted because of the angular limitations mentioned at the beginning of this section. The rapidity distribution of protons as compiled by Lilletthun shows an increased accumulation of protons in the region around $y_{\text{max}} - y = (0.4 \sim 1.5)$ for $P_1 = 0.4$ GeV/c, while the increase is much less conspicuous for $P_1 = 0.7$ GeV/c (see Table II). Another possible information on the Aleph may be seen in the results of CERN-Holland-Lancaster-Manchester collaboration as shown in Figs. 12(a) and 12(b) of Ref. 13). They measured the $X$ distribution of the leading proton at 40 mrad in coincidence with pions at $90 \pm 14^\circ$. One sees a considerable increase of protons above $X \geq 0.99$ which are not associated with the large-angle pions. The simplest interpretation would be that these protons just excite the other proton diffractively. With this supposition, one can calculate the mass $M$ of the recoiling excited state by the formula $M^2 - M_{\pi}^2 = (1 - X)s$. The mass $M$ thus calculated turns out to be less than 3 GeV. Combined with the data on the same experiment at smaller $s$, one arrives at the conclusion that the mass of the recoiling excited state cannot exceed 2 GeV much. This is in good agreement with the mass of Aleph given in Table II as well as the masses in Table I.

§ 4. Discussion

We seem to find some evidence for the Aleph, though it is still fragmentary. Obviously, the spectrometer data at angles down at least to 10 mrad or so are necessary for proving or disproving the alleged dominant role at ultra high energy of the Aleph. Another possibility would be to analyse the fragmentation of target proton hit by the highest energy beam of NAL.

It is admittedly still premature to think of the question whether the Aleph is simply an aggregate of collimated proton, kaons and pions or a definite isobar. The author is inclined to think that it is a definite isobar not only because of the observed invariant mass distribution but also because of the following consideration.
That is, the \( s \)-dependence of the slope parameter of \((p-p)\) elastic scattering becomes less steep between 150 GeV\(^2\) to 900 GeV\(^2\). Let us accept the view that elastic scattering is essentially governed by the shadow of the dominant inelastic processes and proceed to estimate the \( s \)-dependence of the slope parameter along the line of Van Hove.\(^{23}\) The Pionization component with an \( s \)-insensitive angular distribution would give it only a mild increase with \( s \) only due to the slow increase of its multiplicity. The effect of the Aleph is quite different according to whether it is simply an aggregate of particles or it is a single entity which decays into those particles. Namely, when it is just an aggregate the cone of these particles would shrink as \( s \) increases thereby causing the slope parameter to increase considerably with \( s \). When it is in fact a definite isobar, the slope parameter would not increase anymore once it becomes the dominant process and one is left with a mild \( s \)-dependence only due to the Pionization component. The existing data seem to favor the latter view and if this is indeed the case one may be able to see the Aleph already at 450 GeV interaction.

Concerning the Pionization component or "MVR"\(^{s}\), it is rather surprising that the simple formula (1), (2) and (3), with essentially two adjustable parameters \( \alpha \) and \( \beta \) can reproduce so many data and the author is inclined to think that it actually is representing a physical entity. From Eq. (2') we see that the angular distribution in its rest system is proportional to \((1/\sin \theta) \cdot d(\cos \theta)\). It is not easy to find a theory which can explain this particular form of the angular distribution but we can certainly reinvestigate the classical papers of Fermi,\(^{24}\) Landau\(^{25}\) and Heisenberg\(^{26}\) for a possible explanation.

The author at present has the following picture. That is, when the nucleon with its meson cloud receives a sharp enough impulse, the core, the Aleph, shakes off its meson cloud\(^{29}\) which in the form of a vortex ring moves slowly as a whole. It can be easily shown that vortex ring motion can give the above angular distribution, \((1/\sin \theta) \cdot d\Omega\), in its rest frame for the evaporated mesons. The description for this physical entity may be found in the theory of viscous fluid rather than in the theory of elementary particles. Note that we may be already in the asymptotic energy range and we are to deal with a cloud of many low energy mesons interacting strongly with each other. We need to consider a viscous fluid, otherwise we cannot produce a vortex ring; recall Lagrange-Helmholz theorem.\(^{27}\) Furthermore, when the shaking-off of the meson cloud by one of the two colliding nucleons occurs independent of the shaking-off of the other nucleon, we have a natural explanation of the experimental observation by Pisa-Stony Brook that the multiplicities in the two hemispheres are independent of each other. In Fig. 7 an artist, hopefully, view of the picture of meson vortex ring, "MVR", is shown.

The observation of a large \( K/\pi \) ratio of 0.49\(^{29}\) for the large momentum particles at 90° may be explained by the large-angle deflection of the produced Aleph. The reported positive excess would then be due to the decay proton,
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because the Aleph produced at ISR would be mostly of positive charge as long as the iso-scalar vacuum trajectory is mostly operative at ultra-high energies.

Things seem to look so far so good with this picture. There are however a few disturbing points we admit. Namely, the behavior of $K^-$ and $\bar{p}$ at large $X$, or small $(\gamma_{\text{max}} - \gamma)$, seems to deviate from our “MVR” fitting. The observed $K^+/K^-$ ratio larger than unity is certainly related to this effect. A remedy for this would be to consider either that it is the effect of the overall energy momentum conservation or that the $y$-distribution of the $K$’s and $\bar{p}$’s pair-produced in a “MVR” has a form different from that of the $\pi$’s, i.e., getting closer to the isotropic distribution as the mass of the particle becomes larger. If we do accept this remedy, the resulting excess $K^+$ in the region $(\gamma_{\text{max}} - \gamma) < 2$, see Fig. 6, would have to be associated with the baryon. The only possibility would be to consider the process (“$N^*$”)$\rightarrow Y^+ + K^+$). In this connection, we note the fact that at 200 GeV incident energy the inclusively produced $\Lambda$ seems to show a relative increase at $X = \pm 0.5$ as compared with the results at lower energies.

This observation, however, does not invalidate our conclusion on the Aleph. It simply means that, besides our Aleph, such “$N^*$” are also produced.

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**Note added:** The original manuscript of this paper was prepared at the time of the International Symposium on High Energy Physics held in Tokyo in July, 1973. The further accumulation of the experimental data made the author feel some additional comments are in order. Most of them were added in the form of footnotes in this article. There are, however, still some more to say.

In this article, we omitted discussion on the two and/or three particle rapidity correlations, the so-called Short-Range-Correlations, because we believe it can be fitted by considering that the original product in "MVR" is an appropriate mixture of vector mesons which subsequently decays into pions, kaons and nucleon-antinucleon pairs.

In fact, for instance, a simple octet mixture of $\rho^+ : \rho^- : \omega^+ : \phi^+ = 1:1:1:1/4:3/4$ would produce a $K/\pi$ ratio close to the observed value.

Since the original manuscript of this paper was prepared in summer of 1973 by using the experimental results reported at the Batavia Conference, a considerable amount of further experimental data have been published. Especially the results on large transverse momentum particles are extremely interesting in the sense described in the text. Among others, the observations of $\phi^0$, $\omega^0$, $\rho^+$, $\pi^+ \pi^- \sim 10^4$, $\mu^+ / \mu^- \sim 1$, $e^+ / e^- \sim 1$, and $e^+ / \mu^+$ slightly larger than 1, are quite compatible with a mixture of $\phi^0$ and $\omega^0$ as the parent. Furthermore, if we consider the propagator $\propto t^{-1}$ thus making the cross-section $d\sigma / dt \propto c^{-4}$, it seems likely to reproduce the relative rate at different angles although at this moment there are not enough experimental data at different angles. In this connection, the author would like to suggest the use of the variable $(r \phi / r^0 \phi^0)$ for the analysis of large momentum mass separated particles.

The conditional particle densities observed around the equator and in the collision axis directions when a high energy $\gamma$, or $\gamma$'s, is detected at 90° are quite consistent with the picture shown in Fig. 7, in which Meson-Vortex-Rings are created in the directions of scattered Alephs. One can see this readily by looking at Fig. 3 of which in this case the rapidity axis is in the direction of the scattered Alephs and its decay products $\phi^0$ and/or $\omega^0$, decayed into $\pi^+ \pi^- \pi^0$ or into the $3\gamma$'s.

There still remains the next question how the scattered Alephs can create Meson-Vortex-Rings and this will be discussed elsewhere.

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