Pion Rescattering Correction to the $^3\text{He}(\gamma, \pi^+)\text{H}$ Reaction near Threshold

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The reaction $^3\text{He}(\gamma, \pi^+)\text{H}$ is attractive for studying pion photoproduction in a complex nucleus. The calculations which have already been made theoretically are divided into two different approaches. As the $^3\text{He}-\text{H}$ system is analogous to the neutron-proton system, Griffiths and Kim have treated $^3\text{He}$ and $^3\text{H}$ as “elementary particles” of spin 1/2, and calculated the differential cross section using the method of current algebra and PCAC hypothesis in the soft pion approximation. The same treatment was improved by Pascual et al. Their results are comparatively in good agreement with the experimental data at low energies, and their approximation is expected to be more accurate at threshold. On the other hand, the calculations using the nuclear wave functions and the impulse approximation are always larger than the experimental results in the photon energy between 180 and 500 MeV. Lazard et al. have suggested that the rescattering corrections are very important in this energy region from a simple estimation. The importance of this effect is related to the complexity of nuclei. In general, the problem of calculating the pion rescattering effects in light nuclei is a difficult one, since it is not easy to construct an effective pion-nucleus potential. In this short note we report the rescattering effects of pion on the reaction $^3\text{He}(\gamma, \pi^+)\text{H}$ near the threshold. This reaction may be considered as the inverse process of radiative pion absorption. We write the interaction density for the leading term in pion photoproduction near the threshold as:

$$H_{\gamma\pi} = (4\pi/\mu) \langle ef/4\pi \rangle (4k_0 \omega_q)^{-1/2} \times \exp[i(k \pm q) \cdot r] \sqrt{2} \cos \alpha \times \frac{\tau \cdot \sigma_- (q) - \tau \cdot \sigma_+ (q)}{1 + \mu}, \quad (1)$$

where $k$ and $q$ are the momenta of the photon and the pion, $k_0$ and $\omega_q$ their energies, $e$ the polarization vector of the incident photon and $\mu$ the pion mass. The operator $\sigma$ and $\tau$ are the usual nucleon spin and isospin operators, and $\sigma_-$ is the operator that annihilates $\pi^-$ or creates $\pi^+$, etc. As we treat the region of pion threshold, we take into account only the $S$-wave as the rescattering interaction. We introduce the non-charge exchange ($H_{\gamma\pi}^{(1)}$) and charge exchange ($H_{\gamma\pi}^{(2)}$) interaction densities as the rescattering interaction. Here $\pi(x)$ is the canonical momentum of pion field $\phi(x)$, and $\lambda_1^0 = (1 + (\mu/m)) \lambda_1^p$ is the constant. In the present calculation we use the parameters $\lambda_1^0 = 0.0068$, $\lambda_2^0 = 0.0487$ which are obtained from the $S$-wave $\pi-N$ scattering phase shifts. For the process in which a single free pion is absorbed by or emitted from a nucleon, $H_{\gamma\pi}^{(1)}$ and $H_{\gamma\pi}^{(2)}$ can be written as follows:

$$H_{\gamma\pi}^{(1)} = (4\pi \lambda_1^0/\mu) (4\omega_q \omega_q)^{-1/2} \times \exp[i(-q + q') \cdot r] \cdot 2 (\sigma_- (q) \sigma_+ (q') + a_+ (q) a_- (q') \cdot (q')),$$

$$+ a_+ (q) a_- (q') + a_0 (q) a_0 (q')}, \quad (2)$$
In perturbation theory, the transition amplitude $T(q,k)$ for the process, including rescattering (Fig. 1), is given by

$$T(q,k) = T_0(q,k) + T_1(q,k) + T_2(q,k),$$

(4)

where

$$T_0 = \sum_{i,j} H_{ss}(i),$$
$$T_1 = \sum_{i,j} H_{ss}(i) (E-H+i\epsilon)^{-1} H_{ss}(j).$$

(5)

Neglecting the excitation of nucleus in the intermediate states, the effective interaction operator can be readily written down as follows:

$$T(q,k) = (4\pi/\mu) \langle \omega_\phi | G_{\phi} \rangle (k_\phi \omega_\phi)^{-1/2} \sqrt{2} \times \left[ \sum_{i} (\sigma_{is}) \tau_{i} - \exp[i(k-q)r_i] \right]$$
$$\times \left\{ \sum_{i,j} \exp[ikr_i - iqr_j] (\sigma_{ij}) \times (2\lambda_i/\mu) \tau_{i+} + (\lambda_j/\mu^2) (\omega_q + k_\phi) \tau_{j0} \tau_{i+} \times \frac{\exp[i\sqrt{\lambda_i^2 - \mu^2} r_{ij}]}{\lambda_i^2 - \mu^2} \right\}.$$  

(6)

We have taken into account only the $S$ and $S'$ states for the initial and the final nuclear states. We assume an admixture of 4\% (= $P_3$) for the $S'$ state. The dominant $S$ state is normalized to $1-P_3$. For the radial wave function we have used the Gaussian form. We neglect the $S'$-$S'$ contribution. It is found that the contributions of the cross terms between $S$ and $S'$ states and of charge exchange terms are exactly zero. Using the cross section $(d\sigma/dQ)_f$ for the photopion production from a free proton, the following differential cross section for the process is obtained:

$$\frac{d\sigma}{dQ} = 0 \left[ F - \frac{4\lambda_i}{\mu} G \right] P_3^2,$$

(7)

where $0$ is the kinematic factor which arises in the evaluation of the cross sections from the matrix elements and $F = \exp[-(k-q)^2/18\alpha^2]$,

$$G = \exp[-(k-q)^2/18\alpha^2] (6\alpha^2/\pi |k+q|)$$
$$\times \int_0^\infty \frac{ldl}{l^2 - (k^2 - \mu^2) - i\epsilon}$$
$$\times \left\{ \exp\left[-\left(l - \frac{1}{2} |k+q|\right)^2/6\alpha^2 \right] \right\}.$$  

The second term in Eq. (6) represents the contribution from rescattering. There is no experimental data near the pion threshold up to the present time. Therefore the rescattering effect in Eq. (6) is evaluated numerically at $E_\gamma = 143$ MeV. It gives the suppression of 5\% at the most to the cross section calculated by the impulse approximation. It is unexpectedly small. This is consistent with the fact that the elementary particle treatment for the $^4$He-$^3$H system is a fairly good approximation in low energy limit. However, we have neglected the contributions of the $P$-wave pion-nucleon interaction and the absorption effect. It seems that they give an important correction to the impulse approximation.

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