Integration of an analytic element model in a stochastic analysis of infiltration into a complex unconfined aquifer system

E. K. Paleologos, T. S. Sarris and M. Tolika

ABSTRACT

A hydrologic model based on the Analytic Elements Method (AEM) has been developed for a phreatic aquifer in the General Separations Area (GSA), of the Savannah River Site (SRS), South Carolina, USA. The AEM is a semi-analytical method that relies on the superposition of individual closed-form solutions of elements representing the main hydrologic features of a site. Our study adopts techniques from the stochastic subsurface hydrology to assess the impact of precipitation (and consequently infiltration) variations on the flow field. The precipitation was considered as a random variable following a statistical model that was obtained from a record of the past 112 years. A measure of the variability of the hydraulic head field is obtained through Monte Carlo simulations and a discussion on the uncertainty in different regions of the model is provided. The ease of model development and the small processing power required by AEM models, make the method applicable not only to initial investigations for the identification of test parameters and boundary conditions but also as a tool that in a stochastic framework can provide initial estimations of uncertainty associated with certain model assumptions.

Key words | infiltration, Monte Carlo, stochastic hydrology

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INTRODUCTION

A hydrologic model based on the Analytic Elements Method (AEM) has been developed for a phreatic aquifer at the Savannah River Site (SRS), South Carolina, USA (Tolika and Paleologos 2004). The AEM is an object-oriented, semi-analytical method that consists of various types of elements, which represent the main hydrologic features of a site. The overall solution is obtained by superposition of the individual closed-form solutions of the analytical elements on a background uniform flow field (Strack 1989).

The modeled area is the General Separations Area (GSA), which covers 15 mi^2 within the central SRS. The site had been developed as a controlled area for nuclear material production and the disposal of several types of hazardous wastes has resulted in groundwater contamination. Our model incorporates the main hydrologic features of the area that include artificial recharge basins, river branches, and waste containment basins. Our current study adopts techniques from the stochastic subsurface hydrology to assess model uncertainty when infiltration variations are considered (Dagan 1989). In past modeling studies, estimates of the average surface recharge had been obtained using precipitation data from 1979 to 1984 (Cahill 1982; Hubbard 1984, 1985; Parizek & Root 1986). However, recent drought conditions in the south-eastern US have resulted in total rainfall estimates (for the period of 1999–2001) that are 20% lower than the corresponding average of the 1982–1984 period that was used in the GSA studies (NOAA 2002). The influence of the recent temporal variability in infiltration rate on the modeling results is the focus of this study, which is performed through a Monte Carlo analysis.

Preliminary statistical analysis of historical data for the state of South Carolina indicates that the average annual precipitation, P, can be considered as a random variable that follows a log-normal distribution. By choosing random values for the average precipitation from the distribution of P, one can obtain the corresponding infiltration rates and by using these as input parameters in the AEM model an ensemble of random hydraulic head fields can be obtained. These flow fields provide us with a measure of the potential variability in the actual flow field as a result of varying climatologic conditions. An important advantage of the AEM is the very low computational expense of a single AEM run, which leads to significant savings in the overall performance of studies that utilize a Monte Carlo framework to analyze climatologic scenaria. The objectives of this paper are to test the incorporation of stochastic concepts in the framework of the analytic element method and to evaluate the applicability of this method to studies of global, regional, or local climate change.

SITE DESCRIPTION

The General Separations Area (GSA) is situated at the central Savannah River Site (SRS), an area of 300 mi^2 in South Carolina. The SRS (SRS 1996) was developed in 1950 as a controlled area for nuclear material production (primarily tritium and plutonium-239) for national defence purposes. Parts of the SRS area have been contaminated by radioactive waste, generally low-level, that leaked during weapons production. The General Separations Area (GSA) is an area of 15 mi^2 within the SRS complex with the main operations taking place there being chemical separations, tritium processing, and receipt of offsite fuel for processing. The main sources of substantial groundwater pollution are two seepage basins that contain a large number of underground storage tanks with high-level liquid radioactive waste that have leaked into the aquifers below the tank farms. Plumes have discharged radionuclides, metals, and nitrates into a nearby creek, the Fourmile creek (Figure 1).

Figure 1 shows that the GSA is partly bounded by three river branches: the Upper Three Runs (UTR) to the north, the Fourmile Branch to the south, and the McQueen Branch to the east. The aquifers that are present in the GSA area are the upper zone of the Upper Three Runs Aquifer (UTRA), the lower zone of the UTRA, and the Gordon aquifer (Figure 2). Underlying the Gordon Aquifer is the Meyers Branch confining unit, which provides this aquifer with a small upward flow. Monitoring wells provide information about the hydraulic head at several locations. A network of 415 wells monitors the upper zone of the UTRA, the lower zone of the UTRA is monitored by 173 wells, and information on the Gordon Aquifer is provided through a network of 79 wells.

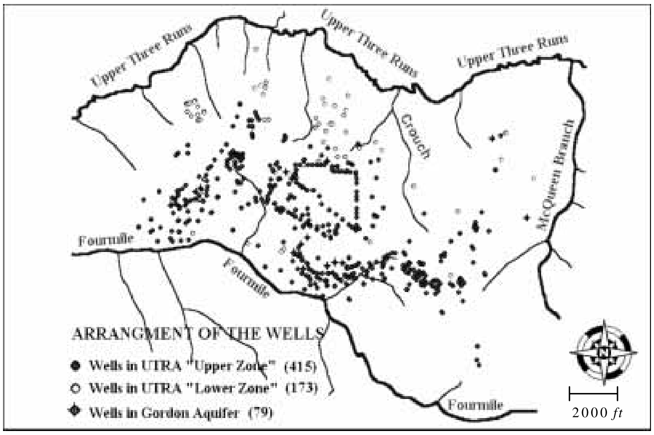

Figure 1 | Plan view of the GSA with the wells of the area.

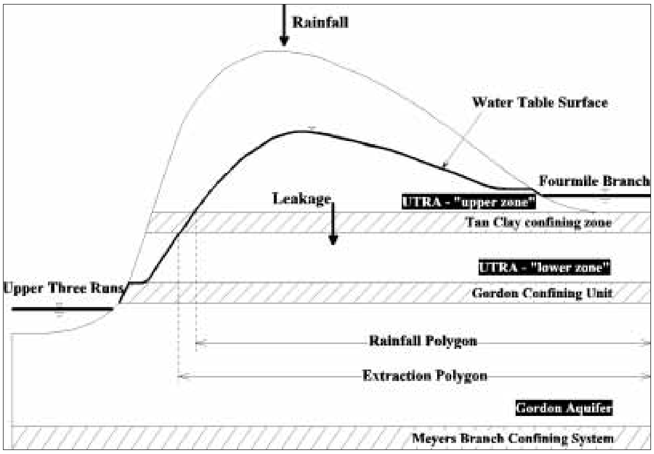

Figure 2 | Idealized cross-section of the GSA aquifer system (modified after Flach and Harris 1999).

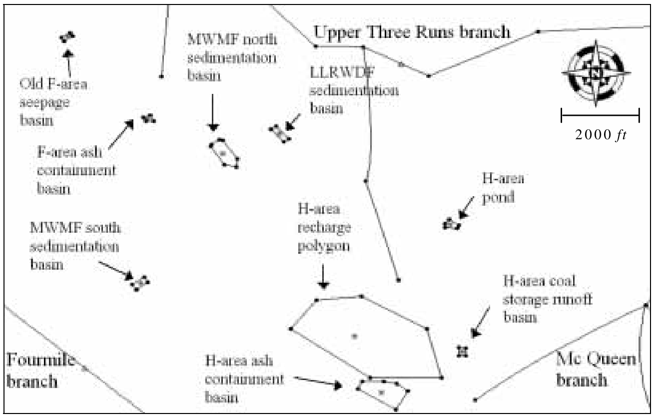

Figure 3 | Artificial recharge sources.

DETERMINISTIC MODEL

The main hydrologic features that were considered in the AEM model were the Upper Three Runs, the McQueen, and the Fourmile branches (as well as some smaller river branches), surface recharge facilities, and infiltration and leakage between the hydrogeologic units (Figure 3). Since only a single layer AEM code was available to us during this study, the focus has been on the upper zone of the UTRA. The SLAEM (Single Layer Analytic Element Method) code was used and appropriate adjustments were made to account for leakage from the upper to the lower zone of UTRA through the Tan Clay confining zone. A uniform base height for the upper zone of UTRA was utilized and a constant infiltration rate of 15.3 in/yr was assumed. The model included some additional hydrologic elements outside the GSA to establish the background regional flow field (the far-field) (Tolika 2001).

In order to account for infiltration and leakage to and from the upper zone of the UTRA, rainfall and extraction polygons were added (Figure 2). The rainfall polygon simulated the infiltration that took place on the surface of the site. Additionally, the various artificial surface sources (infrastructures) that contribute to the aquifer recharge were also represented with polygons approximating their areal distribution. The extraction polygon simulated the leakage that took place from the upper to the lower zones of the UTRA.

The extraction rate did not remain constant along the base of the aquifer because the hydraulic head, as well as the resistance of the confining unit changed spatially. Extraction rates were calculated through the expression

$$N = \frac{\phi - H}{c} \tag{1}$$

where N represents the infiltration rate, φ the value of the hydraulic head in the upper zone of the UTRA, H the value of the hydraulic head it the lower zone of the UTRA, and c the resistance of the Tan Clay confining layer. The resistance to flow was defined as $c = L/K_V$, where L is the average value of the vertical distance traveled by water (the thickness of the Tan Clay confining layer), and K_V is the vertical hydraulic conductivity of the Tan Clay confining layer.

On the surface of the GSA area there exist a number of infrastructures that contribute significantly to the aquifer recharge. These are shown in Figure 3 with the corresponding recharge rates given in Table 1.

The head values calculated with the AEM model are depicted in Figure 4, with the contour levels given in units of feet. A discussion of the model results and the agreement with measured heads can be found in Tolika and Paleologos (2003). In general, model results agreed with field observations, with the average absolute difference between simulated and measured hydraulic heads being about 5 ft. This agreement deteriorated in the eastern part of the model where a limited amount of data was available.

STOCHASTIC MODEL

Several studies have been conducted in the central SRS in an effort to present a detailed hydrologic budget. Hubbard (1984) conducted a three-year lysimeter study for the period between 1980 and 1982 at the SRS burial grounds. For an average precipitation of 47 in the groundwater recharge was estimated to be approximately 15 in or 32% of the average precipitation. Hubbard (1985) extended the previous study to include additional data from another two years of lysimeter measurements and concluded that

Table 1 | Artificial sources: Recharge rates

Source	**Recharge rate (in/yr)**	**Source**	**Recharge rate (in/yr)**
Old F-area seep. basin	85.8	H-area pond	55.9
F-area ash cont. basin	82.8	H-area coal storage runoff basin	62.9
MWMF south sed. basin	71.8	H-area ash containment basin	15.0
MWMF north sed. basin	16.0	H-area recharge polygon	14.0
LLRWDF sed. basin	59.9		

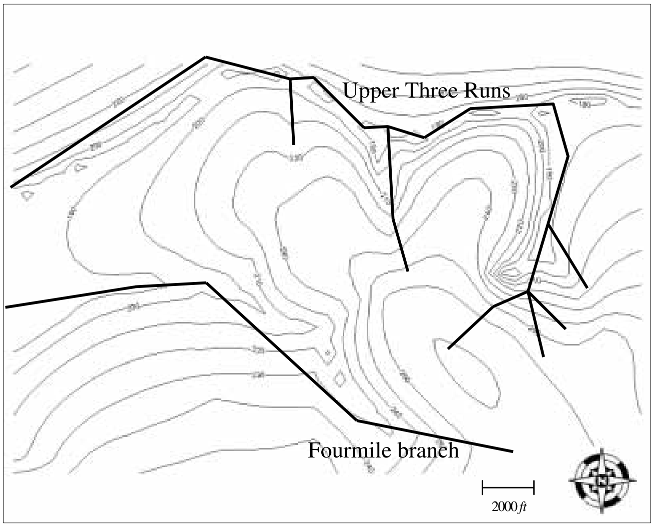

Figure 4 | Deterministic model: Heads in the upper zone of the Upper Three Runs aquifer (in feet).

16 in would recharge in this case the water table (33.3% of the observed precipitation). Similarly, Parizek and Root (1986) conducted a detailed hydrologic budget study of the McQueen Branch Basin, part of which is contained in the GSA area. The authors concluded that the average recharge was 32.7% of the average precipitation of 47.6 in.

Based on the previous studies, we have assumed that approximately 33% of the annual precipitation recharges the groundwater. This assumption requires a linear relation between precipitation and infiltration and neglects the effects of, among others, seasonal and annual energy or moisture fluctuations, vegetation and soil cover changes, or anthropogenic impacts.

Precipitation data for the area have been obtained for the last 112 years. The histogram of these values is plotted in Figure 5. Preliminary statistical analysis indicated that the histogram can be modeled by a log-normal probability distribution function, with a mean of 47.89 in and standard deviation of 6.59 in. Based on our assumption of an

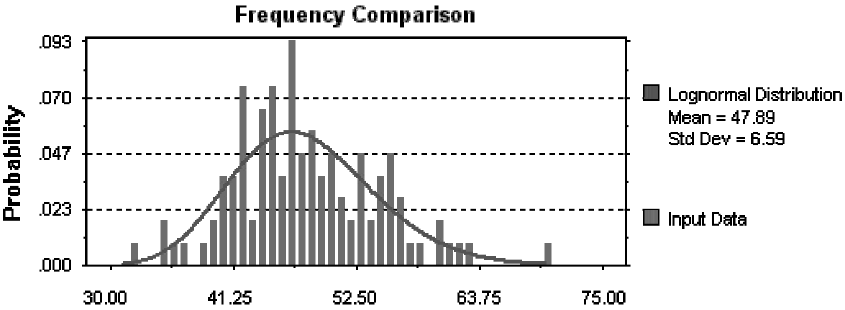

Figure 5 | Histogram and fitted distribution model for precipitation data.

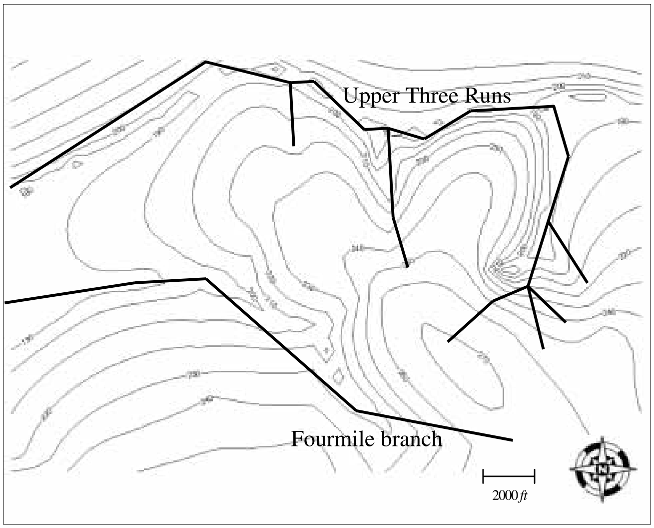

Figure 6 | Mean hydraulic head distribution.

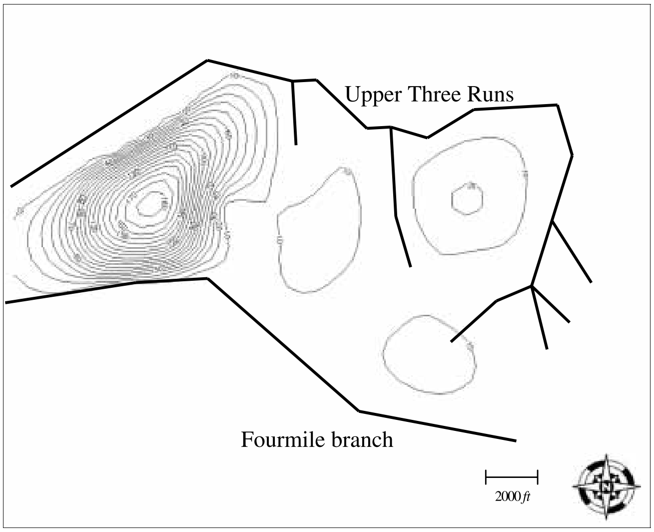

Figure 7 | Variance of the mean hydraulic head distribution.

infiltration rate that corresponds to 33% of the total rainfall, the moments of the infiltration rate (also assumed to follow a log-normal distribution) were taken to be a mean of 15.8 in and a standard deviation of 2.2 in. The Monte Carlo analysis proceeded by drawing a total number of 500 log-normally distributed random values of the infiltration rate and then solving the groundwater flow problem for each value of the infiltration rate. The resulting ensemble of 500 equiprobable flow patterns was averaged and the mean hydraulic head field was obtained. The mean head field, depicted in Figure 6, is almost identical with the one that was obtained through the deterministic model. Very small differences, of the order of 1–5 ft, were observed in the west side of the model between the Upper Three Runs and the Fourmile Branches.

RESULTS AND DISCUSSION

The variance of the mean hydraulic head provides a quantitative measure of the uncertainty about the actual head in a specific location. If the head variance is large (with respect to the mean) then the uncertainty in the mean calculated head value is large. When on the other hand, the variance of the head is small the computed mean head should be stable and close to the observed value at each location, if all the sources of variability are considered (Harter 2000). A contour plot of the head variance that was obtained from the 500 Monte Carlo realizations is shown in Figure 7. The computed variance is very small (the corresponding standard error is of the order of 3 ft) in most of the modeled area. There is however an area, on the west side of the model, south of the Upper Three Runs branch, where the variance takes very large values. On the 200 ft contour line of the mean head field, the variance becomes approximately 183 ft^2 (corresponding to a standard deviation of 13.5 ft). The variance decreases quickly in the north-south direction and becomes zero on the two river branches that run in the east-west direction, where the head was prescribed.

The large variances that were observed in the west side of the model, can be partially attributed to the fact that only a part of the flow domain is bounded by river branches modeled as known head linear elements. In particular, at the west side of the modeling area there exist very few hydraulic constraints, and in general, it is natural to observe large head variances there. This is also demonstrated in Figures 8 and 9 that plot the hydraulic head for the two realizations that correspond to the limiting cases of maximum (22.8 in/yr) and minimum (10.6 in/yr) simulated infiltration rates, respectively. From these two

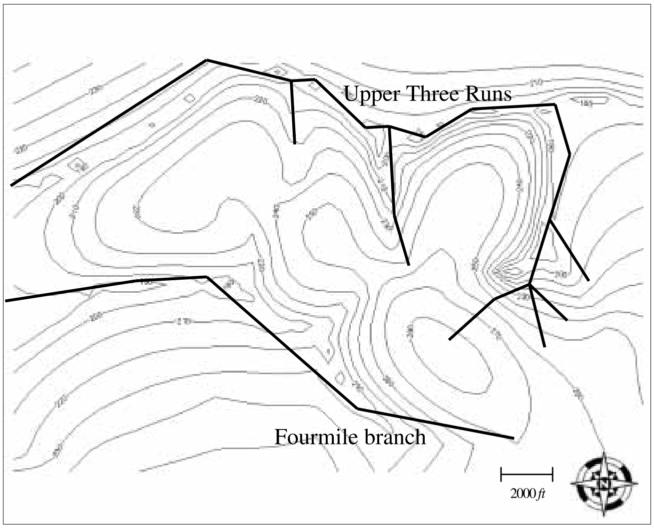

Figure 8 | Single realization: Hydraulic head obtained for the maximum infiltration rate.

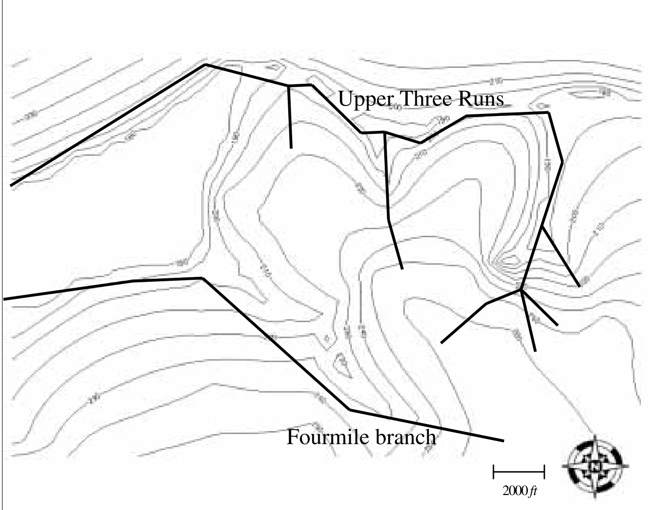

Figure 9 | Single realization: Hydraulic head obtained for the minimum infiltration rate.

figures one can obtain an extreme picture of the variability of the flow field. The smallest differences between the two simulated head fields (approximately 20 ft, or 7%) were observed in the area below the 'H area recharge polygon' (southeastern side of the model). On the west side of the model the difference in head for these two cases was approximately 50 ft (or 22%). Figures 7 and 9 indicate that the high variance in hydraulic head on the west side of the modeling area, for the case of the lowest infiltration rate, might be an artifact of code output, rather than a real effect. The head contour map for lowest infiltration (Figure 9) suggests that the computed head is flat (approximately 180 ft) to the west. This result combined with the fact that the base elevation of the upper zone of the UTRA was taken to be 177 ft indicates that, for the case of lowest infiltration, the code in the west region is reporting the elevation of the Tan Clay confining zone and hence the results there should be read with caution. One should note though that the west side of the model plays but a small role in the flow conditions and prediction of a contaminant plume in the areas of interest. All sources of contamination extend to the far east of the modeling domain and hence a numerical artifact in the west region, for low infiltration rates, should not have any real consequences to flow and contaminant plume prediction.

SUMMARY AND CONCLUSIONS

This paper demonstrates how some concepts taken from stochastic hydrology can be used in models based on AEM, and how the modeler can account and quantify parameter uncertainty. We have applied these concepts in a model developed for an unconfined aquifer in Savannah River Site, South Carolina, USA. We have considered the precipitation rate of the area as a random variable, with the same statistics as the historically available data.

A Monte Carlo analysis indicated that although the hydraulic head in most areas is largely insensitive to infiltration fluctuations, the same is not true for the west part of the model. The standard error in hydraulic head for this area was about 13.5 ft, indicating that predictions of a deterministic (or mean stochastic model) might not correspond to the actual flow behavior in the west part of the GSA. If additional sources of variability are considered the uncertainty of the model may be further increased. The limitation of the AEM to two-dimensional models has also played an important role in decreasing the overall model variability, mainly because of the need to model three-dimensional features in a simplified manner.

The ease of model development and the small processing power that is required, make the method applicable to initial investigations not only as a screening tool for the identification of test parameters and boundary conditions (Hunt *et al*. 1998; Olsthoorn 1999) but also as a tool that in a stochastic framework can provide initial estimations of uncertainty associated with certain model assumptions.

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