The authors are to be praised for their contribution of refinement of gas leakage flows to account for entrance flow effects. They have cleverly reduced the set of differential equations into a single equation with the entrance Mach number as the adjustable parameter. Further the compressible flow equation was transformed such that an existing incompressible entrance length boundary layer numerical analysis technique could be employed for solution of this equation. The authors then greatly enhanced the practical utility of this paper by utilizing viable approximations and generating a set of gas tables. Then the use of the tables was demonstrated with an example problem.

I wish to compare the analysis [8] with the present theory and the conventional gas dynamics theory using the laminar flow friction factor variation of $24/Re$. The comparison is shown in Fig. 6 which is Fig. 2 modified to include analysis [8]. Since the analysis [8] is basically the conventional gas dynamics theory, Fig. 6 shows that the analysis agrees here as it should. Note that the analysis can be made to agree almost identically with the authors' theory providing a suitable entrance loss coefficient is selected. This brings up the question how can the authors' theory be modified to account for a nonuniform entrance velocity profile as is the case in many practical applications?

Also shown in Fig. 6 is surprisingly good agreement between analysis [8] and the present theory for entrance Mach numbers of 0.4 to 0.6 even with isentropic entrance conditions. Analysis [8] used a turbulent friction factor relation—the well established Blasius friction law. The reason for this is that the Reynolds number has exceeded 2300. Since the wall friction is now much greater than the equivalent laminar flow case, the flow accelerates to choking more rapidly. The laminar-turbulent flow transition conditions can be readily found from the conventional gas dynamic relation of the friction parameter with entrance Mach number. The relation is [8]:

$$\frac{dP}{2h^*} = B(M) = \frac{1}{\gamma} \left( \frac{1}{M^*} - 1 + \frac{1}{2} (\gamma + 1) \ln \left[ \frac{\frac{1}{2}(\gamma + 1)M^*}{1 + \frac{1}{2}(\gamma - 1)M^*} \right] \right)$$

since $\dot{f} = \frac{24}{Re}$

and $Re_{entr} = 2300$

It can be readily shown that transition will occur when $P/P_0 = 0.296$ and $M_r = 0.349$. This then says that when the pressure ratio exceeds this ratio along with conditions which result in an entrance Mach number to exceed 0.349, transition to turbulent flow will occur.

The authors' theory should be compared with experimental results. Leakage flow experiments were conducted at the Lewis Research Center for a radial geometry representative of face seals and reported in [9]. Leakage flow was studied for two coaxial rings, having 5.50 in. inner dia and 6.00 in. outer dia separated by a fixed parallel gap of 1.5 mils. A parallel plate model should be valid for this geometry. The reservoir pressure was held fixed and the exit pressure varied. For each reservoir pressure condition the mass flow rate would increase as the exit pressure was decreased until the limiting condition of fluid choking occurred. Fig. 7 shows a comparison of the authors' analysis, analysis [8] with isentropic and 0.6 entrance loss coefficient and the radial flow experiment. The reservoir pressure was 40.0 psia. The authors' theory was applied using the gas table 6. Note the agreement between the theories and experiment. Analysis [8] showed the flow was laminar everywhere. Fig. 9 also shows the same comparison for a reservoir pressure increased to 60.0 psia. The flow is still laminar everywhere and there is once again good agreement. When the reservoir pressure is further increased to 97.3 psia, analysis [8] predicts the flow to be turbulent when pressure ratios are smaller than 0.55. The authors' theory tends to overestimate the leakage flow in this regime but still appears to be satisfactory for engineering purposes.

I have two questions. It should be pointed out that the Reynolds number should be sufficiently high that the boundary layer approximation can be made in the entrance region. Do the authors have an idea what this minimum Reynolds number value is? Also I am puzzled as to why the Mach number exceeds unity in Fig. 3 when $\frac{X_P}{aU} = 2.22$. 

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4 Numbers 8 and 9 in brackets designate Additional References at end of paper.
Authors' Closure

The authors appreciate very much Mr. Zuk's discussion which contributes experimental data and raises several interesting questions.

The agreement between his data and our theory, as shown in Fig. 7, is certainly as good as could be expected. Some of the residual discrepancy may arise from a nonrounded entrance configuration in the experimental apparatus.

Mr. Zuk introduces the question of turbulent flow. The Reynolds number of 2300 is more appropriately used for relaminarization, rather than transition to turbulence. In a developing flow within a duct, one of the authors has observed laminar flow at a Reynolds number as high as 35,000. Actual transition depends on the magnitude of initial disturbances, surface roughness and contours, time-in-duct, etc. The assumption of laminar flow with a rounded entrance, as made in the present paper, tends to overestimate the mass flow—a conservative form of error in most leakage calculations.

In reply to Mr. Zuk's last two questions—No serious error is likely to arise from use of the boundary-layer equations within the duct, even at very low Reynolds number. After all, the boundary-layer equations do contain Reynolds equation as a special case. The Mach number in a duct may locally exceed unity—only in the case of one-dimensional flow would such attainment be precluded.

Despite the results which Mr. Zuk has shown can be attained by judicious selection of loss coefficients (0.8 in Fig. 6 and 0.6 in Fig. 7) the authors believe that direct calculation with Gas Table 6 is simple enough to make its use advisable.