

DISCUSSION

H. Blok¹⁹ and K. Herrebrugh²⁰

We share the author's hope that (quoting) "availability of simple methods will make film-pressure predictions for dynamically loaded journal bearings much more common in the future

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and that experience with such predictions will prove useful in the formulation of realistic criteria of failure based on both film thickness and pressure information."

We have indeed been prompted by this same hope to compute a family of lines for equal values of the film-pressure ratio, P_{max} , as defined by the author. Since we based ourselves on the complete equation of Reynolds, instead of on Ocvirk's simplified equation as used by the author, our results may well be more realistic, particularly at high eccentricities.

Our computational results have been plotted in the following figure which is valid for fairly narrow journal bearings; that is,

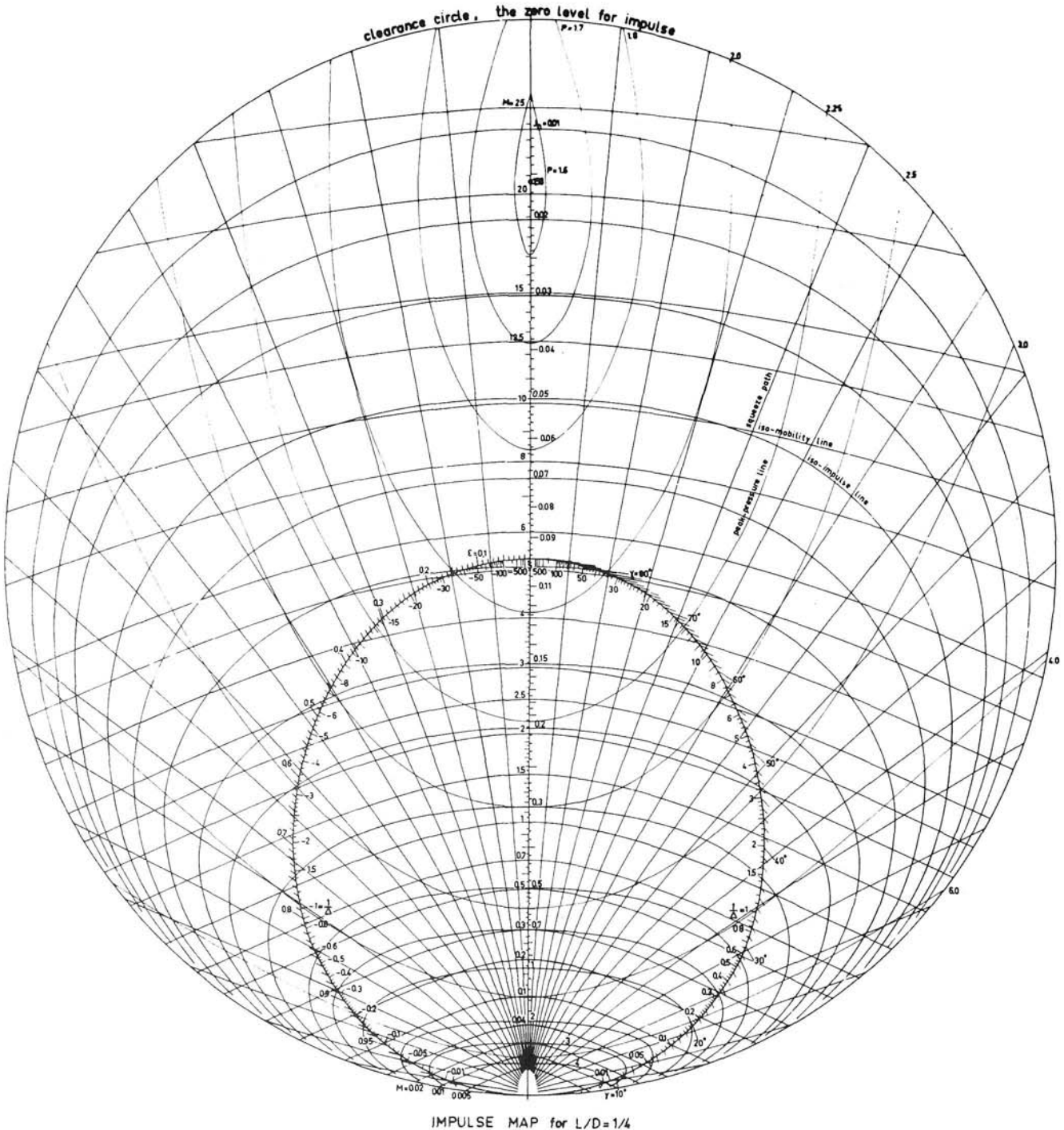


Fig. 5 Family of lines for equal values of film-pressure ratio, P_{max} , in journal bearings with $L/D = 1/4$

those with an L/D ratio equal to $1/4$. This figure is a reproduction of one of the large-scale finite bearing pressure maps referred to by the author (see author's reference [6]).

By comparing the chart, Fig. 5, with the author's Fig. 4 it is seen that there is a fair agreement in the trend of the lines, particularly for the smaller values of P_{\max} (in our chart the symbol P has the same meaning as the author's P_{\max}). But the maximum eccentricities reached by the lines for the higher P_{\max} -values are distinctly greater than the author's. So it would appear that in the most important range, that of high eccentricities, the author's Ocvirk approximation to the P_{\max} -values is on the high, or, say, the conservative side.

Further it can readily be seen that in another respect the lines in Fig. 5 for equal P_{\max} -values are even more similar to those in the author's Fig. 4; that is, at P_{\max} -values greater than about 2.5 the locus for the maximum eccentricities reached on the lines concerned very nearly coincides with the equilibrium locus.

In this connection it is worthy of note that for judging the risk of fatigue failure of the bearing metal it is important to know two characteristics of the maximum film pressure; that is, not only its magnitude as derivable from the author's Fig. 4, but also its location. Through conceiving the maximum film pressure as a rotating vector, both these characteristics could be depicted on an animated movie which we, in collaboration with Moes,²¹ have prepared for illustrative purposes, using a representative polar load diagram.

For a description of the significance of the other features of Fig. 5 including the various scales indicated along the equilibrium locus, the reader is referred to Fig. 4.4 of the author's reference [2]. A more elaborate description will soon be published elsewhere by one of us.²²

Finally, it may be pointed out that readers may be somewhat confused by the wording of the author's footnote.¹⁷ This footnote reads: "This elementary fact is not often noted; see [2]" (unquote), and it relates to the sentence, "It is easily shown that the maximum eccentricity in any cyclical journal center path must occur precisely on the equilibrium locus."

In actual fact, at the time of writing the authors of the paper concerned (reference [2]), as is evidenced by their text, were well aware of our even more general version of the aforementioned elementary fact. This version is as follows. All eccentricity extremes, including not only the maximum but also the minimum eccentricities, in any kind of journal center path, irrespective of whether it be cyclic or acyclic, must occur precisely on the equilibrium locus.

G. B. Grim²³ and J. L. Arneson²⁴

Development of methods for computing journal paths of dynamically loaded bearings, which yield eccentricities and oil film profiles, provide a means of comparing bearing designs on the basis of probability of surface distress during operation: by wiping, scoring, scuffing, and wear. A study of journal paths under dynamic loading also permits an estimate of the probability of cavitation erosion of the bearing. There still remained the need for calculating the maximum stress which a dynamic

load produces in the bearing material. The common concept of maximum load uniformly distributed over the bearing projected area produces, at best, a very rough, inaccurate, and often misleading value of unit load, which does not truly indicate the maximum stress. Since maximum stress is a direct result of maximum film pressure, ability to calculate the maximum film pressure can bring us to the threshold of predicting maximum bearing stress. Total bearing stress consists of the sum of stress due to the oil film pressure and the relatively small contributions due to assembly and deflections under load. Hopefully, the day will come when endurance limits of bearing materials can be determined by conventional evaluation of the properties of the materials, and the information can be applied in the same manner as similar data on other materials are applied in the design of structural components. It will be a happy day when published bearing load capacities will all be derived by a universally accepted test method, and when bearing designers will know what the rated capacities mean and how to apply them.

In the Caterpillar Tractor Co. Research Department, with the help of Professor Booker and his Mobility method, we have developed a bearing analysis procedure which starts with basic engine design parameters (bmep, rpm, cylinder dimensions, bearing dimensions, oil supply conditions, etc.) and computes bearing loads, journal path, oil flow, and bearing friction. From the journal path data we derive an oil film thickness profile.

It occurred to us that we had enough information available to develop a computation for oil film pressures, which we need to determine bearing stress. We had already started on this project and were well along in developing it when Professor Booker's paper on the subject was announced. The author sent us an advanced copy and asked us to comment on it.

Before discussing any details of the paper, we will compare the author's solution to the oil film pressure problem with our own.

We started with the equation for oil film pressure as stated in Ocvirk's short-bearing solution:

$$p = -6\mu \left(\frac{R}{C}\right)^2 \left(\frac{L}{D}\right)^2 \left[1 - \left(\frac{2z}{L}\right)^2\right] [\epsilon \cos \theta + \epsilon(\phi - \bar{\omega}) \sin \theta] / (1 + \epsilon \cos \theta)^3$$

This has the same form as equation (1) in Booker's paper. We easily transformed it to equation (1) by suitable substitution.

As Professor Booker shows, it is easy to locate P_{\max} along the line of $z = 0$, which eliminates the "z" term from the equation. By setting the first derivative of the remaining equation equal to zero, we obtain the equation for determining the angular coordinate of P_{\max} . Without belaboring the mathematical details, a quartic equation evolves. This would be formidable for manual solution, but readily yields to the digital computer. Imaginary angles and those outside the bounds of the positive pressure region are discarded, and the remaining roots are tested to determine the one at which maximum pressure occurs. The solution is correct and not difficult to program. Computer time is acceptable.

When we examined the Booker paper, we found that he had pursued a method which invests heavily in computer time to produce data from "numerical experiments," which can be plotted as maps of P_{\max} versus ϵ and ϕ for the general case. He then fits equations to the curves and has the means for less laborious solutions of specific problems. At the expense of computer time during initial programming, subsequent computer time is reduced. Along the way he obtains the maps, which may be used for graphical solutions. The purely mathematical approach lacks this bonus.

We have compared the two methods critically, and we find neither disagreement between them nor errors in either. Because of its reduced computer time per problem, the Booker method is preferable.

It is our hope that we can find good correlation between maximum oil film pressures and maximum bearing stress, and relate

²¹ At the time Moes was a scientific officer to the Laboratory of Machine Elements and Tribology, University of Technology, Delft, The Netherlands.

²² Blok, H., "The Orbital Hydrodynamics of Dynamically Loaded Self-Acting Journal Bearings, as Simplified by the Impulse/Whirl Angle Method," to be published in Japanese in the *Journal of the Japan Society of Lubrication Engineers*.

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this to the endurance limit and fatigue life of bearings. Oil film profiles, maximum film pressures, temperature rise, and journal path are all important factors in evaluating bearing designs.

Referring to details of the present paper, we question the implication of the statement that "squeeze velocity—and hence the pressure p —is directly proportional to the specific load. . . ." We believe that the proportionality exists independently of the role of squeeze in determining oil film pressure distribution. It is true that both squeeze velocity and pressure are proportional to specific load, but independently, not interdependently as implied.

We see no need for the qualification that $P_{\max} \geq 1$. The maximum pressure must always be at least equal to the specific pressure, since

$$p = \frac{F}{LD} \text{ and } P = \frac{p}{\bar{p}}$$

These last comments are not of major significance, and should not detract from the value of the paper. The author has made a valuable contribution to the science of performance analysis of dynamically loaded bearings. The paper represents a great deal of work which needed to be done. The author makes available a method of solution for those who have ready access to computers, and offers his maps to those who prefer graphical methods.

D. F. Hays²⁵

The author has presented a most interesting method for obtaining fluid-film pressure data in dynamically loaded journal bearings. Such information, especially the maximum pressure ratio P_{\max} and the maximum pressure gradient, should prove to be very helpful to the bearing designer who must be concerned with bearing failure through fatigue or cavitation erosion. This data will also be of value as a check on oil hole and oil groove location. These items are usually placed in the lightly loaded bearing areas as are now determined by the load vector referenced to the journal.

For those who are not already familiar with the author's mobility method, the present paper will pose several difficulties. A question that is fundamental to all material that is presented in the paper is why any point within the clearance circle has associated with it a *unique* pressure ratio distribution P , a distribution that is independent of time. Further comment on this point would be helpful.

The load line example used by the author leads to a pressure distribution with two maxima and this phenomenon is then related to the "folding" exhibited in the clearance circle map of P_{\max} . This could become clearer if the author would elaborate on what is meant by a condition of folding and describe the bearing/journal geometry conditions that led to multiple P_{\max} values.

Although the paper presents a technique for finding the P_{\max} distribution, it does not give an example to show how such a curve would compare with the load-time diagram and minimum film-thickness distribution with time. Such a comparison would be most interesting, especially in the case of an automotive connecting rod bearing where the pressure varies considerably.

A final comment is related to an earlier statement regarding the use of the mobility method in obtaining journal orbits. Persons not familiar with the technique and the principles upon which it is based will not be in a position to use the present paper to determine the pressure distribution. A significant addition to the paper would be a "how to do it" section. Perhaps a simple

bearing example could be used with the author leading the reader through the several steps required to find the P_{\max} curve. Such an example would be a welcome addition to this paper and would give the reader a greater feeling of security in utilizing the information that has been so well presented.

T. Lloyd²⁶

I should like to congratulate Professor Booker for his further, useful contribution to the design of reciprocating engine bearings. Once again, his graphical approach gives a very clear physical picture of what is happening. I am particularly interested in Fig. 4, which shows that, for increasing eccentricity ratio (the region enclosed by the equilibrium locus), the maximum pressure ratio is virtually a function of eccentricity ratio alone. This gives a very simple rule when designing to avoid high peak pressures. Since the peak pressure is invariably associated with the maximum cycle load, this load should be taken at as low an eccentricity as possible. This happens naturally with the big end bearing, when the gas load forces the journal across the clearance space and the maximum load is taken before the movement is completed.

If only high eccentricity ratios are considered, it is possible to compare the infinitely long and the infinitely short bearing theories and give the designer a simple approximate formula for peak pressure ratio in finite bearing.

- 1 Equilibrium locus, infinitely short theory:

$$P_{\max} \rightarrow 2.196/\sqrt{1-\epsilon}$$

$$\epsilon \rightarrow 1$$

- 2 Load line, infinitely short theory:

$$P_{\max} \rightarrow 1.801/\sqrt{1-\epsilon}$$

$$\epsilon \rightarrow 1$$

- 3 Equilibrium locus, infinitely long theory:

$$P_{\max} \rightarrow 0.918/\sqrt{1-\epsilon}$$

$$\epsilon \rightarrow 1$$

- 4 Load line, infinitely long theory:

$$P_{\max} \rightarrow 0.900/\sqrt{1-\epsilon}$$

$$\epsilon \rightarrow 1$$

These figures show that there is roughly a factor of two between the infinitely long and the infinitely short solutions and that the infinitely short theory, equilibrium locus, gives the highest figure for the peak-pressure ratio. Plotting the approximate formula onto Fig. 2 it is found that the results are close to, but slightly greater than, the exact result over the range $0 < \epsilon < 1$. Consequently, the formula $P_{\max} = 2.196/\sqrt{1-\epsilon}$ ought to give a pessimistic estimate for peak-pressure ratio for all conditions.

In conclusion, I am sure that the journal center locus alone is not enough for the design of dynamically loaded journal bearings. Intuitively, I feel that the peak pressure is an important parameter and I have always printed its position and value when running my dynamically loaded bearing computer program. In addition I print the cumulative oil flow and the work done on the oil film, together with the locus of the position of oil film buildup and breakdown. Evaluation of these figures is difficult and based mainly upon the experience gained in looking at the results of a very large number of bearing designs. What is needed is a better understanding of the mechanisms of bearing failure, so that the computer results may be presented and interpreted in a more meaningful way.

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Author's Closure

The finite bearing pressure map of Blok and Herrebrugh is a major contribution and a perfect complement to the present paper; it is fortunate that it could be made available at this time. I hope that we can expect such maps for other L/D ratios (along with the corresponding curve-fits for numerical work) in future publications²² by these workers.

Blok and Herrebrugh note correctly that, although Figs. 4 and 5 are in fair agreement in the lower pressure regions, Fig. 4 (short bearing) predicts distinctly greater maximum pressures than does Fig. 5 (finite bearing) for a given journal position in the higher pressure regions. It should be noted, however, that predicted journal orbits based on the mobility field in Fig. 3 (short bearing) have greater film thicknesses than do those based on the mobility field in Fig. 5 (finite bearing). Thus maximum film pressures may differ very much less than minimum film thicknesses in consistent predictions for a given loading. (Of course both theories underestimate film thicknesses and over-estimate film pressures in heavily-loaded bearings, since both ignore elastic deformations of the bearing surfaces. This should not seriously limit their usefulness as comparators, however.)

I certainly concur with the suggestion of Blok and Herrebrugh that maximum film pressure can be usefully treated as a vector, having both magnitude and direction. Fig. 6²⁷ is a map of the vector field $\mathbf{P}_{\max}(\epsilon, L/D)$, differing from Fig. 4 only in the addition of "direction lines" to which the vector \mathbf{P}_{\max} is everywhere tangent (in a generally downward sense). Along the load-line in the low pressure region this map shows quite clearly the double-valued directions arising from the presence of two relative maxima in the circumferential pressure distribution.²⁸

As a practical matter, however, note that in the high pressure region of Fig. 6 (important in the consideration of fatigue) the direction of \mathbf{P}_{\max} is always very nearly that of the applied load \mathbf{F} .

A more accurate approximation, useful within and near the equilibrium locus, is

$$\beta \approx \tan^{-1} \left(\frac{\eta}{1 + \xi} \right)$$

Figs. 7, 8, and 9 show an application of Fig. 6 to a very simple problem.²⁹ Fig. 7 shows the path followed by the center of a steadily-rotating journal in moving from the top of the clearance space toward a final equilibrium position under a steady non-rotating downward load. (See Example 1 of reference [1] for details of constructing such a locus using Fig. 3.) Fig. 8 shows the consequent action of the maximum film pressure ratio vector, as could be obtained from overlaying Figs. 6 and 7. Fig. 9 displays the magnitudes of the vectors of Figs. 7 and 8 against time, and could have been obtained directly from overlaying Figs. 4 and 7. Corresponding points in time have been labeled ①, ②, etc. in all three figures.

Note that $\dot{\epsilon} = 0$ as the journal path crosses the equilibrium locus³⁰ at points ②, ③, etc. The minimum eccentricity ratio occurs at point ② on the equilibrium locus; the maximum occurs at the initial point ①, however. The possibility of just such trivial initial (or final) extrema was the rationale for limiting the extremum principle on page 4 to cyclical journal paths.

It is easy to imagine the improvements in clarity and understanding that an animated display such as described by Blok and Herrebrugh would make over such static displays as those shown here. Perhaps, though, this very simple example will give the reader some idea of the method of application and the type of re-

²⁷ Note the analogy to Fig. 3, which is a map of the vector field $\mathbf{M}(\epsilon, L/D)$.

²⁸ See p. 535 and Fig. 2.

²⁹ Although these figures show nondimensional quantities, equivalent dimensional plots of film thickness and pressure could be made if clearance and specific load were known.

³⁰ See footnote⁸ of present paper and equation (7) of reference [1] for an explanation of this general result. Additionally, Blok has shown that $\dot{\epsilon} > 0$ and $\dot{\epsilon} < 0$, respectively, within and without the equilibrium locus. See reference [2].

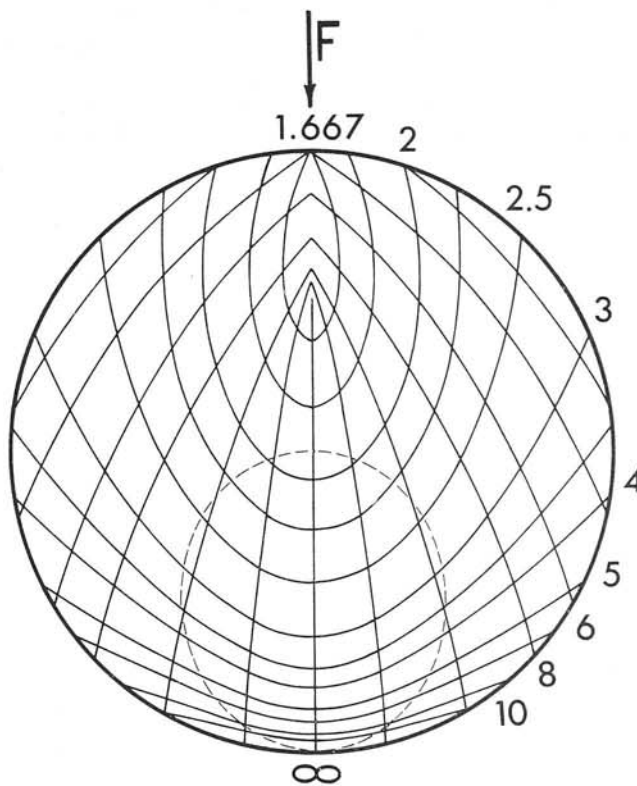


Fig. 6 Clearance circle map of maximum pressure ratio P_i short bearing solution ($0 \leq L/D$)

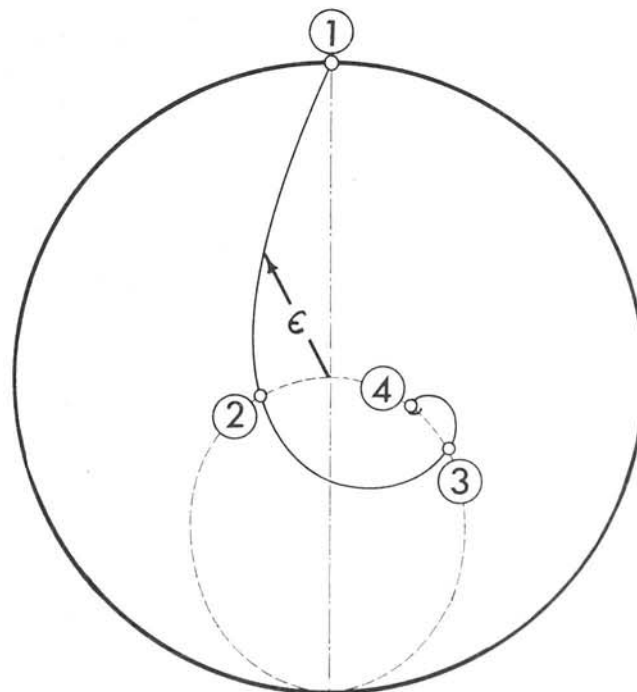


Fig. 7 Path of eccentricity ratio vector

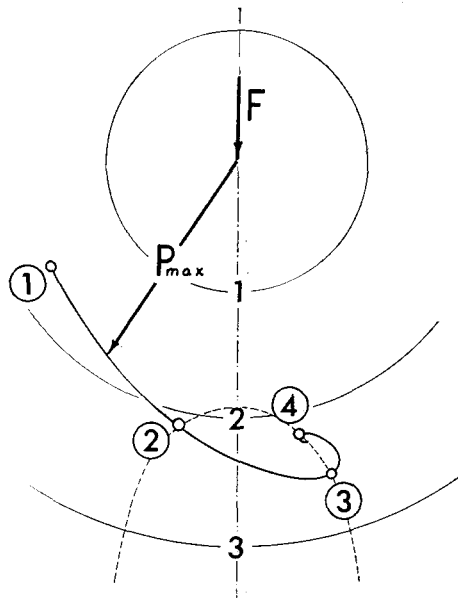


Fig. 8 Path of maximum pressure ratio vector

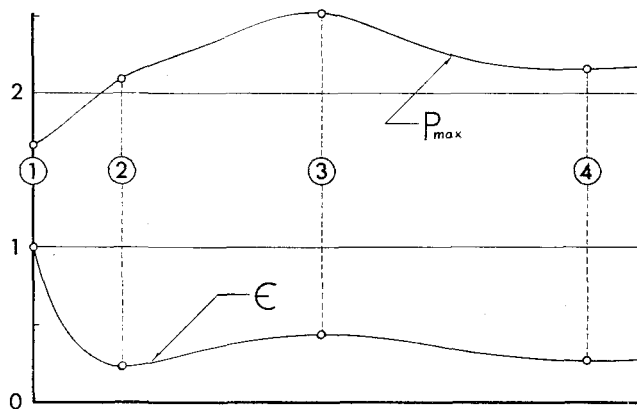


Fig. 9 Eccentricity ratio and max pressure ratio versus time

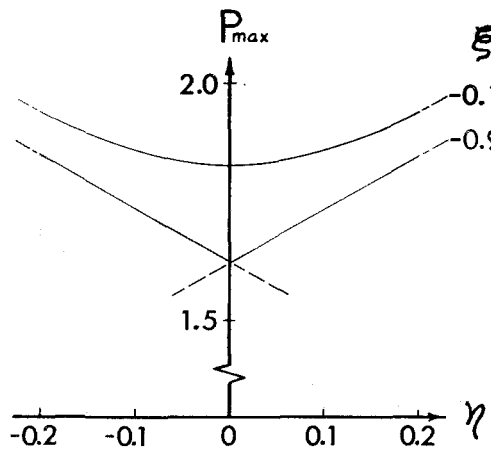


Fig. 10 Maximum pressure ratio
Pi short bearing solution ($0 \leq L/D$)

sults which can be obtained. An early publication will be devoted to numerical application to more complicated problems, such as the reciprocating engine bearings mentioned by several discussors.

Since Grim and Arneson have evidently been working along much the same lines as those detailed in the paper, there is really not very much to say by way of reply. In fact, they seem to have supplied the further comment asked by Hays on the central issue of the very existence of a unique pressure ratio distribution P .

Hays has also questioned the meaning of the term "folding," used to describe a feature shared by both the short and finite bearing pressure maps (Figs. 4 and 5). Fig. 10 shows a series of cross-sections through the "surface" of Fig. 4. The word "folding" was applied on page 4 to the slope discontinuity where branch surfaces describing *different* pressure peaks intersect along the load line ($\eta = 0$). As discussed on page 535, such *double* pressure peaks occur only where the journal is displaced high enough in the clearance space that thicknesses at the sides of the load-supporting film (which occupies roughly the lower half of the bearing) are very much less than those toward the bottom of the bearing. Along the load line this condition is first met (for the short bearing) when the journal is displaced upwards half the

clearance space. For greater displacements upwards the pressure peaks diverge circumferentially, reaching a maximum of ± 60 deg when the journal is at the top of the clearance space.

Lloyd has called attention to the considerable practical importance of zero film thickness as a limiting case and has given a series of useful asymptotic expressions for it. These verify the observation that maximum film pressure ratio is very nearly a function of eccentricity ratio alone in the region of greatest interest. It is also interesting to compare the coefficient (1.801) in his *asymptotic* expression (2) with the coefficient ($6/\pi \approx 1.91$) which results from a *global* approximation¹⁴ which is reasonably valid over the entire load line.

With respect to his comparison of long and short bearing results, I would note again that it isn't entirely consistent to compare different theories *for the same eccentricity ratio*. If one compares two theories *for the same load*, one finds considerably less difference.

The discussors have all shown a lively interest, both academic and industrial, in the role of film pressure in bearing behavior. I hope that as time goes on they and others will be able to report results much more directly useful to the designer than any to date.