Ghost-Free Two-Body Scattering Amplitudes
with Arbitrary External Masses

Yuji NAKAWAKI

Department of Physics, Kobe University, Kobe

(Received November 18, 1974)

Schwarz's method to construct off-mass-shell amplitudes is extended to two-body scattering amplitudes with arbitrary external masses in the Veneziano and Neveu-Schwarz models. It is shown that ghost-free amplitudes are obtainable by replacing linear t-channel trajectories by s-dependent ones in the Veneziano amplitudes and the Neveu-Schwarz ones respectively. It is also shown that transversely polarized integer spin particles can be introduced as one of initial or final two particles. In particular an application of our method to virtual Compton scattering amplitudes reveals that the longitudinal parts vanish identically.

§ 1. Introduction

In constructing realistic dual amplitudes, there still remain a number of difficulties to be solved. These include the problems concerning tachyons and hadronic currents.\textsuperscript{2) As is well-known many attempts to solve these problems have been confronted with ghost states. Recently Schwarz\textsuperscript{3)} has constructed ghost-free off-mass-shell amplitudes with the aid of tree states of the form

\[ V(k_n) \frac{1}{L_n - 1} V(k_{n-1}) \cdots V(k_1) \frac{1}{L_1 - 1} |S(q)\rangle, \]

where \( V(k_i) \) are usual scalar emission vertices with \( k_i^2 = -1 \) \((i = 1, 2, 3, \ldots, n)\). \( |S(q)\rangle \) plays the role of hadronic states produced by an external source such as lepton pairs and satisfies the Virasoro gauge conditions\textsuperscript{4)}

\[ (L_m - L_n + 1 - m) |S(q)\rangle = 0. \quad (m = 1, 2, \ldots) \]

For example scalar analogue of the virtual Compton scattering amplitude is defined to be

\[ \langle S(q_2) \mid \frac{1}{L_2 - 1} V(k_2) \frac{1}{L_1 - 1} V(k_1) \frac{1}{L_0 - 1} |S(q_1)\rangle, \]

where

\[ q_1^2 = q_2^2 = q^2, \quad \text{(virtual scalar photon mass squared)} \]

\[ k_1^2 = k_2^2 = -1. \]

With such a formulation Schwarz has required that the spectrum of hadron states occurring in the processes involving currents should be the same as in purely
hadronic reactions. Then ghosts are eliminated owing to the fact that any state that satisfies the gauge conditions is free from ghosts.\textsuperscript{5} Besides the Compton amplitude (1·3) gives physically desirable features such as Regge poles and fixed poles of the appropriate type and scaling behavior in the Bjorken limit. However there also appear undesirable features in his model. His solution for (1·2) is the scalar state with unit zero intercept in 16 dimensions. In addition tachyons are not eliminated yet. It seems that his solution has no room to remedy those defects and to construct a vector state, which therefore forces us to find other solutions for (1·2) or to alter the formulation from the start.

We examine in this paper an alternative way to construct ghost-free two-body amplitudes in which any mass of 4 external particles can take arbitrary values and any integer spin can be introduced into one of initial or final two particles. Following the amplitude (1·3) which is reexpressed as

\[ \langle \phi(k, q) \big| \frac{1}{L_0 - 1} \big| \phi(k, q) \rangle, \]  

(1·4)

where

\[ |\phi(k, q)\rangle = V(k) \frac{1}{L_0 - 1} |S(q)\rangle \]  

(1·5)

satisfies the Virasoro conditions for \( k^2 = -1 \) and arbitrary \( q^2 \), we construct a two-body scattering amplitude just like (1·4) with a state \( |\psi(k, q)\rangle \) which satisfies the Virasoro conditions for arbitrary \( k^2 \) and \( q^2 \).

\[ (L_m - L_0 - \mu^2 - m) |\psi(k, q)\rangle = 0, \]  

(1·6)

where the spectrum condition is shifted from \( (L_0 - 1) \) to \( (L_0 + \mu^2) \). We introduce integer spin (say \( n \)) into one of final two particles likewise by constructing an \( n \)-th rank Lorentz tensor state. Similarly we extend the method to the dual pion model and require that the \( G \) gauge conditions are satisfied as well. We have found that a state of the following form

\[ e^{-\sqrt{2} \xi (A \cdot Q^{(0)} + B \cdot Q^{(s)})} |0, k + q\rangle \]  

(1·7)

with

\[ Q_\mu^{(+)} = \sum_{\ell=1}^\infty \frac{1}{\sqrt{\ell}} a_\mu^{(+)}(\ell), \]

\[ |0, k + q\rangle = e^{-i(k \cdot q + \pi)\cdot \sigma} |0\rangle \]  

(1·8)

can satisfy Eq. (1·6), specifying two functions \( A \) and \( B \), which were described in § 2. In particular it will be shown that the amplitude given by the state (1·7) is obtainable by replacing linear \( t \)-channel trajectory by an \( s \)-dependent one in the amplitude of Veneziano. In § 3, we construct Lorentz four-vector states of the forms similar to (1·7) to define virtual Compton scattering amplitudes and examine the scaling behavior of inelastic electron-meson scattering amplitudes. It is shown
that longitudinal parts of the Compton scattering amplitudes are proportional to $(L_0 + \mu^2)$ and thus vanish identically and that scaling behavior can be derived by incorporating appropriate excitation form factors. It is also shown that transversely polarized integer spin states can be constructed. Section 4 is devoted to the conclusion.

§ 2. Ghost-free two-body scattering amplitudes for scalars and pions with arbitrary external masses

We determine in the Veneziano model the functions $A^{(n)}$ and $B^{(n)}$ of the state

$$e^{-\sqrt{\frac{1}{2}}(A^{(0)}q^+q^- + B^{(0)}k^+q^-)}|0, q+k\rangle = |\Phi^{(n)}(q,k)\rangle$$

(2.1)

in such a way that the state (2.1) preserves the Virasoro conditions for arbitrary $q^2$ and $k^2$

$$(L_m(p) - L_0 (p^2) - \mu^2 - m)|\Phi^{(n)}(q,k)\rangle = 0,$$

$$m = 1, 2, 3, \ldots$$

(2.2)

where $p = q + k$ and

$$L_m(p) = -\sum_{l=1}^{\infty} \sqrt{l(l+m)} a^+(l) \cdot a(l+m) + \sqrt{2i\sqrt{m}p} \cdot a(m)$$

$$+ \frac{1}{2} \sum_{l=1}^{m-1} \sqrt{l(m-l)} a(l) \cdot a(m-l),$$

$$L_0 (p^2) = -\sum_{l=1}^{\infty} la^+(l) \cdot a(l) - p^2 = R - p^2$$

(2.3)

with

$$[a^+(l), a^+(m)] = -g_{\alpha \beta} \delta_{l,m}, \quad (l, m = 1, 2, 3, \ldots)$$

$$[L_m(p), L_n(p)] = (m-n)L_{m+n}(p). \quad (m, n = 0, 1, 2, \ldots)$$

(2.4)

By virtue of the commutation relations (2.4), we can reexpress Eq. (2.2) as

$$L_m(p) z^{L_0 + \mu^2} |\Phi^{(n)}(q,k)\rangle = \left( z - \frac{d}{dz} \right) z^{m + L_0 + \mu^2} |\Phi^{(n)}(q,k)\rangle$$

(2.5)

which is used to prove that the $n$-th eigenstate of $R$

$$\frac{1}{2\pi i} \oint dz z^{-n} |\Phi^{(n)}(q,k)\rangle$$

(2.6)

is annihilated by the gauge operator $L_m(p)$ when $p^2 = n + \mu^2$, as follows:

$$L_m(p) \frac{1}{2\pi i} \oint dz z^{-n} |\Phi^{(n)}(q,k)\rangle$$

$$= \frac{1}{2\pi i} \oint dz \left( z - \frac{d}{dz} \right) z^{-n +\mu^2} |\Phi^{(n)}(q,k)\rangle = 0,$$
where the integral contour encircles the point \( z = 0 \).

Now from the direct calculation of the left-hand side of (2·5) we obtain

\[
\varepsilon_{n+m-p} \left\{ \frac{d}{dz} - 2 \left( A^{(n)} q \cdot p + B^{(n)} k \cdot p \right) - (m - 1) \left( A^{(n)} q + B^{(n)} k \right) \right\} \varepsilon^{r} | \Psi^{(r)} (q, k) \rangle
\]

which is equated with the right-hand side of (2·5) to derive the following equations:

\[
2 \left( A^{(n)} q \cdot p + B^{(n)} k \cdot p \right) = p^2 - \mu^2 - 1, \\
\left( A^{(n)} q + B^{(n)} k \right) = -1. 
\] (2·7)

Equations (2·7) are satisfied by

\[
A^{(n)} = \frac{1}{2p^2} \left( p^2 - \mu^2 - 1 \mp 2k \cdot p \sqrt{\left( \frac{p^2 - \mu^2 - 1}{p^2 - k^2 + q^2} \right) - 4q^4 p^2} \right), \\
B^{(n)} = \frac{1}{2p^2} \left( p^2 - \mu^2 - 1 \pm 2q \cdot p \sqrt{\left( \frac{p^2 - \mu^2 - 1}{p^2 - k^2 + q^2} \right) - 4q^4 p^2} \right). \tag{2·8}
\]

When \( q^2 = -1 \) \((= \mu^2)\) and \( k^2 = \mu^2 \)) the plus (minus) sign solution of \( A^{(n)} \) is equal to 1 (0), while the one with minus (plus) sign of \( B^{(n)} \) is equal to zero (1) and the state (2·1) reduces to the usual state

\[
e^{-\sqrt{2} \varepsilon \cdot q |0, q + k \rangle = V(q) |0, k \rangle} \quad (V(k) |0, q \rangle).
\]

In this way we can introduce initial or final two particles on the equal footing; this property is not seen in the usual string formulation.\(^{\ast} \)

Similarly the method is extended to the dual pion model. It is easily verified that the state

\[
| \Psi^{(r)} (q, k) \rangle = \sqrt{2} \left( A^{(n)} q \cdot H^{(r)} + B^{(n)} k \cdot H^{(r)} \right) V^{(r)} (q, k) |0, q + k \rangle, \tag{2·9}
\]

where

\[
H^{(r)} = \sum_{m=1/2}^{m} b_{m}^{r} (l), \quad \{ b_{m}^{r} (l), b_{m}^{r} (m) \} = -g_{m} \delta_{l, m} \quad (l, m = 1/2, 3/2, \ldots)
\]

and

\[
V^{(r)} (q, k) = e^{-\sqrt{2} \varepsilon \cdot q |A^{(n)} q \cdot H^{(r)} + B^{(n)} k \cdot H^{(r)}|} \\
\]

with

\[
A^{(n)} = \frac{1}{2p^2} \left( p^2 - \mu^2 - 1 \mp 2k \cdot p \sqrt{\left( \frac{p^2 - \mu^2 - 1}{p^2 - k^2 + q^2} \right) - 4q^4 p^2} \right), \\
B^{(n)} = \frac{1}{2p^2} \left( p^2 - \mu^2 - 1 \pm 2q \cdot p \sqrt{\left( \frac{p^2 - \mu^2 - 1}{p^2 - k^2 + q^2} \right) - 4q^4 p^2} \right). \tag{2·10}
\]

satisfies the Virasoro conditions. Note that the commutation property of \( H^{(r)} \)

\(^{\ast} \) A similar treatment is first considered by T. Kunimasa of Osaka University.
Y. Nakawaki

\[
\{L_m - L_0, H_\mu^{(+)}\}|0\rangle = \frac{m}{2} H_\mu^{(+)}|0\rangle
\]  

(2.11)

has changed the second equation of (2.7) into \( (A^{(e)} q + B^{(e)} k)^2 = -1/2 \) and hence Eq. (2.2) into

\[
\left( L_m(p) - L_0(p^\dagger) - \mu^2 - \frac{m}{2} \right) V^{(e)}(q, k)|0, p\rangle = 0.
\]  

(2.12)

Note also that when \( q^2 = -1/2 \) and \( k^2 = \mu^2 \), the state (2.9) with the plus-sign solution of \( A^{(e)} \) and the minus-sign one of \( B^{(e)} \), reduces to the usual one

\[
\sqrt{2} q \cdot H^{(+)} e^{-\sqrt{2} \kappa q \cdot k}|0, p\rangle.
\]

In the dual pion model the state has to satisfy another gauge conditions, i.e., G gauge conditions. To verify that (2.9) really satisfies them, it is convenient to reexpress the state (2.9) with the aid of \( G \) operator as

\[
|\psi^{(e)}(q, k)\rangle = G_s(p) V^{(e)}(q, k)|0, p\rangle,
\]  

(2.13)

where

\[
G_s(p) = \sqrt{2} p \cdot b(s) + i \sum_{l=1}^{\infty} (\sqrt{l} a^+(l) \cdot b(l+s) - \sqrt{l+s-\frac{1}{2}} a(l+s-\frac{1}{2}) \cdot b^+(l-\frac{1}{2}))
\]

\[-i \sum_{l=1}^{s-\frac{1}{2}} \sqrt{l} a(l) \cdot b(s-l). \quad (s=1/2, 3/2, 5/2, \ldots)
\]

Equation (2.13) has no \( s \)-dependence as should be. Then the G gauge conditions are verified as follows:

\[
G_r(p) |\Psi^{(e)}(q, k)\rangle = G_r(p) G_s(p) V^{(e)}(q, k)|0, p\rangle
\]

\[
= (L_0(p^\dagger) + r + \mu^2) V^{(e)}(q, k)|0, p\rangle,
\]  

(2.14)

where we have used (2.13) firstly, the anticommutation relations

\[
\{G_r(p), G_s(p)\}_+ = 2L_{r+s}(p)
\]

with \( r = s \) secondly, and finally the relation (2.12).

Next we construct the two-body scattering amplitude with the aid of (2.1). We define it as follows:

\[
\left< \Psi^{(e)}(q_2, k_2) \left| \frac{1}{L_0(s) + \mu^2} \Psi^{(e)}(q_1, k_1) \right> \right|^1
\]  

(2.15)

and calculate it straightforwardly

\[
\left< 0, q_2 + k_2 | e^{\sqrt{2} t (A^{(e)} q_2 + B^{(e)} k_2)} \frac{1}{L_0(s) + \mu^2} e^{-\sqrt{2} t (A^{(e)} q_1 + B^{(e)} k_1)} |0, q_1 + k_1 \right>
\]

\[
= \int_0^1 dz \, z^{n-1}(1-z)^{-a(s,t)-1} \frac{\Gamma(-\alpha(s)) \Gamma(-\alpha(s,t))}{\Gamma(-\alpha(s) - \alpha(s,t))},
\]  

(2.16)
where
\[
\alpha(s) = s - \mu^2,
\]
\[
\alpha(s, t) = -1 - \frac{(s - \mu^2 - 1)^2}{2s} + \left\{ t - q_1^2 - q_2^2 + (s - k_1^2 + q_1^2)(s - k_2^2 + q_2^2) \right\}
\times \frac{1}{2s} \left\{ (s - \mu^2 - 1)^2 + 4s \right\}^{1/2} \left\{ (s - k_1^2 + q_1^2)^2 - 4q_1^2 \right\}^{1/2} \left\{ (s - k_2^2 + q_2^2)^2 - 4q_2^2 \right\}
\]
\[\text{(2.17)}\]
A remarkable feature of (2.16) is that it is obtainable by replacing the linear \( t \)-channel trajectory by the \( s \)-dependent one in the Veneziano amplitude (the same property is also true of the \( \pi \pi \) two-body amplitude constructed with the state (2.9)). The complicated \( s \)-dependence of \( \alpha(s, t) \) can be simplified if \( t \) is expressed in terms of the scattering angle in the center-of-mass system as
\[
\alpha(s, t) = -1 - 2 \left\{ \frac{(s - \mu^2 - 1)}{2\sqrt{s}} \cdot \frac{(s - \mu^2 - 1)}{2\sqrt{s}} \right\} - \frac{\sqrt{(s - \mu^2 - 1)^2 + 4s} \cdot \sqrt{(s - \mu^2 - 1)^2 + 4s} \cos \theta}{2\sqrt{s}}
\]
\[\text{(2.18)}\]
which exhibits the original Veneziano form except that the scattering angle between two tachyons is composed of that of particles with arbitrary masses. It is easily noted from this that there are no ghosts and no ancestors in direct channel and odd daughter states decouple only for \( \mu^2 = -1 \). It is also noted from (2.17) that the high energy behavior is not altered since \( \alpha(s, t) \sim t + 1 \) for large \( s \). An undesirable feature is that the \( s \)-dependent \( t \)-channel trajectory produces \( s \)-dependent \( t \)-channel poles. Since the \( s \)-dependent factor has originated from the functions \( A^{(n)} \) and \( B^{(n)} \) of the state (2.1), it is necessary to reconsider the model to remedy the defect. However, since the amplitude has similar features of the original Veneziano amplitude in \( s \)-channel physical domain, some useful applications may be made by using the amplitude in physical domain of \( s \)-channel or by applying residue function of \( s \)-channel resonances. In this connection we notice that Igi and Shimada have already utilized the \( s \)-channel resonance residue functions of the Neveu-Schwarz amplitude when they tried to realize experimental features of \( \pi \pi \) and \( \pi K \) elastic scattering. We examine in \( \S \) 3 the structure functions of electron-meson inelastic scattering as such an application.

\section*{§ 3. Virtual Compton scattering amplitudes}

Using the operator
\[
P_\mu^{(+)} = \sqrt{2i} \sum_{l=1}^{\infty} \sqrt{l} a_\mu^+(l)
\]
\[\text{(3.1)}\]
and the state
\[ V^{(\nu)}(q, k) |0, p\rangle = \exp(-i(A^{(\nu)}q \cdot p + B^{(\nu)}k \cdot p)) |0, p\rangle \]

with
\[
A^{(\nu)} = \frac{p^2 - \mu^2}{2p^2} \left( 1 + \frac{2k \cdot p}{\sqrt{(p^2 - \mu^2 + q^2)^2 - 4q^2p^2}} \right),
\]
\[
B^{(\nu)} = \frac{p^2 - \mu^2}{2p^2} \left( 1 + \frac{2k \cdot p}{\sqrt{(p^2 - \mu^2 + q^2)^2 - 4q^2p^2}} \right),
\]

we consider the Lorentz four-vector state by starting with
\[ |\Psi^{(\nu)}_\mu(q, k)\rangle = \{(2k + q)_{\mu} + k_{\mu}(C_{12}q \cdot p + C_{13}k \cdot p) + q_{\mu}(C_{14}q \cdot p + C_{15}k \cdot p) \}
\]
\[ V^{(\nu)}(q, k) |0, p\rangle, \]  \hspace{1cm} (3.2)

where \( k^2 = \mu^2 \), target mass squared and \( q^2 \), virtual photon mass squared. The first term \((2k + q)_{\mu}\) is specified to describe the coupling of lowest mass state, and \( C_{1} \sim C_{5} \) are in general functions of \( q \) and \( k \) and are to be fixed so as to satisfy the Virasoro conditions and current conservation. Operation of \( L_{m}(\rho) \) to the state
\[ |\Psi^{(\nu)}_\mu(q, k, z)\rangle = z^{m} |\Psi^{(\nu)}_\mu(q, k)\rangle \]

yields the following result:
\[ L_{m}(\rho) |\Psi^{(\nu)}_\mu(q, k, z)\rangle = z^{m} \left( z \frac{d}{dz} + m + \mu^2 - p^2 \right) |\Psi^{(\nu)}_\mu(q, k, z)\rangle \]
\[ + 2mz^{m} \left[ C_{1}p_{\mu} + k_{\mu}(C_{12}q \cdot p + C_{13}k \cdot p) + q_{\mu}(C_{14}q \cdot p + C_{15}k \cdot p) - \frac{1}{2} (2k + q)_{\mu} \right] \]
\[ + m(m - 1)z^{m} \left[ C_{1}A^{(\nu)}_{\mu}q_{\mu} + B^{(\nu)}_{\mu}k_{\mu} \right] + (C_{2}k_{\mu} + C_{3}q_{\mu}) (A^{(\nu)}q \cdot k + B^{(\nu)}k \cdot q) \]
\[ + (C_{5}q_{\mu} + C_{6}k_{\mu}) (A^{(\nu)}k \cdot q + B^{(\nu)}k \cdot q) \]  \hspace{1cm} (3.4)

The Virasoro conditions require the terms in the square bracket to vanish. Equating the terms proportional to \( k_{\mu} \) and \( q_{\mu} \) with zero respectively in the first and second terms of the square bracket, we obtain the following relations for \( C_{1} \sim C_{5} \):
\[
\begin{cases}
C_{1} + C_{12}q \cdot p + C_{13}k \cdot p = 1, \\
C_{1} + C_{14}q \cdot p + C_{15}k \cdot p = 1, \\
C_{2}B^{(\nu)} + C_{4}(A^{(\nu)}q^2 + B^{(\nu)}k \cdot q) + C_{3}(A^{(\nu)}q \cdot k + B^{(\nu)}k \cdot p) = 0, \\
C_{1}A^{(\nu)} + C_{4}(A^{(\nu)}q^2 + B^{(\nu)}k \cdot q) + C_{3}(A^{(\nu)}q \cdot k + B^{(\nu)}k \cdot p) = 0.
\end{cases}
\]  \hspace{1cm} (3.5)

Equations (3.5) are solved as
\[
\begin{align*}
C_{2} &= \frac{q \cdot k}{(q \cdot k)^2 - \mu^2q^2} (q \cdot k A^{(\nu)} + \mu^2 B^{(\nu)} - C_{1}), \\
C_{3} &= \frac{q^2}{(q \cdot k)^2 - \mu^2q^2} (C_{1} - \frac{q^2 A^{(\nu)} + q \cdot kB^{(\nu)}}{q^2 (A^{(\nu)} - B^{(\nu)})}),
\end{align*}
\]
with which we can rewrite (3.3) as

\[
|\Psi^{(\nu)}(q, k)\rangle = \{c_1 G^{(\nu)}(q, k) P^{(\nu)} + (2k + q)_\nu \left( p^2 - \mu^2 + A^{(\nu)} q \cdot P^{(\nu)} + B^{(\nu)} k \cdot P^{(\nu)} \right) \} V^{(\nu)}(q, k)|0, p\rangle
\]

where

\[
G^{(\nu)}(q, k) = g^{(\nu)} q^2 \left( k - q, k \right) \left( \frac{A^{(\nu)} q \cdot k + B^{(\nu)} k \cdot q}{q^2} \right) \left( \frac{k - q, k}{q^2} \right) \left( \frac{A^{(\nu)} q \cdot k + B^{(\nu)} k \cdot q}{q^2} \right).
\]

A noticeable feature of (3.7) is that the current conservation is satisfied without making any imposition since the divergence equation for the current holds as follows:

\[
q^\mu |\Psi^{(\nu)}(q, k)\rangle = -(L_\mu + \mu^2) V^{(\nu)}(q, k)|0, p\rangle.
\]

Similarly we extend the method to the dual pion model. We add to the bracket part of (3.7) the following terms:

\[
G^{(\nu)}(q, k) \left( H^{(\nu)} + d_2 k \cdot H^{(\nu)} \right) + (d_2 q \cdot k \cdot H^{(\nu)} + d_4 k \cdot k \cdot H^{(\nu)})
\]

and determine \(d_1 \sim d_4\) so as to satisfy the \(G\) gauge conditions. After straightforward calculations we obtain

\[
d_1 = C_1 A^{(\nu)}, \quad d_2 = C_1 B^{(\nu)}, \quad d_3 = d_4 = 0
\]

and thereby the state

\[
|\Psi^{(\nu)}(q, k)\rangle^{(\nu)} = \left\{ C_1 G^{(\nu)}(q, k) \left( P^{(\nu)} + [H^{(\nu)} + A^{(\nu)} q \cdot H^{(\nu)} + B^{(\nu)} k \cdot H^{(\nu)}] \right) \right\}
\]

\[
- \left( \frac{2k + q}_\nu \left( L_\mu + \mu^2 \right) \right) V^{(\nu)}(q, k)|0, p\rangle.
\]

The first terms of (3.7) and (3.12) are transverse polarization terms. Longitudinal polarization terms are included in the second terms. We can understand these states as reasonable ones. In order that the Virasoro conditions are satisfied, the annihilation operators in the commutator \([L_m - L_0, P^{(\nu)}]^{(\nu)}\) have to commute with \(V^{(\nu)}(q, k)\). The transverse annihilation operators commute with \(V^{(\nu)}(q, k)\) by independence and the longitudinal ones by \((A q + B k)^2 = 0\).
From this observation we may construct transversely polarized integer spin states
\[ \sum_{n} \epsilon_{\mu_{1}...\mu_{n}}^{(2)}(q, k) \epsilon_{\nu_{1}...\nu_{n}}^{(2)}(q, k) P^{(+)}_{\nu_{1}}...P^{(+)}_{\nu_{n}} e^{-\sqrt{2} i \langle A^{(n)} Q^{(n)} + B^{(n)} K^{(n)} \rangle |0, p \rangle}, \]
\[ (n = 1, 2, 3, \ldots) \tag{3.13} \]
where \( \epsilon_{\mu_{1}...\mu_{n}}^{(2)}(q, k) \) are \( n \)-th symmetric transverse polarization tensors satisfying
\[ q^\mu_{1} \epsilon_{\mu_{1}...\mu_{n}}^{(2)}(q, k) = k^\mu_{1} \epsilon_{\mu_{1}...\mu_{n}}^{(2)}(q, k) = 0, \]
\[ g^{\mu_{1}...\mu_{n}} \epsilon_{\mu_{1}...\mu_{n}}^{(2)}(q, k) = 0, \tag{3.14} \]
which give the commutation relations
\[ [L_{n} - L_{m}, \epsilon_{\mu_{1}...\mu_{n}}^{(2)}(q, k) P^{(+)}_{\nu_{1}}...P^{(+)}_{\nu_{n}}]|0\rangle = nm \epsilon_{\mu_{1}...\mu_{n}}^{(2)}(q, k) P^{(+)}_{\nu_{1}}...P^{(+)}_{\nu_{n}}|0\rangle \tag{3.15} \]
and hence
\[ A^{(n)} = \frac{1}{2p^2} \left\{ (\mu^2 - \mu^2 + n - 1) \pm 2k \cdot p \sqrt{\frac{(\mu^2 - \mu^2 + n - 1)^2 - 4(n - 1) \mu^2}{(\mu^2 - k^2 + q^2)^2 - 4q^2 p^2}} \right\}, \]
\[ B^{(n)} = \frac{1}{2p^2} \left\{ (\mu^2 - \mu^2 + n - 1) \pm 2k \cdot p \sqrt{\frac{(\mu^2 - \mu^2 + n - 1)^2 - 4(n - 1) \mu^2}{(\mu^2 - k^2 + q^2)^2 - 4q^2 p^2}} \right\}. \tag{3.16} \]

Besides these we can construct a Lorentz four-vector state possessing transversely polarized spin 1 state \( G_{\mu\nu}(q, k) a^{+\nu}(1)|0, q\rangle \)
\[ e^{-\sqrt{2} i \langle A^{(n)} Q^{(n)} + B^{(n)} K^{(n)} \rangle} G_{\mu\nu}(q, k) a^{+\nu}(1)|0, p\rangle, \tag{3.17} \]
where \( A^{(n)} \) and \( B^{(n)} \) are obtainable by inserting \((\mu^2 + 1)\) instead of \( \mu^2 \) in \( A^{(n)} \) and \( B^{(n)} \) of (2.8). The state (3.17) reduces for \( q^2 = 0 \) and \( k^2 = -1 \) to the usual one where the vector particle is scattered by the external scalar potential.

Here we remark that form factors are incorporated in our model. When we introduce form factors, it is necessary to produce no ghosts. Therefore they are required to obey the gauge conditions. We can easily confirm that the following is the only one satisfying the requirement:
\[ |\Phi_{\mu}(q, k)\rangle = \int d\sigma \rho(q^2, \sigma)(y(q^2, \sigma))^{\mu} |\phi_{\mu}(q, k)\rangle, \tag{3.18} \]
where the state \( |\phi_{\mu}(q, k)\rangle \) stands for (3.7) or (3.12) or (3.17) and we can obtain form factors such as vector dominance type ones by specifying the functions \( \rho(q^2, \sigma) \) and \( y(q^2, \sigma) \). We include \( C_{1} \) of (3.7) and (3.12) into \( \rho(q^2, \sigma) \).

Now let us evaluate virtual Compton scattering amplitudes defined as
\[ T_{\mu\nu} = \langle \Phi_{\mu}(q_3, k_3) \left| \frac{1}{L_{\mu}(s)} + \mu^2 \right| \Phi_{\mu}(q_1, k_1) \rangle, \tag{3.19} \]
where
\[ q_1^2 = q_2^2 = q^2 \text{ and } k_1^2 = k_2^2 = \mu^2. \]
Firstly we insert (3.7) and (3.17) into (3.19) with form factors introduced.
according to (3·18) and after a straightforward calculation obtain the following result:

\[ T_{\mu\nu}^{(s)} = \frac{4}{t} (\alpha^{(s)}(s, t) - 1) G_{\mu\nu}(q_1, k_1) q_1^a \]

\[ \times G_{\sigma\tau}(q_2, k_2) q_2^a \int_0^s d\sigma_1 \int_0^s d\sigma_2 \int_0^1 dz \frac{\rho_1 \rho_2 \zeta^{n-1} \cdot (z y_1 y_2)^2}{(1 - z y_1 y_2)^{s+\alpha(s,t)}} \]

\[-2G_{\rho\lambda}(q_1, k_1) G_{\tau\lambda}(q_2, k_2) \int_0^s d\sigma_1 \int_0^s d\sigma_2 \int_0^1 dz \frac{\rho_1 \rho_2 \zeta^{n-1} \cdot y_1 y_2}{(1 - y_1 y_2)^{s+\alpha(s,t)}} \]  

where

\[ \gamma_1 = y(q^a, \sigma_1), \gamma_2 = y(q^a, \sigma_2), \rho_1 = \rho(q^a, \sigma_1), \rho_2 = \rho(q^a, \sigma_2) \]

and

\[ \alpha^{(s)}(s, t) = \begin{cases} 
1 + \frac{(s - \mu)^2}{(s - \mu^2 + q^2)^2 - 4q^2 t} & \text{for (3·7)}, \\
1 + \frac{(s - \mu^2 - 2)t + 4s}{(s - \mu^2 + q^2)^2 - 4q^2 t} & \text{for (3·18)}. 
\end{cases} \]

We note that the second term of (3·7) has no contribution to \( T_{\mu\nu}^{(s)} \) which results in decoupling the lowest mass state in \( T_{\mu\nu}^{(s)} \). The \( \tau \)-channel trajectory changes depending on the targets except when \( \mu^2 = -1 \) and \( q^2 = 0 \). Similarly the dual pion state (3·12) brings

\[ T_{\mu\nu}^{(s)} = -\frac{4}{t} (\alpha^{(s)}(s, t) - 1) G_{\mu\nu}(q_1, k_1) q_1^a \]

\[ \times G_{\sigma\tau}(q_2, k_2) q_2^a \int_0^s d\sigma_1 \int_0^s d\sigma_2 \int_0^1 dz \frac{\rho_1 \rho_2 \zeta^{n-1} \cdot y_1 y_2}{(1 - y_1 y_2)^{s+\alpha(s,t)}} \]

\[-2\alpha^{(s)}(s, t) G_{\rho\lambda}(q_1, k_1) G_{\tau\lambda}(q_2, k_2) \int_0^s d\sigma_1 \int_0^s d\sigma_2 \int_0^1 dz \frac{\rho_1 \rho_2 \zeta^{n-1} \cdot y_1 y_2}{(1 - y_1 y_2)^{s+\alpha(s,t)}} \]  

with \( \alpha^{(s)}(s, t) \) given in the first line of (3·21).

Next we discuss the scaling behavior in the Bjorken limit and some related properties. Since at \( t=0 \) the terms of (3·20) and (3·22) vanish, it is enough to examine the imaginary parts of their second terms with the tensor factors eliminated:

\[ W = \sum_{n=1}^{\infty} \delta(s - n - \mu^2) \Gamma(n + \alpha^{(s)}(s, t), G(n, q^a))^\gamma \Gamma(1 + \alpha^{(s)}(s, t))^{-1} \]

\[ = \frac{\Gamma(s - \mu^2 + \alpha^{(s)}(s, t))}{\Gamma(s - \mu^2)} (G(s - \mu^2, q^a))^\gamma \Gamma(1 + \alpha^{(s)}(s, t))^{-1} \]  

for scalar, 

\[ \Gamma(\alpha^{(s)}(s, t))^{-1} \]  

for pion.

\[ (3·23) \]
where

\[ G(n, q^2) = \int_0^\infty d\sigma \rho(q^2, \sigma) (y(q^2, \sigma))^n \]  \hspace{1cm} (3.24)

and the summation over \( n \) is replaced by an integral. We notice that the scaling behavior is valid only at \( t=0 \) since nonscaling \( t \)-dependent factor \((\sigma q^2)^{(1-n)/2n}\)

with \( \sigma = 2q \cdot k/(\sigma q^2) \) arises in the Bjorken limit with fixed \( t \). Therefore provided that \((G(s-\mu^2, q^2))^2\) behaves in the Bjorken limit as

\[ (G(s-\mu^2, q^2))^2 \rightarrow f(1) \]  \hspace{1cm} (3.25)

we get the scaling structure functions

\[ W_1 \rightarrow f(1) (\omega - 1), \quad qkW_2 \rightarrow 2f(1) \frac{\omega - 1}{\omega}. \]  \hspace{1cm} (3.26)

Unfortunately we have not yet found vector dominance type form factors which show the behavior (3.25). If \( f(1) \neq 0 \), the power behavior near \( \omega = 1 \) agrees with that expected from the Drell-Yan relation for meson with elastic form factors falling off asymptotically like \((\sigma q^2)^{-1}\).

\section*{4. Conclusion}

In this paper extending Schwarz method, we have examined the alternative way to construct ghost-free two-body scattering amplitudes which have properties

i) initial or final two particles have equal footing in the processes such as scalar-scalar and \( \pi-\pi \) scattering;

ii) any mass of 4 external particles can take arbitrary values;

iii) transversely polarized integer spin particles can be introduced as one of initial or final two particles;

iv) form factors can be introduced.

We have found features inherent to our model as well as a feature of more general origin. The former features are that

v) \( t \)-channel trajectories come to possess \( s \)-dependence which hence produces \( s \)-dependent \( t \)-channel poles and breaks \( s-t \) symmetry of the dual amplitudes;

iv) \( t \)-channel trajectory changes depending on the targets in the Compton scattering amplitudes;

vii) longitudinal parts of the Compton scattering amplitudes vanish identically which results in decoupling of the lowest mass state.

The latter feature is that

viii) zero intercept of \( t \)-channel trajectories is not altered. This property results from the factor \((1-z)^{-1}\) of forward amplitudes (two-point function). Since Schwarz' trajectories show the same feature, we speculate that this property is common to \( t \)-channel trajectories of all amplitudes constructed with the states
which satisfy the Virasoro conditions.

To remedy the undesirable features v) and vi) and to incorporate the lowest mass state, we have to reconsider our model from the start. We should find other ways to eliminate ghosts. Here we note briefly the possibility to introduce an infinite number of satellite resonances. Consider the following off-mass-shell vertex:

$$(q \cdot P)^{k+1} V(k),$$

where $V(k)$ stands for the ordinary scalar emission vertex and

$$P_\mu = 2p_\mu + \sqrt{2i} \sum_{l=1}^{\infty} \sqrt{l} \left( a_\mu^+ (l) - a_\mu (l) \right),$$

$$q^2 = 0, \quad q \cdot k = 0, \quad k^2 \leq 0.$$  

The operator $(q \cdot P)^{k+1}$ is well-defined by virtue of light-like vector $q_\mu$ and has conformal spin $(k^2 + 1)$ assuring that the vertex satisfies the Virasoro gauge conditions. To satisfy $q \cdot k = 0$, it is necessary that $k_\mu$ is not a time-like vector. We can calculate the two-body amplitude as usual, which turns out to be a series of functions with linear trajectory originated from the Taylor expansion of $(q \cdot P)^{k+1}$. We leave the detailed study along this line for a future publication.

**Acknowledgements**

It is with great pleasure that the author acknowledges constant guidance and encouragement of Dr. Tosaku Kunimasa of Osaka University. The author also wishes to thank Professor Yasutaka Tanikawa for hospitality extended to him at Kobe University.

**References**

2) Some of the papers on this subject are:
5) As for the proof of no ghosts, see: