Hirayama’s and Muta’s Relations as a Finite Mass Difference Condition

Hiroshi KATSUMORI
Department of Applied Physics
Chubu Institute of Technology, Kasugai
Nagoya-Sib. 487
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As a condition for making the nucleon electromagnetic mass difference finite, Hirayama\(^1\) has found a relation between the structure functions in the Bjorken scaling limit by using a scaling variable \(\bar{\omega}\) instead of \(\omega\),

\[
F_1(\bar{\omega}) - (1/\bar{\omega}) F_2(\bar{\omega}) = 0, \tag{1}
\]

where

\[
1/\bar{\omega} = 1/\omega + 1/2\nu + O(1/\nu^{1+\epsilon}) \quad (\epsilon > 0) \tag{2}
\]

and \(\omega = q^2/\nu\) is the original Bjorken scaling variable, \(-q^2\) is the virtual photon mass squared, \(\nu = -p\cdot q/M\) is the photon laboratory energy, \(p\) is the nucleon momentum and \(M\) is the nucleon mass.\(^2\)

On the other hand, Muta\(^3\) has shown another expression as the finite mass difference condition,\(^4,5\)

\[
R = \omega^2/2q^2, \tag{3}
\]

where \(R = \sigma_\perp/\sigma_\parallel\) is the ratio of virtual photon cross sections. The purpose of this note is to show that Hirayama’s and Muta’s relations are identical although they have been derived on seemingly different grounds.

According to Bloom and Gilman\(^6\) and also to Hirayama,\(^1\) it is assumed that the structure functions \(W_1(\omega, q^2)\), \(W_2(\omega, q^2)\) scale more rapidly as \(q^2\) increases when the scaling variable \(\bar{\omega}\) instead of \(\omega\) is used;

\[
W_1(\omega, q^2) = F_1(\bar{\omega}) + O(1/(q^2)^{1+\epsilon}), \quad (\epsilon > 0) \tag{4}
\]

\[
\nu W_2(\omega, q^2)
\]

Equation (1) can then be written as

\[
W_1(\omega, q^2) + O(1/(q^2)^{1+\epsilon})
\]

\[
- [1/\omega + \omega/2q^2 + O(1/(q^2)^{1+\epsilon})] \times [\nu W_2(\omega, q^2) + O(1/(q^2)^{1+\epsilon})] = 0.
\]

Putting the familiar relation

\[
W_1(\omega, q^2)/W_2(\omega, q^2) = (1 + \nu^2/q^2)/(1 + R)
\]

into Eq. (4), we have

\[
1/(1 + R) \left[ -1/\omega + \omega/2q^2 \right] W_2(\omega, q^2) = O(1/(q^2)^{1+\epsilon}). \quad (\epsilon > 0)
\]

When we assume a non-trivial Bjorken limit in the deep-inelastic region \((q^2 \to \infty, \omega \text{ fixed})\), we find Muta’s relation

\[
R \approx (\omega^2/2q^2) [1 + O(1/q^2)]. \tag{6}
\]

Finally we note that although the finite mass difference condition, Eq. (1) or Eq. (3), has not yet been experimentally checked because of large error bars,\(^7\) the possibility that the condition may hold could not be excluded. In our opinion, however, as described in Ref. 4), an appearance of the logarithmic divergence in the self-energy expression seems more plausible rather than the finiteness of self-energy by means of an accidental cancellation, unless we introduce a finite size effect of the constituents,\(^8\) or an anomalous dimension parameter.\(^9,10\)

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2) Note that we use notations \(q^2\), \(\nu\), \(\omega\) as defined in Ref. 4) below, which differ from those of Ref. 1).
3) T. Muta, Prog. Theor. Phys. 48 (1972),
Note that Eq. (3) provides a sufficient condition. The necessary and sufficient condition can be seen from Eq. (3·6) of our previous work: H. Katsumori, Phys. Rev. D6 (1972), 1438. That is,

\[ \lim_{q^2 \to 0} \int \frac{2q^2 R(q^2, \omega)}{\omega} W_s(q^2, \omega) \, d\omega = 0, \]

in which vanishing of the integrand leads to Muta's relation.


H. Katsumori, to be published.