Pion Production in Proton-Proton Collision and a Unified Description of Inclusive Spectra

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The inclusive reactions $pp \to \pi^+, \pi^-$ and $\pi^0 X$ are systematically analyzed in terms of a unified formula for single-particle distribution function. These processes are characterized by two components of production mechanism, i.e., fragment and central production mechanisms which are mapped on the $H \otimes P$-type and $P \otimes P$-type bond diagrams. The former and latter mechanisms are dominant in the large and small energy fraction regions, respectively. The ratio $\pi^+/\pi^-$ on the entire Peyrou plot is naturally explained.

§ 1. Introduction and summary

Recently, from the viewpoint of composite structure of hadrons a unified description of the single-particle distribution function has been proposed. Hadrons are described by the minimum urbaryon number (valence urbaryon number), and reaction mechanism is characterized by the flow of valence urbaryons which is mapped on the bond diagrams. The unified description is given by the whole-region formula, which incorporates smooth transition between soft collision at small $p_T (p_T < 2 \text{GeV/c})$ and hard collision at large $p_T (p_T > 2 \text{GeV/c})$, where $p_T$ denotes the transverse momentum.

In this paper, we present the systematic analyses of inclusive reactions $pp \to \pi^+, \pi^-$ and $\pi^0 X$ in terms of the unified description of the single-particle spectrum. These processes are characterized by the two components of production mechanism, i.e., fragment and central production mechanisms which are described by the $H \otimes P$-type and $P \otimes P$-type bond diagrams as shown in Fig. 1. At small $p_T$ regions, the $H \otimes P$-type mechanism exhibits the limiting behaviour and the $P \otimes P$-type one the belated Feynman scaling with central plateau in rapidity space. The increase of spectrum at large $p_T$ regions is explained by the $P \otimes P$-type mechanism.

The relative magnitude of the amplitude is given by a simple counting rule. The good agreement of the model with the data on $pp \to \pi^+, \pi^-$ and $\pi^0 X$ is obtained.
It is shown that the $H\otimes P$-type mechanism is dominant in the large energy fraction region and the $P\otimes P$-type one in the small energy fraction region. This behaviour explains the ratio $\pi^+/\pi^-$ at both the small and large $p_T$ regions.

In § 2, we discuss the notations and the model. In § 3, the comparison with experiments is given. In § 4, discussion is given.

## § 2. The model

### 2-1 Notations

We define the notations. We consider the inclusive reaction $ab\to cX$ and denote the four momenta of particles $a$, $b$, and $c$ as $p_a$, $p_b$, and $p_c$, respectively. Hereafter we adopt the center-of-mass frame. The total energy square is $s=(p_a+p_b)^2$. When $E_c$, $p_\perp$, $p_T$, and $m_c$ denote energy, longitudinal momentum, transverse momentum and mass of the produced particle $c$, respectively, the scaling variables are given as

$$x = 2p_\perp/\sqrt{s} \quad \text{and} \quad \bar{x} = 2E_c/\sqrt{s},$$

where $x$ and $\bar{x}$ represent the Feynman variable and energy fraction, respectively. Furthermore, it is convenient to define the light-like variables $(x_\pm)$ and other ones as follows:

$$x_+ = \frac{1}{2}(\bar{x} \pm x),$$

$$x_T = \frac{2m_T}{\sqrt{s}},$$

where $m_T = \sqrt{p_\perp^2 + m_c^2}$.

### 2-2 Model

We consider the inclusive reaction $pp\to \pi X$. This reaction is specified by the two components of production mechanism, i.e., fragment and central production mechanisms. From the view point of composite structure of hadrons, these mechanisms are characterized by bond diagrams as shown in Fig. 1.

According to the whole-region formula for single-particle distribution function $\rho_{pp}$, we have

$$\rho_{pp} = \rho_{pp}(H\otimes P) + \rho_{pp}(P\otimes P),$$

$$\rho_{pp}(H\otimes P) = \frac{g_1^* g^*}{1-x} \left\{ F(m_T^p) \left[ \left( \frac{1-x}{1-x_+} \right)^2 \left( 1-x \right) \right]^{2r(p_T^p)} \left( \frac{1-x}{1-x_-} \right)^2 \right\},$$

$$\rho_{pp}(P\otimes P) = \frac{g_2^* g^*}{1-x} \left\{ F(m_T^p) \left[ \left( \frac{1-x}{1-x_+} \right)^2 \left( 1-x \right) \right]^{2r(p_T^p)} \left( \frac{1-x}{1-x_-} \right)^2 \right\},$$

where $g_1^*$ and $g_2^*$ are the normalization constants of each production mechanism.

The function $F(m_T^p)$ is parametrized as follows:
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\[ F(m_T^2) = \left( \frac{m_0^2}{m_T^2 + c \sqrt{s}} \right)^2, \quad (1) \]

where \( m_0 \) is the scaling parameter. The correction term \( c / \sqrt{s} \) is introduced to fit the data from \( p_T \approx 6.6 \text{ GeV}/c \) to 1500 \text{ GeV}/c and contributes to distribution function at low energies and at small \( p_T \) regions. At high \( s \) and large \( p_T \) regions, we obtain \( F(m_T^2) \sim p_T^{-4} \).

The dynamical parameter \( \gamma(p_T^2) \) is parametrized as follows:

\[
\gamma(p_T^2) = 1 - \beta(p_T^2), \\
\beta(p_T^2) = \frac{M_0 e^{-m'T}}{m_T + m'}, \quad (2)
\]

where \( A \) and \( m' \) are free parameters and \( M_0 \) is scaling parameter which is chosen to be 1 \text{ GeV}. Then we assume the smooth transition mechanism of underlying dynamics between small and large \( p_T \) regions. The parameter \( \gamma(p_T^2) \) approaches 1 as \( p_T \) increases. When we use \( A=0.055 \text{ GeV}^{-1} \) and \( m'=1.45 \text{ GeV} \), the structure of \( \gamma(p_T^2) \) is shown in Fig. 2.\(^*\)

The coupling constants \( g^s(H \otimes P) \) and \( g^s(P \otimes P) \) are specified by the simple counting rule.\(^\circ\) These relative values for \( \pi^+, \pi^- \) and \( \pi^0 \) in proton-proton collision are given in Table I. The relative ratio of \( g_s^s \) to \( g_s^s \) prescribes the relative weight between fragment and central production and is phenomenologically determined.

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\[\text{Table I.}\]

<table>
<thead>
<tr>
<th>(pp \rightarrow \pi^- X)</th>
<th>( g^s(H \otimes P) )</th>
<th>( g^s(P \otimes P) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^+ X )</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>( \pi^0 X )</td>
<td>3/2</td>
<td>1</td>
</tr>
</tbody>
</table>

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\(^*\) In the previous analyses,\(^\dagger\) the parametrization of \( \gamma(p_T^2) \) was used as follows:

\[
\gamma(p_T^2) = \begin{cases} 
1.6 - \frac{1}{1 - 4/15 p_T^2} & \text{for } p_T^2 < 2.5 (\text{ GeV}/c)^2, \\
1 & \text{for } p_T^2 \approx 2.5 (\text{ GeV}/c)^2.
\end{cases}
\]

Its parametrization is different from the one of the smooth transition mechanism.

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\(^\circ\) In this section we compare our model with experiments up to ISR and clarify the role of each production mechanism.
3-1 Rapidity distribution at small $p_T$ regions

We study the rapidity distribution at small $p_T$ regions. First, we analyze the data on $pp \rightarrow \pi^- X$ at 6.6 GeV/c$^0$ in order to fix the parameters. Then we have

$$g_1^* = g_2^* = 0.3 \text{ mb (GeV/c)}^{-2},$$

$$c = 0.25 \text{ GeV}^2,$$

$$m_0^* = 1.45 \text{ GeV}^2.$$

The relative ratio $g_1^*/g_2^*$ is phenomenologically chosen to be 1. The values of parameters $A$ and $m'$ are given in the preceding section. The result is shown in Fig. 3.

Next, we use the same parameters and compare our model with the data on $pp \rightarrow \pi^+ X$ from 12 GeV/c to 1500 GeV/c$^6$. The results are shown in Fig. 4 and are consistent with experiments.

Each contribution of the $H\otimes P$-type and $P\otimes P$-type mechanisms to single-particle spectrum is shown in Fig. 5. The $H\otimes P$-type mechanism exhibits the limiting behaviour and decreases with energy at $y^* \approx 0$ where $y^*$ denotes the rapidity in the center-of-mass system. The $P\otimes P$-type mechanism exhibits the central plateau and the belated Feynman scaling.

Also we consider the ratio $\pi^+ / \pi^-$ in order to make clear the role of the coupling constant. The ratio $\pi^+ / \pi^-$ at $p_T = 0.4 \text{ GeV/c}$ is shown in Fig. 6. Here, its maximum value is 2 because of the simple counting rule. That is, it given by the ratio of the numbers of proton and neutron urbaryons in proton. It means that the simple counting rule holds as the first approximation.

3-2 $p_T$ distribution at $y^* = 0$

We analyze the $p_T$ distribution at $y^* = 0$. Each production mechanism exhibits the following behaviour:

$$\rho_{pp}^* (H\otimes P) \sim 2g_1^* g^* (H\otimes P) \frac{F(m_T^2) x_T}{(1-x_T)^2} \frac{(1-x_T)^8}{(1-(x_T/2)^8)^{y^*}}$$

$$\rho_{pp}^* (P\otimes P) \sim g_2^* g^* (P\otimes P) \frac{F(m_T^2)}{(1-x_T)^2} \frac{(1-x_T)^8}{(1-(x_T/2)^8)^{y^*}}.$$

We fit the data on $pp \rightarrow \pi^0 X$. The result is shown in Fig. 7 and is consistent with data. Also, the contribution of each production mechanism to the spectrum is given in Fig. 8. The $H\otimes P$-type mechanism exhibits the increase and decrease
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Fig. 4. The rapidity distribution of (a) $pp \rightarrow \pi^- X$ and (b) $pp \rightarrow \pi^+ X$, where $y_{\text{Lab}}$ denotes the rapidity in the laboratory system. The data are taken from Ref. 6).

Fig. 5. The rapidity distribution of each production mechanism, i.e., $H \otimes P$-type and $P \otimes P$-type mechanisms. The process is $pp \rightarrow \pi^+ X$.

Fig. 6. The rapidity dependence of the ratio $\pi^+ / \pi^-$. The data are taken from Ref. 7).
with energy at $p_T < 3 \sim 4 \text{ GeV}/c$ and $p_T > 5 \text{ GeV}/c$, respectively. The $P\otimes P$-type mechanism exhibits the rapid increase with energy. Therefore the main part of the strong increase at large $p_T$ regions of recent ISR data results from the $P\otimes P$-type mechanism.

However, we note the following relations at very large $p_T$ regions:

$$\rho^*(P\otimes P) > \rho^*(H\otimes P) \quad \text{at} \quad p_T < p_0 \text{ GeV}/c,$$

$$\rho^*(P\otimes P) < \rho^*(H\otimes P) \quad \text{at} \quad p_T > p_0 \text{ GeV}/c,$$

where $p_0$ denotes an adequate value. For example, $p_0$ is about 14 GeV/c when $\sqrt{s} = 63$ GeV in Fig. 8. This behaviour is made clearer by means of analyzing the $p_T$ and/or $x_T$ dependence of the ratio $\pi^+ / \pi^-$ at $y^* = 0$. We show the results in Fig. 9. It is expected that the ratio $\pi^+ / \pi^-$ approaches to 2 as $x_T \rightarrow 1$. It is similar to the behaviour of the ratio as $x \rightarrow 1$ at small $p_T$ regions. Thus, we conclude that the $H\otimes P$-type mechanism

Fig. 7. The transverse momentum distribution of $pp \rightarrow \pi^0 X$ at $y^* = 0$. The data are taken from Ref. 8.

Fig. 8. The transverse momentum distribution of each production mechanism, i.e., $H\otimes P$-type and $P\otimes P$-type mechanisms at $y^* = 0$. The process is $pp \rightarrow \pi^0 X$. The symbol $\uparrow$ denotes the tendency of the energy dependence.

Fig. 9. The ratio $\pi^+ / \pi^-$ at $y^* = 0$. (a) Its $p_T$ dependence and (b) its $x_T$ dependence. The data at $p_T = 300 \text{ GeV}/c$ are taken from Ref. 9.)
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is dominant in the region with the large energy fraction and the $P \otimes P$-type one in the region of the small energy fraction for $pp \rightarrow \pi X$. The ratio $\pi^+ / \pi^-$ on the entire Peyrou plots naturally explained.

§ 4. Discussion

(1) In this paper, we have restricted our analyses to $pp \rightarrow \pi X$. When we use the whole-region formula for single-particle spectrum, we may analyze the other inclusive processes in proton-proton collision in the similar way. Then, we must consider the SU(3) breaking of $\hat{g}(P \otimes P)$, which is approximately prescribed by the CGZ-scheme.\textsuperscript{10} The coupling constant $\gamma(H \otimes P)$ is given by the simple counting rule.\textsuperscript{4} In the previous analyses\textsuperscript{13} by the interpolation formula, we considered the SU(3) breaking and the unknown effects for $\hat{g}(P \otimes P)$ in order to explain the other particle production ratio. But in this case we need not consider such effects. It means that the whole-region formula is good. We shall investigate this point in detail in a forthcoming paper. On the basis of the analyses of this paper, we shall also investigate the two-particle correlation in terms of our approach.

(2) The form of $F(m_T^2)$ in § 2 gives the following behaviour at large $x_T$ regions:

$$ f_{pp}^{(1)} \sim s^{-2} g(x_T), $$

where we have used $\gamma(p_T^2) \approx 1$. This is inconsistent with the recent NAL data\textsuperscript{9} which show the behaviour $s^{-x} g(x_T)$. This point is an open question. However, our model may explain the $p_T$ dependence of the power of $s$ by the structure of $\gamma(p_T^2)$.

(3) We have assumed the smooth transition of underlying dynamics between small and large $p_T$ regions. If we consider an ideal limit of the transition, the previous analyses\textsuperscript{13} of single-particle distribution function at small $p_T$ regions suggest that $\gamma(p_T^2)$ is nearly equal to 0.5, while the recent field theoretical models\textsuperscript{10} with composite view give such a result that $\gamma(p_T^2)=1$ at large $p_T$ regions. We may consider that the value of $p_T$ at the transition point is nearly 2 GeV/c from Fig. 2. Also, the parameter $d\gamma / dp_T$ seems to correspond to the so-called ordered parameter in phase transition.

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