VELOCITY MEASUREMENTS IN NEAR-SURFACE FORMATIONS*

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ABSTRACT

Procedures are described for measuring various seismic velocities at shallow depths in the earth, and examples of the resulting logs are presented. Velocities in Austin chalk and Eagle Ford shale show that these formations are not isotropic, and velocities in loose sand are seen to increase smoothly with depth except for an abrupt increase in compressional speed at the water table. Elastic constants for chalk and shale are computed. A discussion is given of the literature dealing with a packing of spheres as a model for loose sand, and an approximate theory is presented which includes tangential forces between spheres.

INTRODUCTION

Detailed elastic descriptions of near-surface earth materials can be of substantial help in the interpretation of various types of experiments with seismic waves. For example, a knowledge of near-surface layering provides a basis for comparing observed properties of Rayleigh waves with appropriate theoretical treatments. Procedures have been devised which provide suitable elastic descriptions for these purposes, and it is felt that the properties of near-surface earth materials obtained in this way are of some fundamental interest in themselves.

In the experiments to be described, an effort was made to confine the measurements to a small enough region that an assumption of uniformity of earth properties would be justified. Highly localized disturbances are detected by means of geophones or pressure gauges, and velocities are computed as ratios of detector spacings to time intervals obtained from visual comparisons of the observed waveforms. Boreholes permit a systematic variation in the depth at which the measurements are made, and thus a log of earth properties to depths of two or three hundred feet can be obtained.

FIELD PROCEDURES

One experiment requires the use of only one borehole. A blasting cap is fired in the borehole fluid, and the resulting pressure is measured at two shallower depths by crystal gauges. The gauges are usually 15 and 25 feet above the cap. Typical records obtained are shown in Figure 1. The earliest signal shown in Figure 1A indicates a vertically-travelling compressional wave in the formation. The later signal, which causes the trace to go off-scale, is a compressional wave travelling directly up to the borehole fluid. Examples of fluid-borne signals are shown in Figures 1B and 1C.

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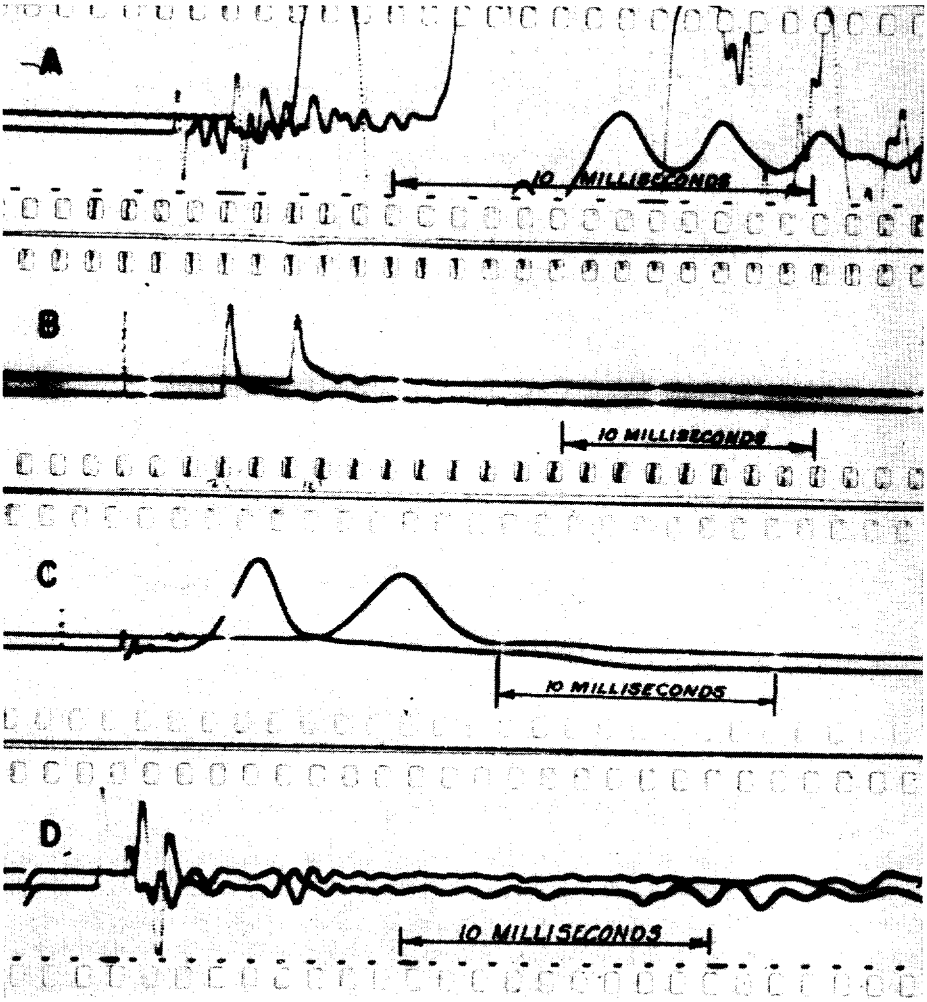

FIG. 1. Typical records from blasting caps. A. Vertically-travelling compressional waves. B. Water pulses in a high-rigidity medium (Austin chalk). C. Water pulses in a lower-rigidity medium (Eagle Ford Shale). D. Horizontally travelling compressional waves.

A second type of measurement requires three boreholes, usually spaced along a line with 10-foot separation. The disturbance due to a blasting cap fired at some depth is observed by means of crystal gauges at the same depth in the adjacent boreholes. Typical records shown in Figure 1D indicate a horizontally-travelling compressional wave in the formation and nothing corresponding to the fluid-borne signal.

The third type of measurement could be done in three holes arranged as above, but frequently five holes in a line with 10 foot spacing have been used. A

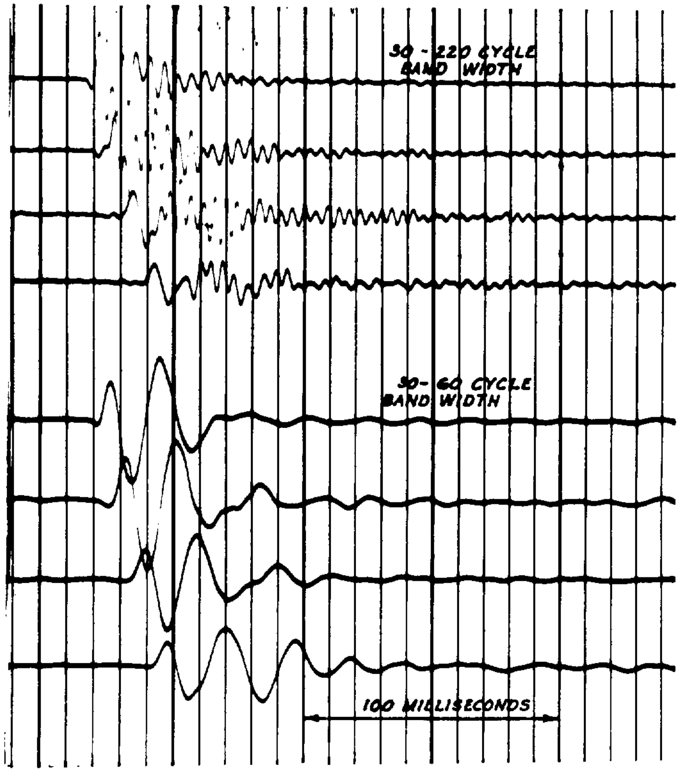

FIG. 2. Records from falling-weight SV generator.

cylindrical container is spiked to the borehole wall at some depth. A 20-pound weight is allowed to fall a foot or so onto the bottom of the container. A force is transmitted to the formation which initiates a horizontally-travelling *SV* shear wave. Vertically-sensitive geophones spiked at the same depth in the other boreholes give signals such as those shown in Figure 2. Although the total waveform from each geophone is rather complex, it is usually possible by means of filters to pick out a portion of the signal which travels at a low speed in good agreement with other indications of *SV* shear speed.

SOME GENERAL CONSIDERATIONS

It is recognized that disturbances created and measured by the procedures described above may result in very complicated records, depending on the nature of the surrounding earth. An effort was made to obtain simple records, first, by confining the experiment to such a small volume that the assumption of homogeneity is generally valid, and second, by choosing sources expected to result in simple elastic waves. Rayleigh[1] described the sound field due to a point compres-

[1] Rayleigh, "Theory of Sound," *Dover Publications*, New York (1945), 418 and 424.

sional source and to a force applied at a point, both applicable to a homogeneous, isotropic solid of infinite extent. These descriptions would certainly be expected to serve as guides for more complex solids, although equivalent treatments of localized sources are not available.

Some aspects of seismic wave propagation in more complex solids have been treated. Stoneley[2] describes a "transversely isotropic" solid in which all properties are symmetrical with respect to a fixed direction. Shales apparently fit this description, where the line of symmetry is perpendicular to the laminations. Stoneley shows that plane waves in any direction must travel at one of three speeds, one of which reduces to compressional speed and the other two to shear speed as the degree of anisotropy is reduced to zero. This kind of solid is described by five elastic constants and the density. Assuming the density is known and that symmetry is referred to a vertical line, one elastic constant is determined by the speed of vertically-travelling compressional waves; another by the speed of horizontally-travelling compressional waves; another by horizontally-travelling *SV* waves; and a fourth by horizontally-travelling *SH* waves. The fifth elastic constant could be obtained from velocities for directions of travel other than vertical or horizontal. It is felt that plane wave propagation in this type of solid offers a reasonable basis for interpreting records in terms of compressional and shear arrivals and for computing elastic constants from the observed velocities. Stoneley shows which elastic constants influence Rayleigh waves and Love waves.

Wave propagation in another type of material has been treated at some length. This material is a mathematical model for loose sand, consisting of elastic spheres packed in a regular way and pressed together by gravity. Gassmann[3] computed the curved ray-paths to be expected for this situation. A local disturbance certainly does not radiate as a spherical wave, and the error in velocity measurement for a given source and detector spacing can be estimated from Gassmann's computed curve. Horizontal velocities determined between two detectors 10 and 20 feet from a source should be pretty good for depths greater than 20 or 30 feet. For shallower depths, measured velocities would be too high unless the spacing is suitably reduced. There is no error in vertical velocity determinations due to curvature of path.

It is, of course, true that horizontal velocities near an interface may be controlled by a refraction path rather than a horizontal path at the depth of measurement. This situation can easily be recognized by inspection of the complete velocity log.

The manner in which the speed of signals travelling in the borehole fluid de-

[2] Robert Stoneley, "The Seismological Implications of Aeolotropy in Continental Structure," *Monthly Notices of the Royal Astronomical Society*, Geophysical Supplement, Vol. 5, No. 8 (1949), 343–353.

[3] Fritz Gassmann, "Elastic Waves Through a Packing of Spheres," *Geophysics*, XVI (1951), 673–685.

pends on the properties of an isotropic solid was treated by Lamb.[4] He derived the result that at low frequencies,

$$c = [\mu B/\rho(\mu + B)]^{1/2} \tag{1}$$

where c is the borehole speed,

B is the bulk modulus of the fluid,

ρ is the density of the fluid,

and μ is the rigidity of the solid.

Borehole speeds are thus measures of the shear rigidity of the formation in which the borehole is drilled if the formation is isotropic. If the formation density is known, the speed of shear waves in the formation can be computed from the rigidity determined in this way. Lamb's result for isotropic solids can be derived very simply from static elasticity. Borehole velocity for a transversely isotropic solid is derived below by this simple method.

Although the results of the procedures described here may at times need interpretation, it is felt that useful descriptions are obtained of the earth materials in which the measurements are made.

Borehole Speed in a Transversely Isotropic Solid

Consider a fluid-filled cylinder of radius a with its axis in the direction of symmetry of a transversely isotropic solid of infinite extent. If the fluid pressure is increased to a value P, the motion in the solid may be assumed to consist of a radial expansion of the form $u_r = Q/r$. This assumption can be checked by substitution into stress-strain relations, and the constant Q can be evaluated by equating stresses at the wall of the cylinder. The appropriate stress-strain relations from Stoneley's article are:

$$\begin{aligned} X_x &= A\frac{\partial u}{\partial x} + (A - 2N)\frac{\partial v}{\partial y} \\ X_y &= (A - 2N)\frac{\partial u}{\partial x} + A\frac{\partial v}{\partial y} \\ Y_x &= X_y = N\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right). \end{aligned} \tag{2}$$

The $x-$ and $y-$ components of the radial displacement are $u = Qx/r^2$ and $v = Qy/r^2$. Equating stresses at the wall of the borehole leads to the following expressions:

$$X_x x^2/a^2 + Y_y y^2/a^2 + 2X_y xy/a^2 = -P. \tag{3}$$

If the stresses, including the appropriate partial derivatives, are substituted into

[4] H. Lamb, "On the Velocity of Sound in a Tube, as Affected by the Elasticity of the Walls," *Manchester Memoirs*, XLII, No. 9 (1898).

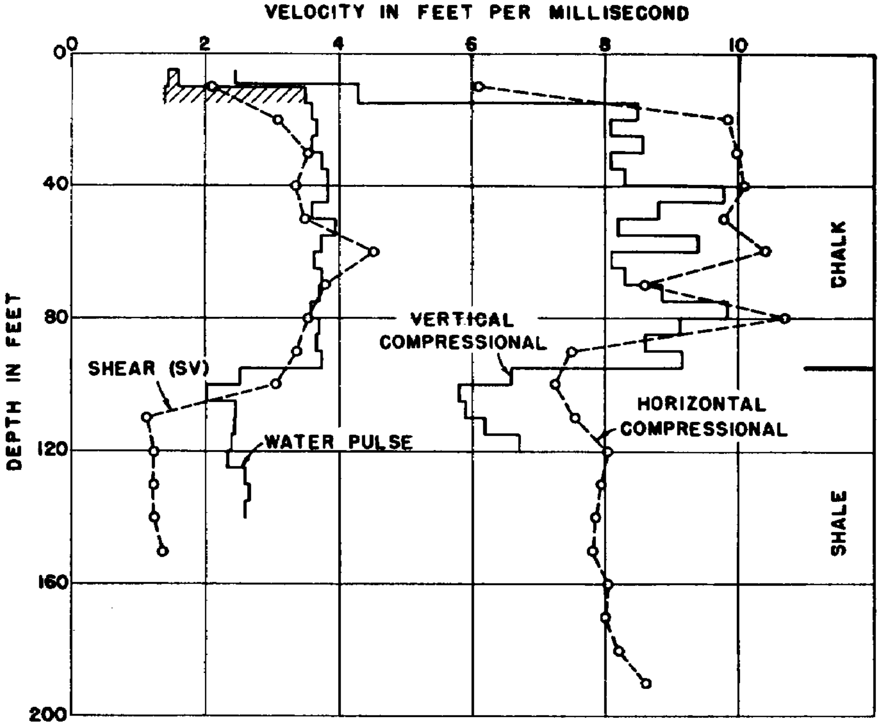

Fig. 3. Velocity logs at the laboratories grounds, Dallas County, Texas.

this equation, the original assumption is seen to be correct if $Q = a^2 P/2N$. N is the elastic constant which controls the speed of horizontally-travelling *SH* waves in the solid. Hence,

$$u_r = a^2 P/2Nr. \tag{4}$$

If the pressure in the fluid is increased by an amount P, a section originally of length l will be shortened by an amount dl. An elastic constant K opposing this contraction is defined by

$$\pi a^2 dl/\pi a^2 l = - P/K.$$

The section will expand radially, and the net change in volume is the difference between contraction and expansion. The net volume change divided by the original volume depends on the pressure change and the incompressibility B of the fluid.

$$(\pi a^2 dl + 2\pi a l P a^2 2/Na)/\pi a^2 l = - P/B.$$

These two relations give

$$K = NB/(N + B), \tag{5}$$

Whence

$$c = [NB/\rho(N + B)]^{1/2}. \qquad (6)$$

This is the same as Lamb's result with N replacing μ.

DISCUSSION OF EAGLE FORD SHALE AND AUSTIN CHALK

Figure 3 shows velocity measurements made in boreholes at the Field Research Laboratories, where Austin chalk at the surface is underlain by Eagle Ford shale. Horizontal compressional speeds are higher than vertical compressional speeds in both chalk and shale. In the shale, the *SV* shear speed is seen to be significantly lower than the borehole fluid speed, the latter being only slightly higher than the derived speed for *SH* shear waves. This difference between *SV* and *SH* speeds for Eagle Ford shale was observed at other locations, an example of which is shown in Figure 4. The abrupt increase in compressional speed at a depth of 10 feet corresponds to the ground water level. The driller's log and the change in shear properties at a depth of 60 feet suggest that the shale has been considerably modified by weathering action to this depth.

Average velocities for Austin chalk and Eagle Ford shale are listed in Table I.

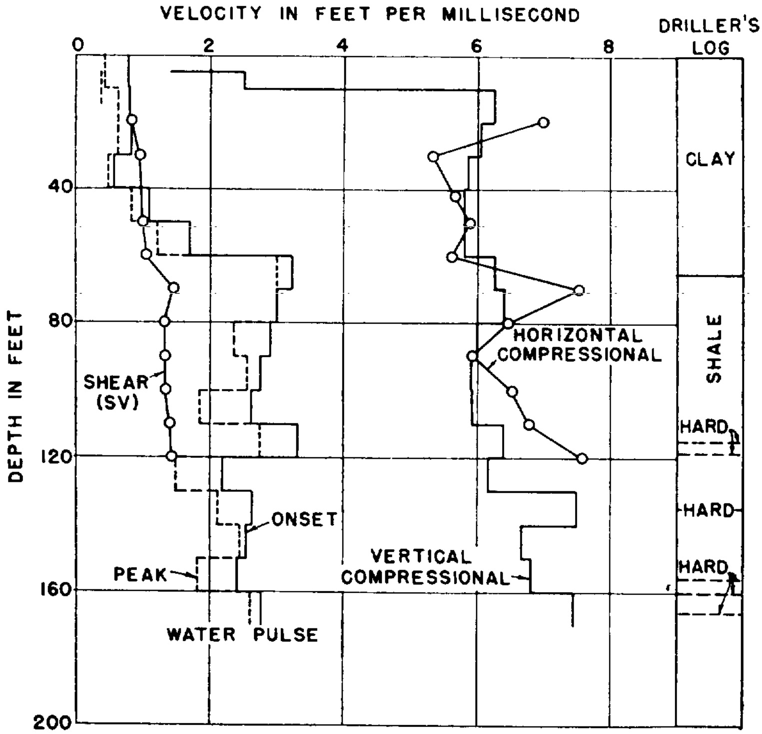

FIG. 4. Velocity logs at Corn Valley Road, Dallas County, Texas.

TABLE I

	Eagle Ford Shale	Austin Chalk
Density	2.0 gm/cm³	2.2 gm/cm³
Horizontal compressional speed	8,000 ft/sec	10,000 ft/sec
Elastic constant A	12×10^{10} dyne/cm²	22×10^{10} dyne/cm²
Vertical compressional speed	6,000 ft/sec	8,500 ft/sec
Elastic constant C	6.5×10^{10} dyne/cm²	14×10^{10} dyne/cm²
SV shear speed	1,250 ft/sec	3,500 ft/sec
Elastic constant L	0.28×10^{10} dyne/cm²	2.4×10^{10} dyne/cm²
Fluid speed	2,500 ft/sec	3,700 ft/sec
Elastic constant N	0.75×10^{10} dyne/cm²	3.1×10^{10} dyne/cm²
SH shear speed	2,000 ft/sec	3,900 ft/sec

Densities obtained from cuttings are also given. Three elastic constants were obtained by multiplying density by the square of the appropriate velocity, and a fourth constant was computed from borehole fluid speed by use of the expression derived above. The derived *SH* speed is also given.

Velocities for both chalk and shale show variation with depth, but when average values are drawn, both formations are obviously anisotropic. All the observations fit the assumption that each of these materials is a transversely isotropic solid, and four of the five elastic constants needed to describe such a solid can be computed. Further measurements might show that even this representation is inadequate to describe the average properties of Austin chalk and Eagle Ford shale.

DISCUSSION OF LOOSE SAND

At several locations where the near-surface material is loose sand, all velocities are found to be very low at the surface, increasing smoothly with depth. Examples are shown in Figures 5 and 6. The rough agreement between horizontal and vertical compressional speeds indicates that there is no marked anisotropy. The *SV* speed shown agrees with *SH* speeds derived from logs of borehole fluid speed, a further indication that loose sand is substantially isotropic.

A plot of derived *SH* speed is given in Figure 6. The ground water level at 50 feet causes an abrupt increase in compressional speed with no perceptible influence on shear properties, a phenomenon which has been observed many times. The solid curves in Figures 5 and 6 are the appropriate velocities of vertically-travelling shear and compressional waves in a hexagonal packing of quartz spheres, computed from Gassmann's expressions with a minor correction discussed below. The constants used for this computation are: density 2.65 gm/cm³, Young's modulus 10^{12} dyne/cm², Poisson's ratio 0.15, all for quartz; and density 1.0 gm/cm³, bulk modulus 2×10^{10} dyne/cm³ for water. The excellent agreement with measured values is somewhat fortuitous, since Gassmann's speeds for horizontally-travelling waves would not agree at all well. However, the dependence of both speeds on the sixth root of the depth and the abrupt increase in compressional speed when the space between the spheres is filled with water

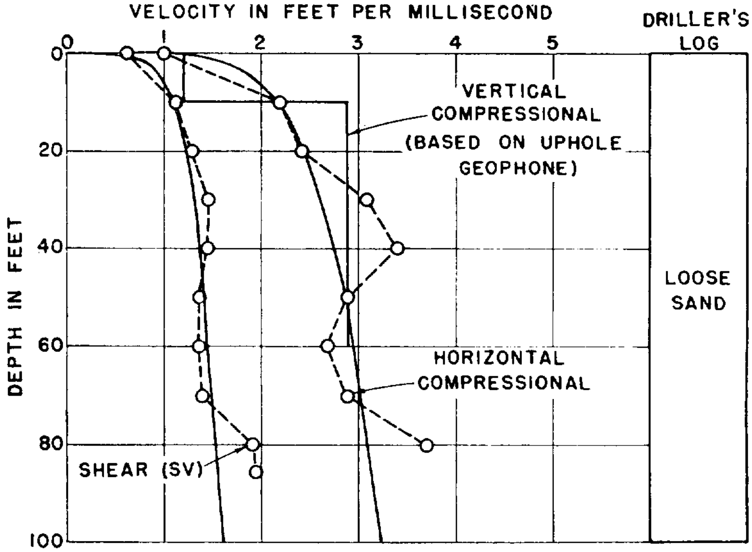

FIG. 5. Velocity Logs at SP 308, T-218 Prospect, Henderson County, Texas.

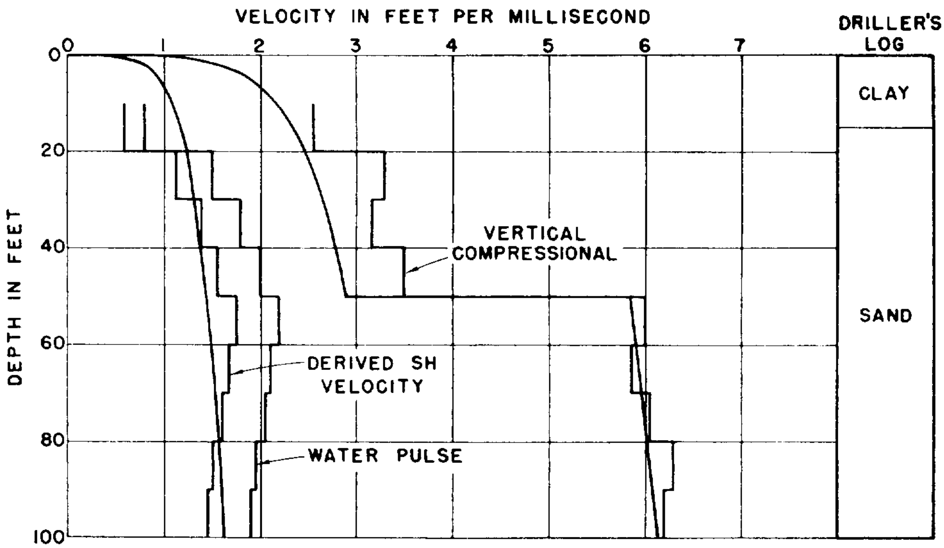

FIG. 6. Velocity Logs at SP 30 A, T-218 Prospect, Henderson County, Texas.

show that a model based on a packing of spheres has a close kinship with loose sand as it occurs in the ground.

THEORIES OF WAVE PROPAGATION IN PACKINGS OF SPHERES

Hara[5] discussed the velocity of sound in a packing of spheres, but all his expressions for velocity are dimensionally incorrect. Iida[6] included the effect of gravity and compared laboratory measurements on columns of sand with derived expressions for velocity, but his expressions are unsatisfactory in some respects. Gassmann recently published a derivation of both shear and compressional speeds in a hexagonal packing of spheres pressed together by gravity, with the pore spaces either empty or filled with fluid.

All of these theories make use of a basic relationship which was worked out by Heinrich Hertz prior to 1881 and reprinted in his collected works. The source used here is Timoshenkos' discussion of Hertz's theory.[7] A force P causes the centers of two identical spheres to approach each other by an amount S. The spheres are characterized by Young's modulus E, Poisson's ration ν, and density ρ. The area of contact is shown to be a circle of radius a, and the following relationships are derived:

$$a = [3(1 - \nu^2)RP/4E]^{1/3}, \tag{7}$$

$$S = [9(1 - \nu^2)^2P^2/2E^2R]^{1/3}. \tag{8}$$

The latter represents a nonlinear spring, since the displacement is not directly proportional to the force. For incremental changes around a preloaded value, the stiffness of this spring is the ratio of incremental force to incremental displacement.

$$\frac{dP}{dS} = [3E^2RP/4(1 - \nu^2)^2]^{1/3}. \tag{9}$$

Gassmann assumed that spheres in contact remain spherical except for the actual circles of contact, where they are flat. As a matter of fact, the spheres are deformed in the neighborhood of the circles of contact. The displacement between centers of spheres for a given load is just twice as large as Gassmann assumed, and for this reason, Gassmann's derived speeds are too high by $\sqrt{2}$.*

The above expressions are exact mathematically and form a basis for stress-strain relationships in a granular substance when the forces act normal to the spherical surfaces at the points of contact. Such forces would exist in a simple cubic packing of spheres for a plane seismic wave traveling along one of the cubic axes. In general, however, forces and displacements would not be in the direction

[5] G. Hara, "Theorie der Schwingungs-ausbreitung in Gekornten Substanzen und experimentelle Untersuchungen in Kohlepulver," Elektrische Nachrichten Technik, Vol. 12 (1935), 191–200.

[6] Kumizi Iida, "Velocity of Elastic Waves in a Granular Substance," Bull. of the Earthquake Research Institute, Vol. 17, part 4 (1939), 783–807.

[7] S. Timoshenko, "Theory of Elasticity," McGraw-Hill Book Co., New York (1934).

* Editor's note: see p. 269 this issue.

of a line connecting centers of spheres, and tangential components would be expected. Elastic distortions due to tangential forces influence the stress-strain relationships, and these are not described by Hertz. The most nearly applicable expression found in the literature so far is an equation given by Mindlin[8] stating the static displacements in a semi-infinite solid due to a point tangential force acting at the surface of the solid. The tangential forces between spheres in contact are undoubtedly distributed over the circle of contact, and an exact treatment would involve a derivation of this force distribution and of the resulting displacements in the spheres. An approximate expression is derived below, based on assumptions which are evaluated to some degree.

Compressional Waves in a Simple Cubic Packing of Spheres

A cubic packing is not a realistic model of sand because it is not mechanically stable, but velocities derived for such a packing illustrate the viewpoint in a very simple way. The treatment is confined to wave-lengths which are very long compared with grain dimensions. The viewpoint consists of deriving an elastic modulus statically for a representative elementary volume and dividing it by the average density for the same elementary volume. The square root of this ratio gives the appropriate velocity.

Plane compressional waves involve normal stresses in the direction of propagation and displacement occurring only in this direction. For simplicity, this direction is taken to be one of the cubic axes. The cube $2R$ on an edge contains one sphere and can be considered a typical elementary volume. The force per unit area divided by the change in length per unit length is the elastic modulus desired.

$$K = \frac{dP}{dS}/2R = [3E^2P/32(1 - \nu^2)^2R^2]^{1/3}. \tag{10}$$

If the loading force is provided by gravity, then it consists merely of the weight of the spheres directly above the one being considered. If the depth of the sphere is Z, then,

$$P = (4\pi R^3/3)\rho_0 g(Z/2R) = 2\pi R^2\rho_0 gZ/3. \tag{11}$$

Substituting this in (10),

$$K = [\pi\rho_0 gE^2Z/16(1 - \nu^2)^2]^{1/3}. \tag{12}$$

The average density is the mass of one sphere divided by the volume of the cube it effectively occupies, or

$$\rho = \pi\rho_0/6. \tag{13}$$

[8] R. D. Mindlin, "Force at a Point in the Interior of a Semi-Infinite Solid," *Physics*, Vol. 7 (1936), 195–202.

The compressional velocity for cubic packing then is

$$V_p = 3^{1/2}E^{1/3}g^{1/6}Z^{1/6}/2^{1/6}\pi^{1/3}\rho_0^{1/3}(1 - \nu^2)^{1/3}. \quad (14)$$

One notable result is that the velocity approaches zero near the free surface and increases as the sixth root of the depth. It should also be noted that the result does not depend on the radius of the spheres.

Displacements Due to Tangential Forces

A plane shear wave traveling along a cubic axis is characterized by tangential stresses on the contact surfaces and displacement perpendicular to the axis. The

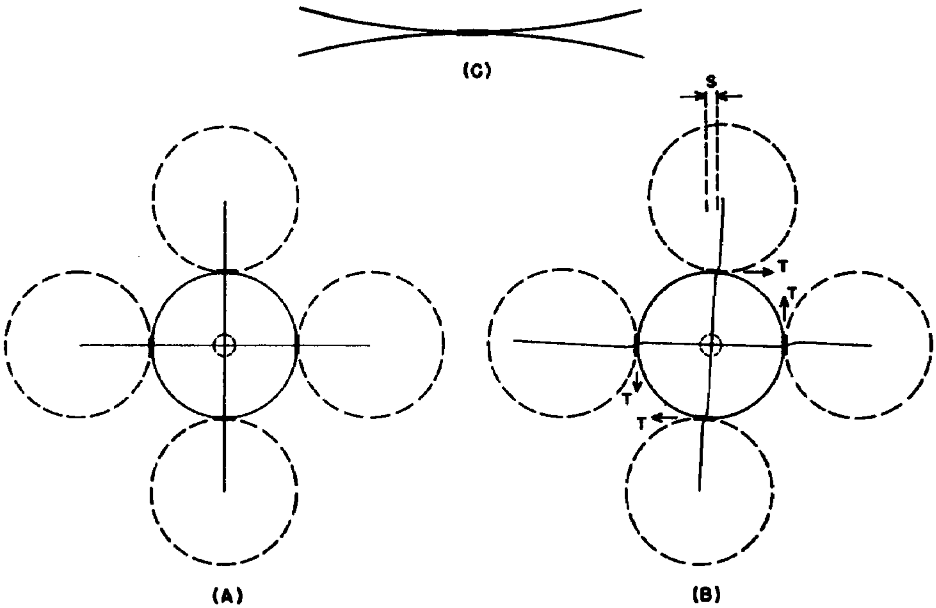

FIG. 7. Geometry for Tangential Forces in a Cubic Packing of Spheres. A. Spheres at rest. B. Spheres displaced by tangential force T. C. Circle of contact at depth of 300 feet.

geometry is the same for compressional waves, and the forces acting on a single sphere are shown in Figure 7. Forces on the front and back contact circles have a negligible effect on the force-displacement relationships. The tangential force at the top must be matched by an equal and opposite force at the bottom for static equilibrium. These forces constitute a couple which can be matched by an opposite couple due to neighboring spheres. Friction between spheres is necessary for these forces to be exerted. An equation is here derived which gives the relative displacement u between the upper and lower contact points as a function of the applied force T.

Mindlin published relations between a force parallel to the surface of a semi-infinite solid applied at a point in the solid and the resulting displacements. When the force is applied at the free surface, the component of displacement parallel to the force and existing at other points on the free surface simplifies to the following equation:

$$\mu = T(1 - \nu \sin^2 \theta)/2\pi\mu r. \tag{15}$$

T acts at the origin in the positive r direction for $\theta = 0$. μ is shear modulus.

If a rigid disc of radius a is assumed to be in slipless contact with the free surface and a total force T is applied to the disc, a displacement U results which is constant over the area of the disc because of its assumed rigidity. Some stress distribution over the disc is set up which may depend on r' and θ' and which must integrate over the disc to the total force T. This distribution of tangential force per unit area should be multiplied by Equation (15) and integrated to give the displacement U. No exact distribution was found by trial and error, so the following approximate one is described. First, Poisson's ratio is considered small enough that $(1-\nu \sin^2 \theta)$ in Equation 15 can be considered unity. This is pretty good for quartz spheres, where ν is about 0.15. This requires that the force distribution be independent of angle, which suggests that a term is being neglected which is the same order of importance as the $\nu \sin^2 \theta$ term above. Then, the force is assumed to vary as the square of the radius,

$$t = Ar'^2.$$

A is evaluated by integrating over the area of the disc and equating to the total force.

$$\begin{aligned} T &= \int_0^a \int_0^{2\pi} Ar'^2 r' d\theta' dr' = \pi a^4 A/2, \\ t &= 2Tr'^2/\pi a^4. \end{aligned} \tag{16}$$

The displacement at any point can be obtained from the following integration:

$$u = \int_0^a \int_0^{2\pi} 2Tr'^2 dr' r' d\theta'/2\pi^2 a^4 \mu r,$$

where r' and θ' are coordinates with the origin at the center of the disc and r is measured from the point in question as an origin. Graphical integration showed that the displacement is not constant, showing the assumed stress distribution to be incorrect. However, the departure is small enough that a useful result is obtained by accepting the stress distribution as approximately correct. The displacement at the center of the disc is

$$U = \int_0^a \int_0^{2\pi} Tr'^2 dr' d\theta'/\pi a^4 \mu = 2T/3\pi\mu a. \tag{17}$$

This is a linear spring with a stiffness constant given by

$$\frac{dT}{dU} = 3\pi\mu a/2. \tag{18}$$

Shear Waves in a Simple Cubic Packing of Spheres

A sphere in a cubic packing is in contact with six neighbors as shown in Figure 7. The circles of contact are much smaller than shown, the radius being about 3 percent of the radius of the sphere at a depth of 300 feet and, of course, being smaller at shallower depths. Figure 7C gives the proportions in the extreme case. Because the circle is small, an assumption is made which is undoubtedly good at shallow depths and is probably pretty good to a depth of 300 feet or more. This assumption is that tangential forces applied over the circle of contact cause displacements relative to the body of the sphere which are the same as if the circle were on a plane rather than a curved surface. This implies that the spheres maintain their shape except for distortions in the immediate neighborhood of each circle of contact.

If a tangential force T is applied on the top contact circle as shown in Figure 7B, a local displacement U results and the sphere moves a local displacement of U at this point, also. The sphere rotates until the couple due to top and bottom forces is balanced by forces T on the side circles as shown, and this results when a displacement of U has been achieved at each circle. The relative displacement between the top and bottom circles is U at the top plus U at the bottom plus $2U$ due to rotation, or a total of $4U$.

Using Equation (18),

$$\frac{dT}{dS} = 3\pi\mu a/8. \tag{19}$$

To get an elastic modulus,

$$K' = \frac{dT}{dS}/2R = 3\pi Ea/32(1 + \nu)R. \tag{20}$$

Making use of Equations (7) and (11), the following expression is derived which is comparable with (12) for compressional waves.

$$K' = [3\pi(1 - \nu)/16][\pi E^2\rho_0 gZ/16(1 - \nu^2)^2]^{1/3}. \tag{21}$$

The velocity is obtained from (21) and (13).

$$V_s = 3\pi^{1/6}(1 - \nu)^{1/2}E^{1/3}g^{1/6}Z^{1/6}/4(1 - \nu^2)^2 2^{1/6}\rho_0^{1/3}. \tag{22}$$

This is the speed of shear waves in a simple cubic packing.

Effect of Water Saturation

If one assumes that enclosed fluid and sand particles move together in a compressional wave, the speed can be computed very simply from static considerations. Displacement of one face of an elementary cube causes volume compression of the fluid and of the solid grains and also deforms the matrix of grains in contact. Hence there are two stiffnesses opposing the motion. The first is

$$B_c = \frac{1}{\dfrac{(1-n)}{B_g} + \dfrac{n}{B_l}}, \quad \text{where} \tag{23}$$

n is porosity, B_g is bulk modulus of the grains, and B_l is bulk modulus of the liquid. The second (for cubic packing) is the same as (12) except for substitution of $(\rho_0 - \rho_l)$ for ρ_0 because of the buoyant effect of the fluid,

$$K_l = \left[\pi(\rho_0 - \rho_l)gE^2Z/16(1-\nu^2)^2\right]^{1/3}. \tag{24}$$

The appropriate compressional modulus is the sum of the two above stiffnesses.

$$K_s = B_c + K_l.$$

The density of the saturated medium is

$$\rho_s = (1-n)\rho_0 + n\rho_l,$$

and the speed is

$$V_p \text{ (saturated)} = [K_s/\rho_s]^{1/2}.$$

For cubic packing, $n = (1 - \pi/6)$, and the speed for compressional waves is:

$$V_p \text{ (sat)} = \left[\frac{\dfrac{1}{\dfrac{(\pi/6)}{B_g} + \dfrac{(1-\pi/6)}{B_l}} + \dfrac{\pi(\rho_0 - \rho_l)gE^2Z^{1/3}}{16(1-\nu^2)^2}}{(\pi/6)\rho_0 + (1-\pi/6)\rho_l}\right]^{1/2}. \tag{25}$$

The more refined derivation by Gassman makes the same assumption of grains and fluid moving together and comes out with the expression for hexagonal packing equivalent to the above equation for cubic packing.

Usefulness of This Model

It must be significant that the computed dependence of speed on the sixth root of the depth and the increase in compressional speed at the water table agrees so well with measurements. However, none of the treatments of packings of spheres is satisfactory in detail. Tangential forces must be a factor in the behavior of sand particles. Such forces result in sliding between spheres near the peripheries of the circles of contact, thus providing a means of introducing sliding friction as a source of attenuation. Although the treatments discussed here can hardly be extrapolated to the depths and types of materials discussed by Faust,[9]

[9] L. Y. Faust, "Seismic Velocity as a Function of Depth and Geologic Time," *Geophysics*, Vol. XVI (1951), 192.

his dependence of velocity on the sixth root of the depth suggests that some extension of this type of approach should be applicable to sands and shales at great depths.

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