DISCUSSION

J. Raabe[2]

The computation of impeller flow presented in this article is used only for recalculation of the flow field in an impeller of given vane geometry, usually defined as the "direct problem." It should be stressed that the method presented, can be applied also to the solution of the "indirect problem" as the design task. This has been described in the proceedings of the fifth conference on fluid machinery in Budapest in the paper "On Design of Blade Cascades by Prescribed Distribution of Contour Velocity" by M. Popov, K. Varsamov, and I. Belberov, Bulgaria. Moreover, these authors have extended their method to a linear cascade of a more general and realistic case with profiled vanes. It should also be mentioned that the method for linear cascades, as presented, has long been known as a common topic of textbooks, which a look in a textbook like W. Traupel, *Thermische Turbomaschinen*, Vol. 1, Springer, Berlin, Heidelberg, New York, second edition, 1966, may convince the reader.

The reduction of the resting circular cascade to the linear one has been mentioned under others also by myself in a textbook published in 1968 (J. Raabe, *Grundlagen der hydraulischen Strömungsmaschinen*, VDI-Verlag, Düsseldorf, 1968). The authors are extending this reduction to the circular cascade rotating with constant angular velocity ω. One must be aware that any treatment of any cascade flow by the author's relation (equation (17*a*) and (17*b*)) is linked to a coordinate system fixed to the vanes. Thus only the steady relative flow is described with the aide of these relations. These relations contain the sum of resulting induced velocities of bound elements of potential vortices carried by an arbitrary carrier of singularities. Contrary to the mentioned approximations given for example by Schlichting and Scholz in form of Birnbaum response. Glauert series for the γ distribution, located at the vane chord, the authors use as vortex carrier the profile itself, a method which has been introduced by Martensen for arbitrary profiles. (E. Martensen: 1. Praktisches Verfahren zur Berechnung der Druckverteilung an stark gewolbten dunnen Profilen, Bericht 57 A 05 der Aerodynamischen Versüchsanstalt Göttingen, 1957. 2. Berechnung der Druckverteilung bei dicken Gitterprofilen mit Hilfe Fredholmscher Integralgleichungen, 2. Art., Mitteilung des Max-Planck-Instituts für Strömungsforschung der Aerodynamischen Versuchsanstalt, Nr. 23, Göttingen, 1959).

Contrary to the plane potential flow treated in the aforementioned articles the here considered relative flow of the rotating cascade is rotational. In the case, assumed, the rotation equals—2ω. As shown by myself in textbooks and publications the distribution of the relative velocity within an impeller in case of the two dimensional flow field can be reduced to a *linear* inhomogeneous differential equation of the relative velocity. The inhomogeneous term is given by the vorticity of the relative whirl, which is reduced in the case, considered to—2ω. Only in this assumed simplified two dimensional flow field the resulting velocity can be obtained by addition of the induced velocities, the vortex bound velocity and the undisturbed throughflow velocity. According to the velocity scale expressed in equation (11)) in consequence of the conformal mapping, also the relative flow through the linear cascade is vortex bound. Therefore the relation 13) for V_{y2} which is restricted to plane potential flow can not be applied to the relative flow of the conformally mapped linear cascade. Contrary to the linear cascade with potential flow the circulation Γ (according to equation (16)) can not be used in the case, considered, for the computation of a lift by means of Kutta's and Jonkowsky's theorem. That means Γ is a practically useless figure in the case, considered.

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A simple example may highlight the difference between the linear cascade, considered, and that in plane potential flow. Imagine a lattice consisting of straight vanes as profiles in the mapping plane. For such a lattice in a potential flow the direction of shockless inlet coincides with the zero lift direction, the outlet direction and the vane direction. Therefore we would have no deflection of the relative flow and no lift in case of the shockless inlet, considered here. Actually we have also a lift under shockless inlet. This is due to the nonvalidity of the aforementioned theorems of a linear cascade in plane potential flow.

One reasonable reason in favour of the conformal mapping into the w-plane may be the fact that the resulting, induced velocity of linear elementary vortex row due to equations 17(*a*) and 17(*b*)) is well tabulated.

Nevertheless it should be mentioned that the computation in the physical plane, with respect to the induced velocities of analagously located vortex elements needs—contrary to the straight cascade—only a finite sum.

The authors' use of a vane skeleton instead of a more realistic profile as carrier of the singularities γ deprives them of the advantage in having with the γ distribution simultaneously also the distribution of the contour velocity in the mapping plane. For getting this needed figure $\gamma/2$ has to be added to the tangential components, with respect to the skeleton, of the induced velocities of the neighbouring profiles 19(*a*) and 19(*b*) and the undisturbed throughflow velocity. Moreover, the conformal mapping needs a retransformation of the so obtained contour velocity into the physical plane. From this the wanted pressure is obtained.

Contrary to early authors as Birnbaum, Glauert, Schlichting and Scholz, the authors are using, instead of a Birnbaum-Glauert series or a polynomial for γ-distribution, the vortex distribution γ as the unknown variable. This leads to a linear integral equation for γ, which can be replaced by a system of linear inhomogeneous equations. Moreover, γ is located at the contour of the vane. This procedure reflects the original idea of the bound vortex. It has to replace the velocity step near the contour from the exterior of the ideal flow to the zero flow in the interior of it by a continuously distributed vortex layer along the contour. It is a pity that the authors are applying this general method of solution only in the very simple example of vane skeletons with a constant angle β (logarithmic spirals) and to a shockless flow rate. Such an example could also be reproduced very well by a Glauert expansion for γ with two to three terms. Moreover, the application of such terms would have reproduced more exactly, the velocity distribution near the leading inlet edge of the vane insofar as they take into account very exactly the relevant boundary conditions, such as shockless inlet and Kutta condition.

Moreover, there exists no rational reason why to use 10, 20, or 30 finite vortex elements and the herewith connected extended computation to reproduce the relative flow around a logarithmic sprial as vane. This case has been treated theoretical exactly by Busemann (reference [1]) and Sörensen (reference [2]). Both have done this by mapping conformally the k-logarithmic spiral like vanes into a unit circle. By the way, the conformal mapping for a basic treatment is not a new method which allows to gain the actual velocity distribution like the singularity method. It includes immanently the velocity step by crossing the contour which equals mathematically exactly and essentially the action of a vortex layer. It is also not clear, why the authors suppose a priori very large values for the contour velocity near the vane inlet and why they omit therefore their results for the case of shockless inlet. I have the suspicion that these high values have been generated artificially by a wrong and too near position of the station for the satisfaction of the boundary conditions to the stations of the finite vortex elements. Thus unexpected high velocities may have been induced.

Moreover, it must be stated that the here omitted inlet region is of high interest with respect to the high cavitational sensitivity of the vane there, provided these vanes have a realistic finite thickness. The theoretical results near the inlet region are the more of interest as the influence of boundary layer can be omitted in this range. On the other hand in the impeller eye the curvature of the streamsurface, neglected by the authors, should be taken into account. Axisymmetric stream surfaces may be a good approximation.

From all the diagrams, presented by the authors, it follows that the rational solution of Acosta applied to logarithmic spiral formed impeller vanes coincides well with the more sophisticated method of the authors.

Simultaneously local large gaps can be seen between the measured and computed pressure distribution of the authors and Acosta as well. This discrepancy between the doubtless correct measurements of Acosta and the predictions of both theories holds especially for the impeller inlet and outlet region.

Since the rotoscopic observations of an impeller by Fischer and Thomas (Fischer K., Untersuchung der Strömung in einer Zentrifugalpumpe, Mitteilungen des Hydraulischen Instituts der TH München, Bd. 4, 1931), made in the Institute of Hydraulic Engines and Plants, we know evidently that usually the theoretically expected velocity maximum near the impeller outlet is shifted from the suction faces to the pressure faces. This dominating effect depends somewhat on the flow rate. It is caused by two secondary flows rotating against another, which are originated by the interaction of the pressure in the sound flow on the boundary layers of the inner and outer shroud. They accumulate boundary layer material of the shroud on the suction face near the impeller outlet. Simultaneously they shift the a priori high speeded fluid parts from the suction face towards the pressure face. Thus a big wake at the suction face and the impeller outlet is originated. The effect is strongly influenced by the design of the diffusers (vane number, toroidal casing, spiral casing). It is mainly responsible for the fact that the mass averaged relative flow near the impeller outlet does not move along the vane. Thus a slip is caused which consists in an angular deflection of the relative flow along the vane to the relative flow far behind the impeller. This slip determines essentially the characteristic and performance of an impeller pump. In the worst case, a wrongly selected slip factor may cause back flow, brake operation and turbine operation for an impeller pump, connected with a pipe line of given pressure. Therefore it should be stressed that good predictions for the pressure distribution at the impeller outlet and for the herewith connected slip may be of more weighty value than recalculated velocity distribution along the impeller channel aside the inlet and outlet. Also losses, spatial vane forms, variable breadth and arbitrary forms of the shrouds should be included in a future computation. For the inlet region with respect to cavitation the mixed character of the flow must be taken under consideration. A first step in this direction I have given in my paper: "The Calculation of the Meridional Velocity Distribution in a Mixed Flow Impeller With Respect to Spatial Shape and Losses," J. Raabe, International Conference on Design and Operation of Pumps and Turbines, National Engineering Laboratory, East Kilbride, Glasgow, September 1–3, 1976.

J. W. Railly[3]

The authors have demonstrated good agreement between their general method and the Acosta predictions wich are related to the Busemann theory for log-spiral blades. A general method, applicable to thin rotating blade cascades lying on a general surface of revolution, has been developed previously, Railly [19] describing a design procedure and Railly et al. [20] describing an analysis procedure. In each case blade thickness and streamsheet thickness variation were included. The design method has been in use for some years by the National Engineering Laboratory, Glasgow, for the development of mixed flow pump and fan sections. Recently an extension of the Martensen method by Lewis, et al. [21] has provided an exact solution for blades of arbitrary shape. However, the use of a thin blade theory has proved of great advantage in design work, Neal [22] in view of the very short computation time.

The treatment of the Authors of the singularity (through the use of equation (18)) is more sound than the wholly numerical treatment of the problem in the above mentioned work. A further improvement may be effected by using a substitution for the chord-wise (or axial) coordinate, x, as follows:

$$x = {}^1/_2(1 - \cos\theta)$$

This substitution has the advantage that the (unknown) vortex distribution $\gamma(x)$ may be replaced by $\gamma(\theta)$ which remains finite at the leading edge. This has the additional advantage that equal increments of θ automatically "bunch" the points near the leading edge which is specially valuable at nonzero incidence conditions.

Additional References

19 Railly, J. W., "The Design of Mixed Flow Cascades by Ackeret's Method," *The Engineer*, London, Vol. 224, (5827), 1967, pp. 405–416.

20 Railly, J. W., Houlton, J. M., and Murugesan, K., "A Solution to the Direct Problem of Flow in a Mixed Flow Turbomachine," National Engineering Laboratory, Report 413, 1969.

21 Lewis, R. I., Fisher, E. H., and Saviolakis, A., "Analysis of Mixed-Flow Rotor Cascades," Aero. Res. Council. G. Britain, R & M 3703.

22 Neal, A., unpublished report, National Engineering Laboratory, 1971.

Authors' Closure

The authors wish to thank Professor Railly and Professor Raabe for their expressed interest in this work.

The authors agree with Professor Railly that the treatment given in the present work has certain advantages over the methods mentioned by him, particularly when it is to be adopted for solving a design problem.

Conversion of the variable x into θ has, as pointed out, the advantage that for equal increments in θ the points near the leading and trailing edges will be closer. However, the necessity of such a non-uniform distribution of the points depends upon the method of numerical integration used and the total number of points chosen. It is felt that with the numerical procedure adopted, the advantage of using the variable θ in place of x may not be significant.

As mentioned by Professor Raabe the relative motion in the impeller is rotational and exactly for this reason, the absolute velocity, which can be assumed irrotational, is considered in this analysis. The conformal mapping gives rise to a vector field $\mathbf{V}$ in the x, y-plane which is also irrotational. There is no mathematical necessity to physically interpret this vector $\mathbf{V}$ as the velocity vector of a fictitious flow. It is well known that the vector $\mathbf{V}$ satisfies a linear differential equation and hence mathematically it is permissible to represent it as the sum of two irrotational vectors $\mathbf{V}_\infty$ and $\mathbf{V}^*$. Further, as $\mathbf{V}$ is discontinuous across the lines obtained by the transformation of the thin blades, it can be considered that the field of one of the component vectors, i.e. $\mathbf{V}^*$, has singularities. As such the field of $\mathbf{V}^*$ can be mathematically modelled by an appropriately chosen system of singularities, provided the resulting vector $\mathbf{V}$ satisfies the necessary boundary conditions, which in turn are derived from the boundary conditions to be satisfied by the absolute velocity

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vector in the impeller. Hence, the authors are of the opinion that the validity of the equations (17*a*) and (17*b*) cannot be judged on the basis whether $\mathbf{V}^*$ represents a relative or absolute flow velocity.

The Kutta-Joukowsky's condition and the circulation Γ are actually considered in the physical region, but for convenience, they are expressed in terms of the parameters of the vector field in the mapped region which obviously is permissible.

Professor Raabe seems to have missed the point that the analysis given is valid not merely for logarithmic spiral blades, but for any arbitrary blade geometry. The example of an impeller with logarithmic spiral blades is chosen only to gain the advantage of comparing the results of the analysis with the experimental data available in literature. It is true that similar methods of analysis are available in the case of logarithmic spiral blades but they cannot be used for blade geometries generally encountered in practice.

A doubt is expressed regarding the large contour velocities near the blade inlet. It is well known that when the flow past a flat plate kept in a stream with an angle of attack is analyzed neglecting the blade thickness, large velocities will occur near the inlet tip. Because of the similarity of this flow problem with the one under consideration, one can expect high contour velocities near the blade inlet tip. Further, the results of this method of analysis are compared with those of Acosta and the agreement is in toto. Of course, such large velocities will not come into existence, if the blade thickness also is taken into consideration.

Thin blade theories have often proved to be of great advantage, particularly in design work and the authors feel that in every situation it is not necessary to consider the blade thickness.