

Fig. 8 $\bar{n} \cong 1.5$. Since the cross-sectional area of the link is determined from $A = \bar{n}\bar{F}/\bar{S}$, it follows that the weight contribution due to the shank of the link is $(1.5)/(2.0)$ of the case where modality is unestablished. If enough parts are to be made, a testing program to prove unimodality can be financed with part of the potential savings. Proof of symmetry (a byproduct of testing for unimodality) allows Fig. 9 to be used and $\bar{n} \cong 1.31$. If the distribution of S can be proved to be of specific identity (Gaussian, for example), then Fig. 10 establishes the design factor as $\bar{n} \cong 1.13$.

It is clear that the more known concerning the distribution of S the smaller can be the design factor \bar{n} to guarantee a specific reliability level. If a few links are to be designed, improving designer's information beyond Bienayme-Chebyshev conditions is uneconomic. If the links are to be mass produced, lack of an experimental program can be difficult to defend.

Example 2

A steel has an ultimate strength of 140,000 psi and exhibits a standard deviation of 7000 psi. The static load on the link exhibits a mean value of 100,000 and a standard deviation of 6310 psi. The probability density function is reasonably uniform. A reliability of 0.99 is required. Because the size of \bar{F} is so large compared to unity, and the range such a small percentage of the mean,

$$\left(\frac{\sigma_{S_i}}{\bar{S}_i}\right)^2 = \left(\frac{6310}{100,000}\right)^2 = 0.0631^2 \cong \left(\frac{\sigma_\psi}{\bar{\psi}}\right)^2$$

From equation (3)

$$\left(\frac{\sigma_n}{\bar{n}}\right)^2 = \left(\frac{\sigma_S}{\bar{S}}\right)^2 + \left(\frac{\sigma_\psi}{\bar{\psi}}\right)^2 = \left(\frac{7,000}{140,000}\right)^2 + 0.0631^2 = 0.081^2$$

With no confirmation of symmetry or unimodality, the Bienayme-Chebyshev conditions of Fig. 6 apply; entering with abscissa of 0.081 and rising to the $R = 0.99$ contour establishes $\bar{n} \cong 5.3$. With proof of unimodality Camp-Meidell conditions of Fig. 8 apply, and the design factor reduces to $\bar{n} \cong 2.15$. If symmetry is established in n , then Fig. 9 reduces the design factor to $\bar{n} \cong 1.63$. In this case, if the distribution of $S\psi$ about its mean is symmetrical, the distribution of n about its mean is symmetrical. If the product $S\psi$ can be demonstrated to be Gaussian, then Fig. 10 yields $\bar{n} = 1.22$.

Example 3

A steel has an ultimate strength of 140,000 psi and exhibits a standard deviation of 7000 psi. The load exhibits a mean of 100,000 psi and a standard deviation of 6310 psi. A reliability of 0.999 is required. From Example 2

$$\frac{\sigma_n}{\bar{n}} = 0.081$$

Fig. 6 shows that a reliability of 0.999 is not attainable. Even if unimodality and symmetry were established, Fig. 7, 8, and 9 show that $R = 0.999$ is not attainable. The reason for the upper limit on reliability is that with $\sigma_n/\bar{n} = 0.081$, the instances of $n < 1$ are produced by outliers in n due to the strength distribution and the load-induced stress distribution. The outliers are sufficiently frequent under Bienayme-Chebyshev and Camp-Meidell conditions to prohibit high reliabilities. The designer has no recourse but to investigate the distributions sufficiently to identify their properties in order to attain, with certainty, a reliability of 0.999.

By adopting the "design load" approach the designer can reduce the abscissa, since he sets $\sigma_\psi = 0$, to 0.05. Under Camp-Meidell conditions with symmetry (Fig. 9), $\bar{n} \cong 3.93$. Higher reliabilities, with certainty, require more information about the probability density function than either Bienayme-Chebyshev or Camp-Meidell theorems demand.

Summary

The design factor used by the engineer is addressed to uncertainties in material properties, effects of size on properties, effects of machining and processing operations on properties, effects of assembly operations, effects of time and environment on properties, effects of mathematical prediction models on induced stress determination, effects of loading, and effects of discontinuities and stress concentrations on induced-stress determinations. Provision for cushion against the presence of unexpected phenomena and unsuspected deviations of mathematical prediction models from reality is very real in the design engineer's design factor decision.

This paper deals only with the necessary minimum design factor to accommodate to uncertainty in the material property of interest, uncertainty in the load, and in the case where design factor is defined as a simple quotient. Rationalizing this component of design factor determination contributes only to the resolution of part of the designer's decision. Using the same approach as exhibited here, more complex design factor equations such as are encountered in models of fatigue failure can be reduced to plots such as Figs. 6, 7, 8, and 9.

It should also occur to the reader that with a part that can fail in different modes (the critical properties are different), different design factors will apply to various regions of the part, for the same reliability.

DISCUSSION

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We have long been in need of a new definition of factor of safety which would account for the distributions of strength and stress and be related to reliability in some manner. Professor Mischke has done an excellent job in meeting this need. The presentation is both rigorous and general and provides a very satisfactory way of predicting the degree of safety, or danger, when the reliability is specified.

In the past we have always thought of factor of safety as a constant which gives the degree of safety beyond which failure will occur in case any of the uncertainties in design were assessed incorrectly. Thus, the idea that the factor of safety can have a distribution and hence a mean and a standard deviation, is new. This may be disturbing to some designers. A large factor of safety provides a comfortable feeling of a job satisfactorily completed. But if this safety factor has a fuzziness about it because of its distribution, then the feeling is not quite so comfortable.

One can define statistical factor of safety in a different manner, provided everything is Gaussian and we are dealing with populations rather than with samples, such that it reveals the degree of safety beyond any specified reliability. The definition is based on the interference of two populations, stress and strength. Define

- s = stress, psi
- S = strength, psi
- μ_s = population mean stress, psi
- μ_S = population mean strength, psi
- σ_s = population standard deviation of stress, psi
- σ_S = population standard deviation of strength, psi
- z_R = standardized variable corresponding to any reliability R

Then, the standardized variable is written

$$z_R = \frac{\mu_S - \mu_s}{\sqrt{\sigma_s^2 + \sigma_S^2}} \quad (8)$$

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One then uses a table of areas under the normal distribution to find the reliability R . For the data of example (9) we have

$$z_R = \frac{140,000 - 100,000}{\sqrt{(7000)^2 + (6310)^2}} = 4.244$$

Using a table of areas one finds $R = 0.99,999$ corresponding to z_R .

Now define *statistical factor of safety* as a number $n \geq 1$ which expresses the degree of safety beyond any specified reliability R . Note that this is not the same design factor defined by Professor Mischke. For any specified reliability n , as defined here, is a unique number. It is not distributed. Thus if we specify a reliability of $R = 0.99,999$ for the data of example (9) then $n = 1.00$ exactly, because there is no safety beyond $R = 0.99,999$.

It is clear then that there is safety beyond a reliability of $R = 0.90$, say. Including this definition of statistical factor of safety, equation (8) becomes

$$z_R = \frac{\mu_S - n\mu_s}{\sqrt{\sigma_S^2 + \sigma_s^2}} \quad (9)$$

or

$$n = \frac{1}{\mu_s} (\mu_S - z_R \sqrt{\sigma_S^2 + \sigma_s^2}) \quad (10)$$

If we specify $R = 0.90$, then from a table of areas under the

normal curve we find $z_{90} = 1.645$. Substituting this and the previous values in equation (10) yields

$$n = \frac{1}{100,000} [140,000 - 1.645 \sqrt{(7000)^2 + (6310)^2}] = 1.245$$

or, in other words, there is a 24.5 percent margin of safety beyond a reliability of 90 percent.

It is also clear that this definition of statistical factor of safety is neither as rigorous nor as general as the one proposed by Professor Mischke. Its use requires that both populations have normal distributions and that the statistics of these populations be known. It is interesting to note, however, that if we specify a reliability of $R = 0.50$, then $z_{50} = 0$, and equation (10) becomes

$$n = \frac{\mu_S}{\mu_s} \quad (11)$$

which is the same as Professor Mischke's equation just above Fig. 5.

It is indeed refreshing to find something new proposed to relate strength to stress. One of the unfortunate facts of life is that there are almost no publications of data on the distribution of stress and strength. Now that a satisfactory way of handling the problem has been devised perhaps these data will be forthcoming.