Journal of Basic Engineering, Trans. ASME, Series D, Vol. 84, No. 2, June 1962, pp. 220–221.

3 Johnson, A. E., Henderson, J., and Khan, B., "Complex-Stress Creep, Relaxation and Fracture of Metallic Alloys," Her Majesty's Stationary Office, Edinburgh, 1962, pp. 22–32.

4 Voorhees, H. R., and Freeman, J. W., "Notch Sensitivity of Heat-Resistant Alloys at Elevated Temperatures," Wright Air Development Center, Technical Report 54-175, Part 2, "Analysis of Notched-Bar Rupture Life in Terms of Smooth-Bar Properties," Jan. 1956.

5 Davis, E. A., "Creep Rupture Tests for Design of High Pressure Steam Equipment," Journal of Basic Engineering, Trans. ASME, Series D, Vol. 82, No. 2, June 1960, pp. 453–461.

6 Kennedy, C. R., Harmes, W. O., and Douglas, D. A., "Multiaxial Creep Studies on Inconel at 1500 deg F," Journal of Basic Engineering, Trans. ASME, Series D, Vol. 81, No. 3, Sept. 1959, pp. 599–609.

7 Garofalo, F., *Fundamentals of Creep and Creep Rupture in Metals*, MacMillan Series in Materials Science, MacMillan, New York, 1965.

8 Kachanov, L. M., "Rupture Time Under Creep Conditions, Problems of Continuum Mechanics" (English Edition), Contribution in Honor of the Seventieth Birthday of Academician N. I. Muskhelishvili, Feb. 1961, p. 202.

9 Abo El Ata, M. M., and Finnie, I., "On the Prediction of Creep-Rupture Life of Components Under Multiaxial Stress," Second IUTAM Symposium on Creep in Structures, Aug. 1970, to be published by Springer Verlag.

10 Robinson, E. L., "Effect of Temperature Variation on the Creep Strength of Steels," Trans. ASME, Vol. 60, 1938, pp. 253–259.

11 Liberman, Y., "Relaxation, Tensile Strength and Failure of EI 612 and 20Kh1F-L Steels," Mettaloved, Term. Obrabotka Metal, Vol. 4, 1962, pp. 6–13.

12 Oding, I. A., and Burduksky, V. V., *Colloquium on Deformation and Flow of Solids*, Madrid, 1955, Sprinrg-Verlag, Berlin, 1956, p. 298.

13 Oding, I. A., and Burduksky, V. V., "Effect of Variable Stress on the Rupture Strength of Steel," *Issledovaniya Po Sharoprochnim Splavam*, Vol. 6, 1960, pp. 77–88.

14 Voorhees, H. R., and Freeman, J. W., "Notch Sensitivity of Aircraft Structural and Engine Alloys," Wright Air Development Center Technical Report, Part II. Further Studies with A-286 Alloy; Jan. 1959, p. 23.

15 Randall, P. N., "Cumulative Damage in Creep-Rupture Tests of a Carbon Steel," Journal of Basic Engineering, Trans. ASME, Series D, Vol. 84, No. 2, June 1962, pp. 239–242.

16 Voorhees, H. R., "The Creep-Rupture Life at Uniform Elevated Temperature of Engineering Structures With an Initial Stress Gradient," PhD thesis, The University of Michigan, 1956.

17 Freeman, J. W., and Voorhees, H. R., "Literature Survey on Creep Damage in Metals," ASTM, STP 391, 1965.

18 Goldhoff, R. M., and Woodford, D. A., "The Evaluation of Creep Damage In a Cr-Mo-V Steel," paper presented at ASTM Anaheim Conference, 1971.

19 Williams, J. A., "A Mechanistic Approach to the Effect of Damage on the High Stress Creep Fracture," Second IUTAM Symposium on Creep in Structures, Aug. 1970, to be published by Springer-Verlag.

DISCUSSION

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The authors have performed a useful service by pulling together published data on creep damage rules. Their own results (although limited in scope) provide an especially valuable addition in that they include a pair of multiaxial-stress tests in which a significant step-change was made in the second principal stress (indeed, a change between tension and compression), but in opposite directions in the two instances.

In contrast to my strictly empirical geometric-mean rule, their summation of a time-fraction function and a strain-fraction function is a logical outgrowth of the separation of creep damage into aspects of nucleation of cracks or voids and of subsequent growth or coalescence of these features. The general improvement in correlation by using this rule supports the soundness of the reasoning on which it was based. But, to achieve more-universal applicability, this additive rule is suggested to probably also need a term to cover property changes associated with time of exposure to creep temperatures.

As the authors state, the limited evidence now available to them does suggest routes for future research. Perhaps release of this paper will finally stimulate more experiments on creep under changing complex stresses.

The broadest study of creep damage would need to consider the most-general histories of load changes, including random reversals of the stress in each of the three principal-stress directions. The continuous interaction of test conditions and properties of the test material would also require attention; otherwise, time-dependent structural changes during creep might result in such large alterations in residual rupture strength that any damage related to other factors could well be masked.

Hopefully, thermally-induced microstructural effects might be effectively subordinated to other aspects of creep damage by testing materials which have already been "stabilized" during earlier creep at or near the intended temperature of testing.

To date, no satisfactory specimen configuration seems to have been devised to permit a uniform pattern of arbitrary triaxial tensions to be introduced at will. The combination used by Abo El Ata and Finnie of internal pressure plus axial load on a cylinder with thin wall does permit random ratios of roughly uniform tangential versus longitudinal stress. The sign of the longitudinal stress can also be reversed by changing the direction of a large-enough axial load. Although the finite thickness of commercial pipe or tube would introduce further variation in the tangential stress across the wall thickness, an acceptable approximation to the desired thin-wall condition should still be attainable with materials in the form and condition most used in the important steam-power and chemical-petroleum industries.

Except for tests in reference [3] of this paper and limited tests of the authors themselves, past studies have employed a fixed condition of loading throughout a given test of creep under biaxial stresses. Problems of interpretation arise from the differing nature of "failure" under the different stress patterns. For example, a test under combined internal pressure and axial load terminates when the first through crack prevents further maintenance of the pressure. A companion test under axial tension alone can continue until the fracture has traversed around an entire cross section, and the specimen has separated into two discrete parts.

This question of how to define failure is not unique to any particular specimen configuration. Even with the common test bar of uniform cylindrical section, surface cracks and grain-boundary voids arise and grow with disproportionate speed at different locations, and necking of the bar may be localized as fracture approaches. After complete rupture of the bar at one location, a smaller bar machined from a resulting piece of the original gage section will still display a finite rupture life at the same test conditions.

Though they offer no absolute measure of remaining life, specimens taken from a pipe after service or prior creep exposure can provide results which correspond directly with test data commonly used for initial evaluation of the pipe material or to supply design data. Furthermore, bars sampled in different directions of a part after prior creep allow direct comparison of existing properties in these directions.

No requirement exists that all stages in a test must be under a multiaxial stress pattern in order to find laws to describe creep rupture behavior for general histories of stress. A completely general relationship to correlate the rate of "consumption" of rupture life under combined-stress creep may not be discovered without at least some portions of some tests being under triaxial stressing. But any valid correlation must also hold for tests with portions under simpler stress patterns.

A correlation established by tests with successive periods under biaxial and under uniaxial stress may be only a special case of some more-general relationship. However, even such a special case, if well substantiated, should find immediate direct applica-

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tion in engineering designs where one principal stress is exactly or very near to zero. Good qualitative guidance should also result for the many other circumstances where one of the three principal stresses is reasonably lower than the other two.

A first step in the proposed tests is determination of the creep-rupture properties in the longitudinal and circumferential directions of the pipe or tube, to provide a reference against which to judge changes in properties to be brought about by creep exposure to different conditions of combined stress. Next, pipe or tube specimens would be subjected to fixed combinations of internal pressure and axial load for times which represent a significant percentage of the rupture life to be anticipated under these conditions. Finally, residual creep-rupture properties would be evaluated, using specimens sampled in the longitudinal and in the circumferential directions from the creep-exposed pipe. (With tubes, properties in the circumferential direction might require the testing of ring-shape specimens instead of the more-usual bars or strips.)

The paper under discussion notes that most interest lies with the largest principal stress and with the effective stress as most likely to correlate creep rupture under complex stressing. But also of interest as a possible parameter for creep damage in a given direction is the amount of creep strain in that particular direction. Experiments summarized in reference [3] of the foregoing paper indicate that the amount of creep in different directions distributes in proportion to the deviator component of stress in each direction. In any principal direction, this deviator component is found by subtracting from the total stress in that direction the average of all three principal stresses.

A critical comparison between different stress criteria for creep damage should result from the fewest possible tests by holding one stress criterion constant in several tests, while giving others a range of fixed values. For a study involving internal pressure, a logical pattern is to make the circumferential hoop tension (S_c) the largest stress, and to hold the level of that stress the same for all tests in a series. Where pipe specimens all have the same diameter and wall thickness, this permits all tests of a series to be run at the same pressure.

Consider thin-wall specimens under the following patterns of stress, in each case relative to unit circumferential stress:

The specific internal pressure and axial loads can be chosen only after the particular creep-rupture properties and dimensions of the pipe have been ascertained. However, assume as an example that a carbon-steel pipe has respective 100-hour and 100,000-hour rupture strengths of 28,000 and 13,000 psi at the test temperature, and that a log-log plot of stress versus rupture time is linear over this range of values. (Note may be taken that the stress axis of this curve could be labeled both "maximum principal stress" and "effective stress," because these two stresses are the same for the usual uniaxial test used to gather the data.) For a life of 10,000 hours, the uniaxial stress in a test bar would need be about 16,800 psi. Let us use that stress as the circumferential stress in our pipe specimens, and fix the internal pressure accordingly.

Now, if the maximum principal stress controls the rate at which the available rupture life is "used up," all pipe specimens after like periods of creep exposure should exhibit an identical extent of lowering of subsequent creep rupture properties—at least in the circumferential direction for which the stress level during the creep exposure was the same in all pipe specimens. But if the effective stress controls, the remaining life after creep exposure should vary noticeably according to the magnitude and sign of the longitudinal stress during exposure to creep.

(Despite the selection of prestabilized pipe for the creep exposures, part of any change found in rupture properties could result from continued microstructural alterations during the exposure period. The most valid comparison of rupture life after creep exposure of pipe specimens should therefore probably be against bars sampled from another pipe specimen exposed at the test temperature to low-stress conditions approximating those of long-time service. Such a "reference" exposure is suggested to be run for at least one test material.)

When enough axial pull is added to make the longitudinal stress equal 16,800 psi, the effective stress equals that value also. But for the pressure-only case, the effective stress in a closed-end pipe approaches only (0.866)(16,800) = 14,500 psi, which corresponds to a rupture life of some 36,000 hours. If enough axial push is applied to make the longitudinal stress (-0.2)(S_c), the resulting effective stress of 18,700 psi corresponds to a rupture life of only about 4,000 hours.

Principal total stress			Effective	Deviator stress		
Circ. S_c	Long. S_L	Radial S_r	stress $\bar{S}$	Circ. S_c'	Long. S_L'	Type of loading on a thin-wall cylinder
1.0	1.2	(0)	1.114	0.267	0.467	Int. pressure + axial pull
1.0	1.0	(0)	1.0	0.333	0.333	Int. pressure + axial pull
1.0	0.5	(0)	0.866	0.500	0.000	Internal pressure only
1.0	−0.2	(0)	1.114	0.733	−0.467	Int. pressure + axial push

Each of the upper three sets of stresses is characterized by a finite ratio between two principal tensions. The ratio of the effective stress to the largest principal stress reaches a minimum of the 0.866 for the pressure-only case where one principal tension stress is double the other. The maximum limit of 1.0 for this ratio is reached for equal principal tensions, and where only one principal stress is not zero.

The effective stress listed for $S_L = 1.2$ exceeds unity only because it is given in comparison to unit value for S_c; it is still smaller than the largest principal stress. In contrast, making one principal stress a compression and the other a tension does give an effective stress larger than 1.0. In fact, $\bar{S}$ then exceeds the larger absolute value of the principal stresses.

Either the first or third set of stresses listed can be eliminated without loss of generality. Each represents biaxial loading under a pair of unequal tensions, but with the direction of the larger and smaller of these stress interchanged. At least for a preliminary study, only the three stress patterns with respective axial stresses equal to 1.0, 0.5, and -0.2 times a constant circumferential stress are suggested to be applied to each of two test materials.

Pipe specimens with $S_L = S_c$ and with $S_L = 0.5\ S_c$ could be run for 3200 hours, and the pipe with $S_L = -0.2\ S_c$ for 1600 hours creep exposure. If the available life is expended at a uniform rate during a rupture test, circumferential bars sampled from the first two of these exposed samples should each show roughly twice as great a drop in residual rupture life as will similar test bars from the third pipe exposed half as long—provided the drop in life for all three pipe specimens is controlled by their common value of maximum principal stress. Moreover, these exposure times should be a large enough fraction of the 10,000-hour rupture life of a uniaxial specimen at this same stress to insure that the resulting change in rupture times can be readily detected above inherent scatter of rupture-time data.

On the other hand, should the effective stress and not the largest principal stress determine the rate of deterioration of residual rupture strength, the 3200-hour exposure time will represent less than 9 percent of the rupture life of the pipe with internal pressure only. Rupture times for bars from that exposed pipe should remain significantly greater than those for comparable tests on bars from the other two pipe specimens, with these having been exposed to 32 and 40 percent of the rupture life of a uniaxial specimen at the respective levels of effective stress employed for the exposures.

The substantial range in magnitude and the reversal in sign for the deviator stress component in the axial direction should amply permit evaluation of the deviator stress as a factor in any observed directional differences in creep-rupture properties after prior creep under differing biaxial-stress patterns.

To preclude extensive creep recovery, pipe specimens from service could be brought to test temperature under an internal pressure (plus an axial load if needed) chosen to roughly duplicate service stresses. After uniform temperature has been achieved, the pressure and axial load would be adjusted to give the desired pattern of biaxial stress for the creep exposure.

During the creep exposure, attempts should probably be made to continuously monitor the circumferential creep. A test could then be discontinued should an increasing rate of creep be encountered before the scheduled end of an exposure.

Authors' Closure

We would like to thank Dr. Voorhees for his comprehensive discussion which forms a valuable supplement to our paper. His comments provide guidance for further work in a field to which he had made extensive contributions.