Journal of Heat Transfer Discussion

Similarity Solution of Combined Convection Heat Transfer From a Rotating Cone or Disk to Non-Newtonian Fluids[1]

P. Mitschka.[2] The problem considered belongs to the class of three-dimensional rotational boundary layer flows of the generalized Newtonian fluids (GNF) of the power-law type. The correct order-of-magnitude analysis of the full equations of motion (see Mitschka et al., 1987, 1989) leads to the conclusion that viscous terms in the transformed Eqs. (12) and (13) of the discussed paper must be formulated as

$$[(F''^2+\underline{G'^2})^{(n-1)/2}F''] \text{ and } [(\underline{F''^2}+G'^2)^{(n-1)/2}G']$$

In this two-gradient (2G) formulation, both dominant velocity gradients, F'' and G', appear in the *invariant* viscosity function $(F''^2+G'^2)^{(n-1)/2}$. This satisfies the rules for correct formulation of GNF models for nonviscometric flows (Bird et al., 1960; Schowalter, 1980).

In the paper discussed, however, the expressions for the viscous terms are given as

$$(|F''|^{(n-1)}F'') \text{ and } (|G'|^{(n-1)}G')$$

i.e., in a form that results from an arbitrary omitting of the underlined terms of the 2G formulation. These expressions (one-gradient (1G) formulation) do not satisfy the requirement of invariant viscosity function and are, thus, from the point of view of contemporary non-Newtonian fluid mechanics incorrect.

The influence of 1G and 2G viscosity functions will be demonstrated on the heat transfer data given by Wang and Kleinstreuer in their Fig. 13, for the uniform surface temperature case.

In Fig. 1, results for this heat transfer problem without ($\lambda_T=0$) and with assisting ($Z=1$, $\lambda_T=2$) buoyancy effects are compared for $\mathrm{Pr}=100$ in the form of dependences of temperature gradients at the wall, $-\theta'(0)$, on the power-law flow index n. As expected, differences in the velocities fields due to the 1G or 2G form of the viscous terms in the boundary layer momentum equations are reflected in the heat transfer characteristics resulting from the solution of coupled energy equation. These differences increase with the increasing pseudoplasticity of the power-law fluid (i.e., with decreasing n) and cannot be, in general, neglected.

Thus, in analyses of this class of rheodynamic and heat/mass transfer problems of GNF, only the correct invariant 2G version of the three-dimensional rotational boundary layer equations should be used.

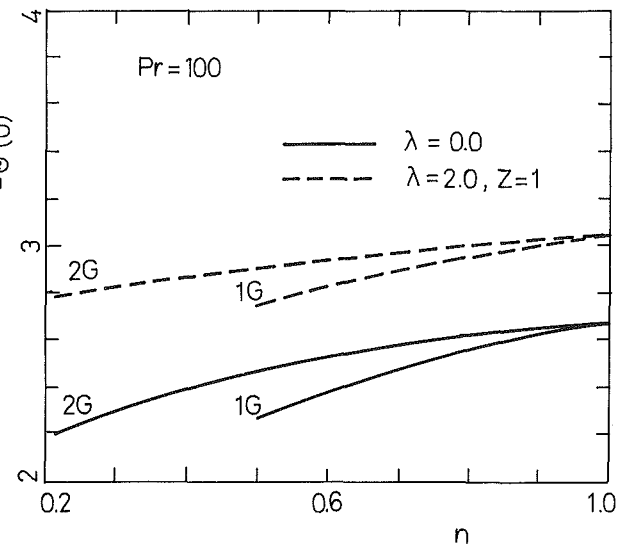

Fig. 1 Comparison of wall temperature gradients in heat transfer from cones and disks rotating in pseudoplastic power-law fluid for 1G and 2G viscous terms

[1] By T.-Y. Wang and C. Kleinstreuer, published in the November 1990 issue of the ASME Journal of Heat Transfer, Vol. 112, No. 4, pp. 939–944.

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Authors' Closure

As stated in the Analysis section of the paper by Wang and Kleinstreuer (p. 939), "the velocity and temperature fields are function of x and y only, and transverse curvature effects can be neglected since $\delta/r \ll 1$." Thus our two-dimensional "viscometric" boundary-layer flow equations based on the premise are correct (cf. pp. 102 and 103 plus associated tables from Bird et al. (1960)). A generalization to three-dimensional, "nonviscometric" boundary-layer flow, as discussed above, constitutes a different problem, which, of course, has a (somewhat) different solution.