against a Newtonian liquid of the same viscosity as the viscoelastic liquid at the given rate of shear. These measurements were done at very low rates of shear of about 10 sec\(^{-1}\) and with a material that was only very slightly viscoelastic under the conditions of experiments so that this only shows the qualitative direction of the expected effects. In sleeve bearings, much higher rates of shear, say, between 1 and 5 million sec\(^{-1}\) occur, where, according to the experiments in Fig. 6, just-mentioned values of the recoverable shear \(s\) of about 40 to 100 are to be expected, whereas in the experiments the value was about 1. It is extremely difficult, not mentioning the large heat evolution, to measure the recoverable shear under such high rates of shear, and it remains to the future to develop measurements for proving or disproving the beneficial effect of viscoelasticity on bearing performance. But with such high a probable, one could reasonably hope for a large beneficial effect, the decrease in viscosity in polymer-containing oils being limited, due to the concentration used, and much smaller than in the present test.

Conclusions

The developed theory together with the experimental proof gives credence to the assumption of the elasticity in steady flow. In any case, this author has not found any other approach that could explain the correlation of such widely different experiments.

Then, naturally, a requires a great importance in the flow of polymer solutions. It has been stated that it is similar in importance to the Reynolds number in the flow of Newtonian liquids. Actually, it is easier to understand as it is directly measurable and not a somewhat abstract calculable number.

The use of the solid theory allows one in many cases a simple mathematical treatment of experimental condition. Often the shear stress is easily calculable from dimensions, whereas the rate of shear is a nonlinear function of the shear stress, which however, can be treated by the differential methods.

References

10 Weissenberg, K., 10 lectures at Columbia University, 1963.

D. B. Cox

As you have observed, there is a tremendous amount of detail in this paper. The work of Dr. Philippoff in this area of normal stress is, of course, extremely well known. However, I suspect anyone unfamiliar with the field would be somewhat confused. I don't mean this as a criticism, but it illustrates a point I'd like to make. I'm going to go off a little obliquely.

I don't recognize the quotation from whoever said there isn't any elasticity in fluids. It seems to me that elasticity in fluids is a well-proven fact. If you don't believe there is elasticity in fluids, it is like believing the world is flat. However, when you get down to studying the quantitative nature of elasticity in fluids, and more to the point, what it has to do with lubrication, there is quite a bit still to be said. For all the tremendous amount of significant work that has been done by Dr. Philippoff and by others in the field, as he said himself in the paper, it is still not really quite clear how viscoelasticity applies to lubrication in a quantitative engineering manner. I don't have the answer either, but I'd like to say something about a viewpoint you should take. A couple of years ago, Prof. Truesdell, who is a continuum mechanics man, gave a speech in acceptance of the Bingham Medal from the Society of Rheology. Prof. Truesdell is a very witty and rather outspoken man and he concluded his talk by challenging the
people who don't like the mathematical approach. He said, approximately, "Don't knock it, learn it." The point is that we have heard some talk already today about, "Is it chemistry or is it physics or is it this or is it that." We shouldn't quibble or set up false boundaries but try to learn it all. I'm a chemist by training and I have been reading Rheology papers like Dr. Philippoff's for years, struggling my way along to some kind of understanding. I've gotten to where I can understand part of what I hear. Perhaps you can't expect one man or any of you as an individual to master all of Rheology and Lubrication in detail. There is too much to learn. There are some ways that you can get around the problem. One of them is to attack the problem—let's say, lubrication—with an interdisciplinary group of people who bring the viewpoints of chemistry, of continuum mechanics, of physics, and so forth to the problem, or you can do something in the way of conferences, as the Lubricant Symposium. You can bring together the people who have an interest in the subject as frequently as possible, hope that they will listen to each other and try to understand what each other said, and thereby rather slowly and painfully arrive at an understanding.

A further point is that the kind of experiment done by Philippoff and Tao a year or so ago is the kind that has to be done for lubrication. That is to physically simulate situations that are present in real lubrication, to make as sophisticated measurements as you can, and then to extract from that information by whatever theory and mathematics you can the basic, fundamental understanding. It is my faith that if you can understand what is going on, it will have practical consequences for lubrication.

My final point is that normal stress measurements by themselves are not enough, chemistry by itself is not enough. You have to wrestle with all these things (which is occasionally painful) to try to put them together. But if your goal is understanding what is going on in lubrication, it is not enough to take one narrow viewpoint and just stick to it.

R. I. Tanner

In view of some previously published calculations on normal stresses (which are related to recoverable shear), is it not the case that these normal stress effects occurring in bearings must be multiplied by a very small number like $k/n!$ in order to assess their true significance as far as load capacity is concerned?

Author's Closure

I wanted to emphasize: that as we saw in Dr. Metzner's talk, the other school, if I may say so, has used two functions for normal stresses, $\sigma_n$ and $\sigma_o$ or sometimes $\sigma_1$ or $\sigma_2$, and another description for whatever other elastic properties you have. In this presentation, I have not introduced any other functions for the normal stresses but consider them as a self-evident conclusion from the one single material constant, the shear modulus $G$. This means also that there is less significance in using the rate of shear as the independent variable but it is more significant to use the shear stress instead for discussing all elastic properties.

In response to Professor Shimarr's question, I would point out that I presented a paper treating elasticity in steady flow last November which will be published soon. The idea is founded on the rotation $\Omega = \frac{1}{6} D$ of the molecule. As I said, the circular frequency of stressing $\omega$ is equal to the rate of shear $D$. We have a Maxwell body in the sense that the same stress acts on an elastic component and on a viscous component. In finite shear, the position of the tensor swivels more in the direction of the deformation and it can be proven that the viscosity will diminish according to the sine of the double angle. So there we have a very clear understanding of what happens: we deform the material in finite shear. The amount of shear is given by the stress tensor and we can determine the modulus of that particular solution, and dependent on the swivelling of that tensor, we get less and less viscosity, until one gets a rate of shear proportional to the square of the shear stress. This in first approximation gives you an ideological combination of linear conditions, a constant viscosity and a constant modulus and still a measured decrease in viscosity determined by geometry. I have not found until now another simple explanation of why a constant viscosity or an elasticity should lead to a non-Newtonian viscosity and for that matter the second order fluid does not calculate any non-Newtonian viscosity at all and molecular theories of viscosity of solutions do not result in a non-Newtonian viscosity without introducing a third constant: the viscosity inside the coiled molecule. If you are interested, I can show you how it was calculated, but essentially this was one of the latest steps that I could make in the direction of coordinating these phenomena under the guiding principle of elasticity.

The $s$ versus $\tau$ curve doesn't flatten out—in practice it has sometimes the tendency of flattening out—but this depends on the structure of the molecule. For polystyrene, the flattening out is much more pronounced than for polyisobutylene and for that matter, for the largest value, which I showed to be about 40 s for the polyisobutylene, there was no trace of flattening out. The reason for this is that the molecule is so large, 15 million molecular weight of about 500,000 chain links, that even the deformation of 40 is small against this enormous number of chain links. This is my current explanation. The Langevin formula that calculates the influence of the finite chain does not give a correction under the actual conditions of the experiments. Why is $s$ proportional to the shear stress for polyisobutylene and not proportional for polystyrene? I don't know.

I do not believe Professor Tanner's question can be answered at present. Of course, the answer depends on the still outstanding solution for the nonstationary flow. It may very well be that for the nonstationary flow, beyond the solution given by Reynolds or by Sommenfeld, the value for the pressure has to be multiplied with this $s$. This means that the pressure is maybe 50 times higher than the one calculated for Newtonian liquids. There are no experiments in the field and this is all conjectural but what one does know from the small amount of results is that the gap (eccentricity ratio) in the viscoelastic liquids under the same viscosity conditions is larger than the ones with normal non-Newtonian liquids.

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