



## Discussion: “Optimum Design of Vibration Absorbers for Structurally Damped Timoshenko Beams” (Esmailzadeh, E., and Jalili, N., 1998, ASME J. Vib. Acoust., 120, No. 4, pp. 833–841)

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### 1 Introduction

In a recent paper by E. Esmailzadeh and N. Jalili the design of optimal vibration absorbers for Timoshenko beams was discussed. While the paper is interesting, the solution seems to be questionable, because it does not satisfy the governing differential equation of motion. There has been no reference made to the corresponding experimental results, or results obtained through the application of other analytical methods. Therefore, the application of the results presented in the paper seems to be misleading.

### 2 Analysis

In the paper, the mode summation procedure has been applied, and the following forms for the transverse deflection  $y(x,t)$  and the orientation of the beam cross-section  $\psi(x,t)$  have been adopted,

$$y(x,t) = \sum_{i=1}^n Y_i(x) \cdot q_{bi}(t) \quad (1)$$

$$\psi(x,t) = \sum_{i=1}^n \Psi_i(x) \cdot q_{bi}(t) \quad (2)$$

It is observed that the same modal amplitudes for deflection and section rotation have been assumed. It will be shown that such an assumption results in a contradiction. For clarity of presentation, let's consider a simple case, where only one vibration absorber system with a single mass,  $m$ , stiffness,  $k$ , and damping,  $c$  is attached to the beam at some location,  $x=l$ . Also the only applied force on the beam is assumed to be  $g(t)$ , which is applied by the

absorber system. This force can be generated, for example, as a result of an initial condition imposed on the beam. Considering the free-body-diagram of an element of the Timoshenko beam, the equations of motion would be,

$$\rho A \frac{\partial^2 y}{\partial t^2} - kAG \left( \frac{\partial^2 y}{\partial x^2} - \frac{\partial \psi}{\partial x} \right) = g(t) \cdot \delta(x-l) \quad (3)$$

$$EI \frac{\partial^2 \psi}{\partial x^2} + kAG \left( \frac{\partial y}{\partial x} - \psi \right) - \rho I \frac{\partial^2 \psi}{\partial t^2} = 0 \quad (4)$$

Now, one can substitute Eqs. (1) and (2) in (3) and (4) to check if assuming similar modal amplitudes for lateral deflection and section rotation is justifiable. Substitution results in,

$$\rho A \sum_{i=1}^n \ddot{q}_{bi}(t) Y_i(x) - kAG \left[ \sum_{i=1}^n q_{bi}(t) (Y_i''(x) - \Psi_i'(x)) \right] = g(t) \cdot \delta(x-l) \quad (5)$$

$$\sum_{i=1}^n q_{bi}(t) \cdot [EI \Psi_i''(x) + kAG (Y_i'(x) - \Psi_i(x))] - \rho I \sum_{i=1}^n \ddot{q}_{bi}(t) \cdot \Psi_i(x) = 0 \quad (6)$$

On the other hand, the free vibration analysis gives,

$$-\rho A \omega_i^2 \cdot Y_i(x) - kAG \cdot (Y_i''(x) - \Psi_i'(x)) = 0 \quad (7)$$

$$EI \Psi_i''(x) + kAG \cdot (Y_i'(x) - \Psi_i(x)) = -\omega_i^2 \rho I \cdot \Psi_i(x) \quad (8)$$

Substitution of Eqs. (7) and (8) in (5) and (6) results in,

$$\rho A \sum_{i=1}^n Y_i(x) \cdot [\ddot{q}_{bi}(t) + \omega_i^2 \cdot q_{bi}(t)] = g(t) \cdot \delta(x-l) \quad (9)$$

$$\rho I \sum_{i=1}^n \Psi_i(x) \cdot [\ddot{q}_{bi}(t) + \omega_i^2 \cdot q_{bi}(t)] = 0 \quad (10)$$

Equations (9) and (10) are to be valid for all  $t$  and  $0 < x < L$ . Thus one concludes from (10) that,

$$\ddot{q}_{bi}(t) + \omega_i^2 \cdot q_{bi}(t) = 0 \quad (11)$$

Substitution of Eq. (11) in (9) results in,

$$0 = g(t) \cdot \delta(x-l) \quad (12)$$

Equation (12) presents a clear contradiction because it would be valid only if  $g(t)$  equals to zero, that is, no absorber system is used at all.

### 3 Conclusion

The method used in the paper for generating a solution for the optimal design of vibration absorbers for Timoshenko beams, is questionable. The assumption of same modal amplitudes for deflection and section rotation results in a severe contradiction. Since the results obtained by the application of this method have

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not been compared with any other method or experimental data, one cannot rely on the results presented in the paper.

## Closure to “Discussion of ‘Optimum Design of Vibration Absorbers for Structurally Damped Timoshenko Beams,’” (1998, ASME J. Vib. Acoust., 120, pp. 833–841)

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Recently we received a discussion by Mohammad R. Rastegar and Mehrdaad Ghorashi on our previously published paper in the *ASME Journal of Vibration and Acoustics*. The commentators claimed, “While the paper is interesting, the solution seems to be questionable, . . . .” They went on to conclude, “Therefore, the application of the results presented in the paper seems to be misleading.” They have also given their own analysis without any fundamental or mathematical basis.

As for us, at first glance, it shows that their intentions for writing such comments are both unprofessional and naïve. We would like to encourage them to be more respectful and knowledgeable when judging others’ refereed research findings and published results without a rational and rigorous analysis. We take their comments to be more the result of hurried opinion and exhausted mind flash rather than sound technical basis.

They have claimed that our paper is “interesting,” but have failed to point out how they have arrived at this conclusion and which part of this paper does indeed interest them most. With regard to their comments “the solution *seems* to be questionable,” we must point out that it is their weak knowledge of basic vibration subjects and the lack of a thorough understanding of the fundamental principles that are most questionable. They then happily sailed home to say, “the application of the presented results *seems* to be misleading.” On the contrary, we believe them to be in error in understanding the basic principles of vibration.

At this stage, we would like to draw the commentators’ attention to how they should have analyzed the contents of our paper; after which they should be able to understand and arrive comfortably at our correct conclusion.

The analysis given by Rastegar and Ghorashi to prove that a contradiction exists in our paper is obviously wrong and has no mathematical basis, as will be shown here. They have arrived at their Eqs. (9) and (10) as

$$\rho A \sum_{i=1}^n Y_i(x) [\ddot{q}_{bi}(t) + \omega_i^2 q_{bi}(t)] = g(t) \delta(x-l) \quad (9)$$

and

$$\rho I \sum_{i=1}^n \Psi_i(x) [\ddot{q}_{bi}(t) + \omega_i^2 q_{bi}(t)] = 0. \quad (10)$$

The paragraph immediately following their Eq. (10) reads “Equations (9) and (10) are to be valid for all time  $t$  and for  $0 < x < L$ . Thus, one concludes from (10) that  $\ddot{q}_{bi}(t) + \omega_i^2 q_{bi}(t) = 0$ .”

Surely, this is an obvious mistake since the summation is over the entire terms and they are not allowed to conclude that every individual term in the summation is equal to zero, i.e.,  $[\ddot{q}_{bi}(t) + \omega_i^2 q_{bi}(t)] \neq 0$ .

Rastegar and Ghorashi have objected to the assumption of using the “same modal amplitudes” in the Timoshenko beam theory used in our paper. We should point out that the use of the same time-functions,  $q_{bi}(t)$  in the Galerkin approximation, used in our paper, is a common practice and the standard assumption that has been utilized by many researchers in other areas of mechanics (for example see [1], [2], [3], and [4] for elastic rods, laminated composite plates, etc.). It will ensure the synchronized nature of the motion of the transverse displacement  $y(x,t)$  of the beam and the orientation of its cross-section  $\psi(x,t)$ . This assumption is also in total agreement with the limiting case for the Timoshenko beam theory, where by assuming  $\psi = \partial y / \partial x$  it will lead to the Euler-Bernoulli formulation. However, by assuming a different set of generalized coordinates it will make this simplified form rather unreachable.

With regard to the requested experimental results, we would like to add that our optimum design technique was successfully implemented in an industrial project supervised by a high-voltage overhead transmission line manufacturer and funded by a governmental department. The absorption of the vibration levels was found to be quite good and within the working range predicted by our simulation study model. Those accepted test results and data were to be treated confidentially under the disclosed agreement signed by both of us with the above-mentioned agency.

## References

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