Partial Wave Amplitude for Low Mass $N\pi$ System Produced in Diffraction Dissociation

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(Received April 14, 1975)

Partial wave amplitudes of the Drell-Hiida-Deck model and its unitarized version by the final state interaction are studied for the low mass $N\pi$ system produced in the diffraction dissociation process. The partial wave amplitude unitarized by the final state interaction can be divided into two components. The one has the minimum and the other the maximum at the resonance position. The resultant amplitudes violate the Gribov-Morrison rule and the conservation of the $t$-channel helicity.

§ 1. Introduction

Partial wave analyses of the $3\pi$ system with small transferred four momentum squared, $t$, at $p_L=5\text{ GeV/c}$ to $p_L=40\text{ GeV/c}$ have revealed the internal structures of a low mass diffraction dissociation (LMDD). These analyses seem to show that the $A_1$ enhancement cannot be a typical resonance with the Breit-Wigner phase. If there are no axial vector mesons at all, our theoretical framework such as composite models or higher symmetries would encounter serious difficulties. It is, therefore, a highly important problem to clarify whether the $A_1$ enhancement in LMDD is really kinematical or not. For this purpose it is very useful to analyse an $N\pi$ system produced diffractively by a process $\pi N\rightarrow \pi(N\pi)$ for an example, because we can compare directly the structure of the $N\pi$ system with that in the $\pi N$ phase shift analyses.

Preliminary partial wave analyses on the diffractively produced $N\pi$ system have recently been started. According to their analyses and other data on the $N\pi$ LMDD, it is indicated that the $N\pi$ mass distribution at the smallest $|t|$ region has a peak at $M_{N\pi}<1.35\text{ GeV}$ which is much lower than that expected from the masses of the resonances. We have already encountered similar phenomena in the pion photoproduction processes at the resonance region or in the $\rho$ photoproduction process. These phenomena are considered to be due to an interference effect between a resonance production term and some kind of background term.

In this paper we study the partial wave analyses of the Drell-Hiida-Deck (DHD) model and of the unitarized version of the DHD model by the final state interaction. We do not use the Reggeized DHD amplitude, because we are interested in LMDD up to the third resonance region at most and its partial wave analysis. (The reason why we do not use the Reggeized DHD and the interpolation of the elementary pion-exchange amplitude to the Reggeized one will be discussed in
§ 5-2.) Since the unitarity condition is an important constraint for the discussion of partial wave analysis in the resonance region, we must unitarize the DHD amplitude with an elementary pion exchange. The full unitarization of the DHD amplitude contains not only the unitarization of the dissociated $N\pi$ system due to the final state interaction, but also the so-called absorptive correction to the DHD amplitude. In this paper we discuss the unitarization due to the final $N\pi$ interaction. (See § 5-2 for the latter absorptive correction.) Such a model was previously studied by Kagiyama and the present author,\(^{13}\) where $pp\rightarrow p(N\pi)$ was studied and it was shown that the final state interaction, which was estimated through the dispersion integral over the physical region, could not give any peak but rather a dip at the resonance position. We follow the same approach with Ref. 13), but the dispersive part of the dispersion integral is treated more phenomenologically. The resultant amplitude can be divided into two components: The one has the minimum and the other the maximum at the resonance position. We tentatively call the former the $B$-term or the background term and the latter the $R$-term or the resonance term. The $R$-term is parameterized phenomenologically, while the $B$-term does not include free parameters. The Gribov-Morrison rule\(^{14}\) and the $t$-channel helicity conservation are shown to be broken. It is to be noted that a similar two-component model has been used by Bowler and Game\(^{15}\) in order to solve the $A_1$ mystery.

In § 2 we give some definitions on the partial waves in the Gottfried-Jackson frame.\(^{16}\) In § 3 characteristic features of the partial wave amplitudes resulting from the DHD model are studied. Section 4 is devoted to presentation of the two-component model and its numerical results. Some related problems are discussed in § 5.

§ 2. Partial wave decomposition of $\pi N \rightarrow \pi (N\pi)$

We consider the process

$$\pi_a + N_b \rightarrow \pi_e + (N_i + \pi_2)_a,$$

(2.1)

where $a$, etc. denote the momenta $p_a$, etc. Independent variables describing the process (2.1) are $s$, $t$, $W$, $\theta$ and $\phi$, where $\theta$ and $\phi$ are the polar and azimuthal angles of $p_i$ at the Gottfried-Jackson (G-J) frame,\(^{16}\) respectively, $s = (p_a + p_b)^2$, $t = (p_e - p_a)^2$ and $W$ denotes the mass of the $(N_i\pi_2)$ system. Other invariant variables used in the following are the energy squared of $(\pi_e\pi_2)$, denoted as $s'$, and the transferred four-momentum squared from $N_b$ to $N_i$, denoted as $t'$.

Helicity amplitudes $T_{\mu\nu}(s, t, W; \theta, \phi)$ in the G-J frame are constructed by the partial waves $T^{P,E}_{\mu\nu}(s, t, W)$, where
$\mu(\nu)$ is the helicity of the final (initial) nucleon, and $J, M$ are the spin of the $N\pi$ system and its polarization, $\sigma$ being the naturality defined as $\sigma = \text{parity} \times (-1)^{J-1/2}$.

\begin{equation}
T_{++}(s, t, W; \theta, \phi) = \sum_{J, \mathbf{M}} \left( \frac{2J+1}{4\pi} \right)^{1/2} \left[ -T_{N,1/2}^s(W) + T_{N,1/2}^s(W) \right] D_{J}^{M+1/2}(\theta, \phi),
\end{equation}

(2.2a)

\begin{equation}
T_{-+}(s, t, W; \theta, \phi) = \sum_{J, \mathbf{M}} \left( \frac{2J+1}{4\pi} \right)^{1/2} \left[ T_{N,1/2}^s(W) + T_{N,1/2}^s(W) \right] D_{J}^{M-1/2}(\theta, \phi),
\end{equation}

(2.2b)

where and hereafter $s$ and $t$ dependence of $T_{N,1/2}^s$ is not written explicitly. Parity conservation gives

\begin{equation}
T_{N,-r}^s(W) = -\sigma(-1)^{J+1}T_{N,r}^s(W)
\end{equation}

(2.3)

and then

\begin{align}
T_{-+}(\theta, \phi) &= -T_{++}(\theta, -\phi), \\
T_{++}(\theta, \phi) &= T_{--}(\theta, -\phi).
\end{align}

(2.4)

Equations (2.2) are converted to give

\begin{align}
T_{N,1/2}^s(W) &= \frac{1}{2} \left( \frac{2J+1}{4\pi} \right)^{1/2} \int d\Omega \left[ -T_{++}(\theta, \phi) D_{J}^{M+1/2}(\theta, \phi) + T_{-+}(\theta, \phi) D_{J}^{M-1/2}(\theta, \phi) \right], \\
T_{N,-1/2}^s(W) &= \frac{1}{2} \left( \frac{2J+1}{4\pi} \right)^{1/2} \int d\Omega \left[ T_{++}(\theta, \phi) D_{J}^{M+1/2}(\theta, \phi) + T_{-+}(\theta, \phi) D_{J}^{M-1/2}(\theta, \phi) \right].
\end{align}

(2.5a)

(2.5b)

Our normalization is given as

\begin{equation}
\frac{d\sigma}{dt dW} = \frac{p_1}{4(2\pi)^{2}(p_Lm)^{2}} \int \frac{d\Omega}{4\pi} \frac{1}{2} \sum_{J, \mathbf{M}} |T_{N}\mu(\theta, \phi)|^2, = \frac{p_1}{2(2\pi)^{2}(p_Lm)^{2}} \sum_{J, \mathbf{M}} \left[ |T_{N,1/2}^s(W)|^2 + |T_{N,-1/2}^s(W)|^2 \right].
\end{equation}

(2.6)

Spherical harmonic moments of $Y_{1m}(\theta, \phi)$, the $\langle Y_{1m} \rangle$'s, can be written by the partial waves. Angular distributions at fixed $t$ and $W$ are expressed in terms of $\langle Y_{1m} \rangle$,

\begin{equation}
\frac{d\sigma}{dt dW d\Omega} = \frac{d\sigma}{dt dW} \sum_{1m} \langle Y_{1m} \rangle \text{Re} Y_{1m}(\theta, \phi),
\end{equation}

(2.7)

\begin{align}
\sqrt{4\pi} \frac{d\sigma}{dt dW} \langle Y_{1m} \rangle &= \frac{p_1}{2(2\pi)^{2}(p_Lm)^{2}} \sum_{J, \mathbf{M}} \left[ \frac{(2J+1)(2J'+1)}{2l+1} \right]^{1/2} c_{JJ'}(0; -1/2, 1/2) \\
&\times (-1)^{M'+J'}c_{JJ'}(m; -M, M') \{e_+ [T_{N,1/2}^s T_{M',1/2}^s]^* + T_{N,-1/2}^s T_{M',1/2}^s]^* \} \\
&- e_- [T_{N,1/2}^s T_{M',1/2}^s]^* + T_{N,-1/2}^s T_{M',1/2}^s]^* \},
\end{align}

(2.8)
where $\varepsilon = (1 + (-1)^{J^p-J^p^{*}})/2$, $\langle Y_{1-m}\rangle = (-1)^m\langle Y_m\rangle$ and $\langle Y_0\rangle = 1/\sqrt{4\pi}$.

We denote the partial wave with the orbital angular momentum $L (L=S, P, \text{etc.})$, total spin $J$ and its polarization $M$ as $L^J_M$. When $M$ is not concerned, $L^J$ is often used.

§ 3. The Drell-Hiida-Deck model

3-1 Kinematical boundary of $s'$ and $t'$ variables

Before proceeding with the partial wave analysis of the DHD model, we discuss the kinematical boundary of $s'$ and $t'$ variables. The $\pi\pi$ energy squared, $s'$, is written as

$$s' = 2m_e^2 + 2E_eE_e + 2p_e^2 \cos \theta_e \cos \phi_e + 2p_e^2 \sin \theta_e \sin \phi_e$$

at the G-J frame, where $\theta_e$ is the angle between the outgoing $\pi_e$ and the initial nucleon $N_e$, and $\phi_e$ is negative. The kinematical boundary of $s'$ and $t'$ variables shown in Fig. 2 at fixed $W$ and $t$, where the incident momentum is 16 GeV/c. The inner regions of the ellipses are allowed kinematically.

From Fig. 2 we can see that at $t = t_{\min}$, $s'$ increases linearly with $t'$ and a similar inclination is also seen at very small $t$, while at large $t$, $t'_{\min}$ becomes large and the range of the $t'$ variation shifts to the left ($=\text{large } |t'|$). The implication of this tendency will be discussed in the next subsection.

3-2 Partial wave analysis of the DHD amplitude

The DHD amplitude can be written as

$$A = F_{x}(s', t'; t')P_x(W^2, t'),$$

$$T_{\pi\pi}(s, t, W, \theta, \phi) = \bar{u}(p_1, \nu) A_{\pi\pi}(p_2, \nu),$$

where $F_{x}$ is the $\pi\pi$ elastic amplitude with the pomeron and $f$-Reggeon exchange and its pomeron exchange part and $f$-Reggeon exchange part are parameterized as

$$F_{x}^{(P)} = i\beta_{x}^{(P)} e^{\beta_{x}^{(P)}} t^{1/2} (s', m_{x}^{2}, t') \left(\frac{s'}{s_0}\right)^{\alpha_{f}^{(t)/2}} \exp\left(-i\alpha_{f}^{(t)/2}\right),$$

$$F_{x}^{(f)} = \beta_{x}^{(f)} \left(1 - \alpha_{f}(t)\right) \cos(\pi\alpha_{f}(t)/2) \left(\frac{s'}{s_0}\right)^{\alpha_{f}(t)/2} \exp\left(-i\alpha_{f}(t)/2\right),$$

respectively, where we take $\beta_{x}^{(P)} = 40 \text{ GeV}^{-2}$ which corresponds to $\sigma_{x}^{(P)}(\infty) = 16 \text{ mb}$, $B_x = 3 \text{ GeV}^{-2}$, $\alpha_{f}^{(P)} = 0.3 \text{ GeV}^{-2}$, $\alpha_{f}(t) = 0.5 + 0.9t$, $\beta_{x}^{(f)} = 30$ which is estimated from the $\rho$-meson width and the exchange degeneracy between the $f^2$- and $\rho$-Reggeon, $s_0 = 1 \text{ GeV}^2$ and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$.

As a pion propagator, $P_x$, we use an elementary pion exchange

$$P_x(W, t') = \frac{G(t')}{m_{x}^{2} - t'},$$

where we assume $G(t') = \sqrt{3} g \exp(b(t' - m_{x}^{2}))$ with $g^2/4\pi = 14.6$. The factor
\( \sqrt{3} \) is needed for the production of the pure \( I=1/2 \) \( \Lambda \pi \) system. (Our amplitude, Eq. (3.2), is identical with \( M_{I=1/2}^{E=0} \) in Ref. 8), where \( I_E \) is the exchanged isospin and \( I_N \) is the isospin of the \( \Lambda \pi \) system.) The additional exponential factor is introduced to suppress the far off-shell part of the pion propagator and the value of \( b \) is taken about 2 GeV\(^{-2} \). The dependence of the results on the value of \( b \) will be discussed in §3-3. (It will be discussed in §5 why we do not use a Reggeized version for the pion propagator.)

3-3 Characteristic features of the DHD model

The partial wave amplitude of the DHD model is obtained by Eqs. (2.5) and is denoted as \( D_{I \ell}^{I_N}(W) \).

The partial cross sections are shown in Fig. 3(a) and the spherical harmonic moments \( \langle Y_{lm} \rangle \) for \( I \leq 2 \) are shown in Fig. 3(b). (It is to be noted that these quantities are calculated only by the partial waves with \( J \leq 5/2 \) and higher partial waves with \( J > 5/2 \) are ignored.) The largest partial cross sections are \( S \) and \( P \) waves with \( J=1/2 \) at small \( t \). As \( J \) increases, the partial cross sections become smaller by one or more orders. The appearance of the parity partner or both naturalities is connected with the fact that there holds a symmetry \( D_{I \ell}^{I_N}(W) = D_{I \ell}^{I_N}(-W) \)\(^{19} \) like the MacDowell symmetry.\(^{19} \)

The negative values of \( \langle Y_{10} \rangle \) and \( \langle Y_{20} \rangle \) at very small \( t \) and small \( W \) are
essentially due to the \( \gamma_5 \) coupling of the \( NN\pi \) vertex. Since the essential part of the cross section of the DHD model is
\[
\frac{d\sigma}{dtdWdQ} \propto \left| \frac{s'}{(m^2_{\pi} - t')} \right|^2 \left| (-t') \exp(2bt') \right|
\]
and \( s' \) increases almost linearly with \( t' \) at very small \( t \) as shown in Fig. 2, the first factor remains a constant and then \( \cos \theta \) distribution should have a forward dip due to the factor \( (-t') \). When \( t \) and \( W \) become large, the variation range of \( t' \) is enlarged, so that the cutoff factor in the second parenthesis causes the backward to be suppressed and then the maximum of the \( \cos \theta \) distribution shifts to the forward region. This pushes \( \langle Y_{10} \rangle \) upwards and makes it positive. The value of \( \langle Y_{10} \rangle \) depends on the value of \( b \) and so does the size of the higher partial waves. Qualitative features, however, do not depend very sensitively on the value of \( b \).

As for the \( \phi \) distribution, an enhancement at \( \phi = 180^\circ \) occurs owing to the factor \( s' \). In general the dependence of the amplitude on \( s' \) breaks the \( t \)-channel helicity conservation.

For the process \( \pi N \rightarrow (\rho \pi) N \), the \( \rho \pi \pi \) vertex does not have an extra \( t' \) dependence and almost constant behavior of \( \left| \frac{s'}{(m^2_{\pi} - t')} \right|^2 \) leads to the large \( S \) wave, that is, the \( 1_s^+ \) state. A similar situation holds for \( \pi N \rightarrow \pi (4\pi) \) and we have a large \( S \) wave, too; that is, the \( D^3 \) state.

\section*{4. Two component model}

\subsection*{4-1 Unitarization of the DHD amplitude}

Since our DHD amplitude does not satisfy the unitarity concerning \( W \), we consider the unitarization of it by the final state interaction. The unitarity equation representing the final state interaction is written as
\[
T_{\rho}(W+i\epsilon) = T_{\sigma}(W-i\epsilon) = \frac{2i\rho(W)}{\rho(-W)} T_{\rho}(-W+i\epsilon) \quad \text{for } W > m_N + m_N,
\]
\[
= \frac{2i\rho(W)}{\rho(-W)} T_{\sigma}(-W+i\epsilon) \quad \text{for } W < -(m_N + m_N),
\]
where \( f^{\rho,s}(W) \) is an elastic \( \pi N \) partial wave with the spin \( J \); the naturality \( \sigma \) and the amplitudes with \( -\sigma \) in Eq. (4.1b) denote those with the opposite naturality to \( \sigma \). \( \rho \) and \( f^{\rho,s} \) are given as
\[
\rho(W) = \frac{p_i}{8\pi W}, \quad (4.2)
\]
\[
f^{\rho,s}(W) = \left[ \frac{\rho^{\rho,s}(W)}{2i\rho(W)} - 1 \right]. \quad (4.3)
\]
Although the unitarity equations, Eqs. (4.1), are the elastic ones, we still use them above the inelastic threshold, assuming that inelasticity can be included in \( f^{\rho,s}(W) \), that is to say, we do not consider conversions from inelastic channels such as \( \pi J \) to the elastic one.
When \( \text{Im} T \) decreases rapidly for large \( W \) over the physical region and \( \text{Im} T \) across the negative \( W \), Eq. (4.1b), is neglected, we have a solution

\[
T_{\text{B}}^{j,s}(W) = D_{\text{R}}^{j,s}(W) [1 + i\rho(W) f^{j,s}(W)] + r_{\text{R}}^{j,s}(W) f^{j,s}(W). \tag{4.4}
\]

Here \( r_{\text{R}}^{j,s} \) has no discontinuity across the physical cut \( W > m_\pi + m_z \) and can be written, if no subtraction is needed, as

\[
r_{\text{R}}^{j,s} = \frac{1}{n^{j,s}(W)} \frac{1}{\pi} \mathcal{D} \int_{m_\pi + m_z}^{\infty} dW' n^{j,s}(W') \rho(W') D_{\text{R}}^{j,s}(W') \frac{W'}{W}, \tag{4.5}
\]

where \( f^{j,s}(W) = n^{j,s}(W)/\rho(W) \) is used. It depends on the asymptotic behaviour of \( f^{j,s}(W) \) and \( D_{\text{R}}^{j,s}(W) \) whether the subtraction is needed or not. Since it is a very difficult problem to know the asymptotic behaviour and to estimate the value of \( r \), we treat the second term phenomenologically.

Hereafter let us call the first term the \( B \)-term or the background term and the second one the \( R \)-term or the resonance term,

\[
B_{\text{R}}^{j,s}(W) = D_{\text{R}}^{j,s}(W) (1 + i\rho(W) f^{j,s}(W)), \tag{4.6}
\]

\[
R_{\text{R}}^{j,s}(W) = r_{\text{R}}^{j,s}(W) f^{j,s}(W). \tag{4.7}
\]

The term, background, comes from the fact that \( B(W) \) gives no peak but a dip at the position of the resonance with \( J \) and \( \sigma \). On the other hand, the term, resonance, comes from the facts that \( R(W) \) gives a peak at the resonance and it resembles a direct resonance production amplitude. If we consider exchange mechanisms other than the one-pion exchange, which are dual to the \( \pi N \) resonances, we can give them as the resonance production amplitudes, because \( W \) lies in the \( \pi N \) resonance region, and we include them into the \( R \)-term.

In order to construct the \( R \)-term with a small number of parameters, we assume that it satisfies the \( t \)-channel helicity conservation (TCHC). The \( R \)-term with the spin \( J \), naturality \( \sigma \) and polarization \( M \) in the \( G-J \) frame is denoted by \( R_{\text{R}, j, \sigma}^{j,s}(W) \) and given as

\[
R_{\text{R}, j, \sigma}^{j,s}(W) = \left[ \delta_{j,1/2} \cos \frac{\phi + \beta}{2} + \sigma \delta_{j,-1/2} \sin \frac{\phi + \beta}{2} \right] R_{\text{R}, \sigma}^{j,s}(s, t; W), \tag{4.8}
\]

where \( R_{\text{R}, \sigma}^{j,s}(s, t; W) \) is the \( t \)-channel helicity amplitude which is the only one independent amplitude because of TCHC. \( R_{\text{R}, \sigma}^{j,s}(s, t; W) \) is parameterized as

\[
R_{\text{R}, \sigma}^{j,s}(s, t; W) = i\lambda_{j,s}(s, m_N^2, m_\pi^2) \left( \frac{s}{s_0} \right)^{a_{j,s} t} \exp \left( -i\pi c_j \sigma t/2 \right)
\times c_{j,s} \exp \left( b_{j,s} t f^{j,s}(W) \right). \tag{4.9}
\]

Two constants \( c_{j,s} \) and \( b_{j,s} \) represent the strength and the slope parameter of the \( R \)-term. \( \phi \) is the \( s-t \) crossing angle of the initial nucleon and \( \beta \) the angle between the directions of the initial nucleon in the CM system and in the rest system of the final \( N \pi \) system.\(^{30}\) When \( s \gg (\text{masses})^2 \) and \( |t| \), they are approximately
**Partial Wave Amplitude for Low Mass N\(\pi\) System**

Table I. Values are taken from the Particle Data. In the effective inelastic channel, "\(\sigma\)" and "\(\rho\)" represent the \(S\)- and \(P\)-wave \(\pi\pi\) resonances, respectively and their effective masses are written in the parentheses.

<table>
<thead>
<tr>
<th>state</th>
<th>(M_{J,\pi}) (GeV)</th>
<th>(\Gamma_{J,\pi}^{\text{el}}) (GeV)</th>
<th>elastic branching ratio</th>
<th>effective inelastic channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_{11})</td>
<td>1.47</td>
<td>0.25</td>
<td>0.60</td>
<td>(N^{\pi} , \sigma^{(400 \text{ MeV})}, l_{\pi}=0)</td>
</tr>
<tr>
<td>(S_{11}^{\pi})</td>
<td>1.53</td>
<td>0.10</td>
<td>0.35</td>
<td>(N_{\pi})</td>
</tr>
<tr>
<td>(S_{1}^{\pi})</td>
<td>1.70</td>
<td>0.20</td>
<td>1.00</td>
<td>(N^{\pi} , \rho^{(500 \text{ MeV})}, l_{\pi}=0)</td>
</tr>
<tr>
<td>(D_{13})</td>
<td>1.52</td>
<td>0.125</td>
<td>0.55</td>
<td>(4\pi, l_{\pi}=1)</td>
</tr>
<tr>
<td>(F_{15})</td>
<td>1.68</td>
<td>0.125</td>
<td>0.60</td>
<td>(4\pi, l_{\pi}=2)</td>
</tr>
<tr>
<td>(D_{33})</td>
<td>1.67</td>
<td>0.125</td>
<td>0.40</td>
<td>(4\pi, l_{\pi}=2)</td>
</tr>
</tbody>
</table>

expressed by

\[
\cos \psi = \frac{(W^2 - m_\pi^2 - t)}{\lambda^{1/2}(t, W^2, m_\pi^2)}.
\]

\[
\cos \beta = \frac{(W^2 - m_\pi^2 + t)}{\lambda^{1/2}(t, W^2, m_\pi^2)}.
\]

(4.10)

4-2 Parameters for the \(\pi N\) resonances

The \(\pi N\) resonances we used in the calculation are \(P_{11}(1470), S_{11}(1530)\) and \(S_{11}(1700), D_{13}(1520), F_{15}(1680)\) and \(D_{33}(1670)\). Since for our purpose we need only approximate forms for these resonances, they are expressed by the Breit-Wigner forms or a sum of them for \(S_{11}\). An elastic width is given as

\[
\Gamma_{J,\pi}^{\text{el}}(W) = \Gamma_{J,\pi}^{\text{el}}(M_{J,\pi}) \left( \frac{p_{1}}{p_{1}^*} \right)^{2l+1} \frac{M_{J,\pi}}{W} \frac{1}{[1 + \frac{(R_{1}p_{1}^*)^{2l}}{W - W_{\text{inel}}}]},
\]

(4.11)

where \(p_{1}^*\) is the value of \(p_{1}\) at \(W=M_{J,\pi}\), the position of the resonance, and \(l\) is the orbital angular momentum corresponding to \(J\) and \(\pi\). A total width is given as

\[
\Gamma_{J,\pi}^{\text{tot}}(W) = \Gamma_{J,\pi}^{\text{el}}(W) + \Gamma_{J,\pi}^{\text{inel}}(W) \theta (W - W_{\text{inel}}).
\]

(4.12)

Similar parameterization is used for \(\Gamma_{J,\pi}^{\text{inel}}(W)\), provided that \(l\) is replaced by an appropriate orbital angular momentum of the effective inelastic channel, which is tabulated in Table I, and \(p_{1}(p_{1}^*)\) by \(p_{\text{inel}}(p_{\text{inel}}^*)\) which is the inelastic momentum at \(W(W=M_{J,\pi})\). For the \(S_{11}\) state two Breit-Wigner forms with the masses 1.53 and 1.70 GeV are added so as to satisfy the unitarity. Parameters are tabulated in Table I, and \(R_{1}\) is taken to be \(6.1/\text{GeV}^{-1}\).

4-3 Numerical results

All calculations are performed for \(J\leq 5/2\) at \(p_{L}=16 \text{ GeV}/c\). Our aim in this paper is not to fit the data but to study the structure of the model.

Two types of interference pattern between the \(B\) and \(R\)-terms appear: The first (second) type is that the interference is constructive (destructive) for \(W\) below the resonance position but becomes destructive (constructive) for \(W\) above
the resonance. The first (second) type is called the constructive (destructive) interference. As a typical example of the constructive interference we depict in Fig. 4 the Argand circles of $T$, $B$ and $R$ of $P_1^1$ at a fixed $t$. For the constructive (destructive) interference the peak of the resonance shifts to the lower (higher) mass side. Since the size of the $B$-term becomes smaller as $J$ becomes larger, the interference is the biggest for the $J=1/2$ states.

When fixing the parameters of the DHD amplitude, only $c^{J,\sigma}$ is left as the free parameters at small $t$. We study two extreme cases where Case I (II) is that there are the $R$-terms with only the natural (unnatural) parity, that is to say, Case I obeys the Gribov-Morrison (G-M) rule and Case II the anti-G-M rule. The parameters we used for the $R$-terms are tabulated in Table II. The value of $b^{J,\sigma}$ is assumed to decrease with $J$, $b^{J,\sigma}=3/(J+1/2)$ GeV$^{-2}$.

We show in Fig. 5 the mass distributions and $\langle Y_{\text{im}} \rangle$ which are integrated for $|t| \leq 0.1$ GeV$^2$ and the partial cross sections at $t=-0.001$ GeV$^2$. The results of both cases are similar for $W<1.5$ GeV. For $W>1.5$ GeV these are different from each other. The bump seen in the mass distribution for $W=1.65\sim1.7$ GeV is due to the $F^3$ state, which comes from the $F_{15}$ resonance, for Case I, but it is due to the $S^1$ state, which comes from the $S_{11}$ resonance at 1.7 GeV, for Case II.

The $t$-dependences of the partial cross sections at fixed $W$ are shown in Fig. 6. We can see that in general the $t$-dependence of the natural parity state is steeper than that of the unnatural one with the same $J$, except for the region dominated by the $R$-term. This property comes directly from the DHD model. Therefore at the low mass region for $W<1.30$ GeV the natural parity states fall off faster than the unnatural parity states. Rough estimate for the partial cross sections integrated for $t=0\sim0.5$ GeV$^2$ at the region for $W=1.20\sim1.30$ GeV are as follows:

![Fig. 4. Argand diagrams for the $P_1^1$ state for Case I. The $B$-term (broken line), $R$-term (dotted line) and the resultant amplitude (solid line) are shown at $t=-0.001$ GeV$^2$.](https://academic.oup.com/ptp/article-abstract/55/1/146/1854581)

<table>
<thead>
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<th>Case II</th>
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<tr>
<td>$S^1$</td>
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<tr>
<td>$D^0$</td>
<td>0</td>
<td>2.96</td>
</tr>
</tbody>
</table>
Partial Wave Amplitude for Low Mass Nπ System

Fig. 5 (a). The mass distribution integrated for $|t| \leq 0.1 \text{GeV}^2$. Case I is drawn by solid line and Case II by broken line. The experimental data, which are taken from Ref. 8, are for the pure $I=1/2 N\pi$ state with $I_F=0$. It should be noted, however, that the data are no t-cut ones. Fig. 5(b). The spherical harmonic moments integrated for $|t| \leq 0.1 \text{GeV}^2$. Lines are the same as in Fig. 5 (a). The experimental points, which are for $|t| \leq 0.1 \text{GeV}^2$, are taken from Ref. 5 for $K^+n \rightarrow K^+(\pi^+p)$ at 12 GeV/c, since we have no data $\pi p \rightarrow (N\pi)^{I_F=0}_{1/2}$ at 16 GeV/c. Fig. 5(c). Partial cross sections at $t=-0.001 \text{GeV}^2$. Lines are the same as in Fig. 5 (a).

$\sigma(P^e) : \sigma(S^e) : \sigma(P^0) : \sigma(D^e) = 30 : 60 : 6 : 1$ for Case I,

$\sigma(P^e) : \sigma(S^e) : \sigma(P^0) : \sigma(D^e) = 16 : 70 : 6 : 1.2$ for Case II.

$\sigma(D^e)$ and $\sigma(F^e)$ are less than 1 for the both cases.
The slope parameters of $d\sigma/dt/\text{d}W$ calculated by using $d\sigma/dtdW$ at $t=-0.001$ and $-0.10 \text{ GeV}^2$ are shown in Fig. 6(c). Increase in the slope near $W=1.7 \text{ GeV}$ for Case II is due to the large $S'$ wave. The $S'$ wave seems to be too large in the whole range of $W$ for Case II.

Since our calculations are limited by the state with $J\leq 5/2$, the results for $W>1.6 \text{ GeV}$ would not be very reliable even within our model. Furthermore our model does not include any contributions from the inelastic channels such as $J\pi$, though the contributions from the $J\pi$ channel would be expected not to be so large except for the $D'$ state near $W=1.45 \text{ GeV}$. In our model the $R$-term is required to satisfy TCHC for the sake of brevity, but it would be better to relax this requirement for fitting the data.

At the end of this section we comment on the results by Ochs et al. They hypothesis I is made from the large $S'$ and $P'$ states whose cross sections are comparable with each other. It is difficult, however, to construct such large $P'$ state from our point of view. Our case I resembles their hypothesis IV2. We point out from our calculations on $\langle Y_{1m} \rangle$ for $l\geq 2$ that the interference term between $J=1/2$ and $5/2$ states cannot be ignored even for $W<1.3 \text{ GeV}$. Our amplitudes
stay in the second quadrant for $W < 1.5\,\text{GeV}$, so that the phases relative to the $J(1236)$ production amplitude stay from $45^\circ$ to $135^\circ$, if the $J$ production amplitude has the simple $\rho$-Regge pole phase.

§ 5. Discussion

5-1 Reggeized version for the DHD amplitude

We did not use the Reggeized version for the DHD amplitude, for it is doubtful that we can use safely the Reggeized pion exchange at low energies up to the third resonance region in addition to the resonance term though the dual nature of the pion exchange amplitude is not necessarily clear. The elementary pion exchange is also used in the charged pion photoproduction at resonance energies.\(^{10}\) If we use the Reggeized pion exchange amplitude, we have inevitably the imaginary part. Where does this imaginary part come from? If it is related to the direct channel resonances in the sense of the duality, it would include much ambiguity to say that an enhancement given by the DHD model is kinematical. On the other hand it is sure that at higher energies the pion exchange should give the Regge behaviour as shown by effective trajectories\(^{10}\) in the charged pion photoproduction and $\rho$-meson production processes. Thus it becomes our task how the elementary pion exchange amplitude is worn out and the Reggeized one is grown.

In order to get the Reggeized amplitude we need to know the mechanism of the growth of the imaginary part and the erosion of the elementary pion exchange as $W$ becomes large and $|t'|$ departs from zero where the shrinkage is conspicuous. As a guess we suppose that the correction due to the final state interaction as discussed in the previous sections and a part of the resonance terms play a role to connect the elementary amplitude to the Reggeized one gradually. Similar cancellation is shown by Barbour, Malone and Moorhouse\(^{18}\) in the pion photoproduction between the electric Born term and the resonance term. In the above we have said a part of the resonance terms, since the whole resonance terms do not contribute only to the pion exchange term but also to other exchange term such as a natural parity exchange term which should be dual to the resonances.

5-2 Absorptive correction

An absorptive correction due to elastic scattering should be taken into account if the full unitarity is required. Adopting the most simple procedure for the absorptive correction,\(^{20}\) the impact parameter profile of the $s$-channel helicity amplitude for the production of the state with $J$, $\sigma$ and $W$ is corrected by multiplying the factor $S_{\text{el}}(b)$ and then it becomes peripheral, where $S_{\text{el}}(b)$ is the elastic $S$-matrix written in terms of the impact parameter $b$. The peripheral nature of the impact parameter profile gives a structure in the differential cross section $d\sigma/d\sigma dW$ when $J$ is small and also can give a strong mass and slope correlation which is one of the characteristic features of the diffraction dissociation.\(^{8}\) The absorptive
correction is also needed to reduce $d\sigma/dt dW$, because our naive $T_{W}^r_{1/2}(s,t,W)$ gives too large cross section at small $W$. This would give a foundation for the more phenomenological approaches to the spin structure of the dissociation system.

5–3 Comments on the $A_1$ problem

Our two-component model is similar to that discussed by Bowler and Game for $A_1$. According to our analysis on the $N\pi$ system produced diffractively, if the true $A_1$ resonance decaying into $\rho\pi$ through $S$-wave does exist, its mass is larger than the observed $1^+_s$ peak when the interference between $B$ and $R$ is constructive. Since the observed $1^+_s$ peak exists at $W\approx 1.15 \sim 1.2$ GeV, the mass of the true $A_1$ is expected to be larger than 1.2 GeV. Even in the case of the charge exchange process, for example $\pi^+n\rightarrow(\rho\pi)^0p$, if the interference between $B$ and $R$ is similar to the diffractive case, the isolated $A_1$ resonance is hardly seen even in the charge exchange process: Since $B$ and $R$ are determined by the $\rho$-Regge exchange, instead of the pomeron exchange, a similar type of the interference would be indeed possible. On the other hand if $B$ and $R$ interfere destructively, the maximum of the cross section shifts to higher mass side.

Recently an interesting observation is obtained that in the process $K^+p\rightarrow K^+K^+K^-p$, which is thought to be a diffractive process, the mass interval $M_{KK}=1.6 \sim 1.9$ GeV, where a $2^-L$-meson lies if it exists, is dominated by the $1^+_sK^+\phi$ state and there is no evidence for the $2^-L$ meson decay. Is there a true $L$-meson which can decay into $3\pi$? If the decay proceeds through two-body channels, there seem to be no candidates: Though $K^+\phi$ and $S^*K^+$ channels are only ones allowed energetically, they have to be in $P$-wave and $D$-wave states, respectively, in order to construct $2^-$ state and there are no peaks in the $2^-P$ and $2^-D$ channels in the case of $3\pi$ system. Therefore the $\phi K^+$ state with $1^+_s$ may be a pure background. It is interesting for the $A_1$ problem to study the process $\pi p\rightarrow(\pi K\bar{K})p$ in a similar sense.

References


Partial Wave Amplitude for Low Mass $N\pi$ System

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15) M. G. Bowler and M. A. Game, Oxford University Preprint 38/74.