A Model for \((d, p)\) or \((p, d)\) Reactions

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A model for deuteron stripping or pick up reaction is proposed which treats the contributions of deuteron breakup in a straightforward way. The model has been applied for \((p, d)\) reaction on \(^{208}\text{Pb}\) and \(^{40}\text{Ca}\) for 22 MeV and 30.5 MeV incident proton energy, respectively and \((d, p)\) reaction on \(^{56}\text{Fe}\) for 23 MeV incident deuteron energy. We find that the adiabatic potential of Johnson et al. and choice of the energy for relative motion of the neutron and the proton of the 'broken deuteron', the same as that in its bound state are poor approximations in our theory. The radial integrals with the wavefunctions of the elastically scattered deuteron and the corresponding ones with the 'broken deuteron' are opposite in sign for low partial waves leading to a new picture of reaction mechanism.

§ 1. Introduction

Johnson et al.\(^{13}\) proposed an approximate treatment of the contribution of deuteron breakup channels to the deuteron stripping or pickup reactions, with a zero radius of cut-off. It constitutes a significant improvement of the earlier version\(^5\) of DWBA theory which uses a finite cut-off radius. Johnson et al.'s "adiabatic theory" was tested with a good degree of success over a wide range of nuclei,\(^3\),\(^4\) and it proved to be more successful than DWBA. In the present paper we propose a theory of \((d, p)\) or \((p, d)\) reaction also with zero cut-off radius, which treats breakup channels more accurately and gives a physical picture of reaction mechanism different from that of adiabatic theory of Johnson et al.\(^{13}\) In § 2, the formalism is developed for \((d, p)\) and \((p, d)\) reactions. In § 3 we discuss the application of our theory, under suitable approximations, for \((p, d)\) or \((d, p)\) reactions on \(^{208}\text{Pb}\), \(^{56}\text{Fe}\) and \(^{40}\text{Ca}\).

§ 2. Formalism

The transition amplitude of \((d, p)\) reaction may be written as \(^{21,22}\)

\[
T_{dp} = \langle \phi_f | V_t + V_{np} | \Psi_i^{(-)} \rangle,
\]

where \(\Psi_i^{(-)}\) is the total wavefunction of the initial state with energy \(E\) and outgoing wave boundary condition. It is an eigenfunction of the total Hamiltonian \(H\) which we write as

\[
H = H_t + V_t + V_{np} + J V_c
\]

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\[ H_f = V_1 + v_{np}, \quad (2\cdot2b) \]

where

\[ H_i = H_T + T_{np} + v_{np} + T_R + V_c(R) \quad (2\cdot2c) \]

and

\[ H_f = H_T + T_n + V_2 + T_p + V_c(r_p). \quad (2\cdot2d) \]

In Eqs. (2·1) and (2·2), \(H_T\) is the target Hamiltonian; \(T_n, T_p, T_{np}\) and \(T_R\) are, respectively, the kinetic energy operators of the neutron, proton and those for their relative- and centre-of-mass motion; \(V_1 = \sum v_{\rho\tau}\) and \(V_2 = \sum v_{\nu\tau}\) where \(v_{\rho\tau}\) and \(v_{\nu\tau}\) are the two-body interactions between a target nucleon “\(i\)” and proton and neutron of deuteron, respectively, the summations in \(V_1\) and \(V_2\) being over target nucleons; \(v_{np}\) is the two-body interaction between neutron and proton of deuteron; \(r\) and \(R\) are the relative and centre-of-mass co-ordinates of neutron at \(r_n\) and proton at \(r_p\); \(V_c(r_p)\) is the Coulomb force between outgoing proton and residual nucleus, while \(V_c(R)\) is that between target and unit charge at \(R\), with \(2V_c = V_c(r_p) - V_c(R)\).

The state vectors \(\phi_i\) and \(\phi_f\) are the eigenfunctions of \(H_i\) and \(H_f\), with energy \(E\); \(E = E_0 + \epsilon_0 + \hbar^2 k_d^2 / 4m = E_R + \hbar^2 k_d^2 / 2m\), where \(E_0, \epsilon_0\) and \(E_R\) are, respectively, the ground state energy of target, deuteron and residual nucleus; \(\hbar k_p\) and \(\hbar k_d\) are momenta of outgoing proton and incident deuteron, respectively, and \(m\) is the mass of a proton.

We may write

\[ |\phi_i\rangle = |\phi_0 \gamma \phi_c^{(+)}\rangle, \quad (2\cdot3a) \]
\[ |\phi_f\rangle = |\phi_{np} \phi_c^{(-)}\rangle. \quad (2\cdot3b) \]

where \(|\phi_0\rangle, |\phi_R\rangle, |\gamma_0\rangle, |\gamma_c^{(+)}\rangle\) and \(|\phi_{np}^{(-)}\rangle\) are, respectively, the state vectors for target ground state, residual nucleus, deuteron ground state, Coulomb-distorted wave of the centre-of-mass of deuteron with outgoing wave boundary condition and Coulomb-distorted wave of outgoing proton with incoming boundary condition.

Let \(|\chi_f^{(-)}\rangle\) be the state vector at energy \(E\) generated from \(|\phi_f\rangle\) by the distorting potential \(Z_f\), with incoming wave boundary condition. Then the transition amplitude of Eq. (2·1) may be written as\(^6\)

\[ T_{fi} = T_1 + T_2, \quad (2\cdot4a) \]

where

\[ T_1 = \langle \chi_f^{(-)} | v_{np} | \phi_i^{(-)} \rangle \quad (2\cdot4b) \]

and

\[ T_2 = \langle \chi_f^{(-)} | V_1 - Z_f | \phi_i^{(-)} \rangle. \quad (2\cdot4c) \]

If we define \(|\chi_i^{(-)}\rangle\) as the state generated from \(|\phi_i\rangle\) by the distorting potential \(Z_i\), i.e.,

\[ |\chi_i^{(-)}\rangle = [1 + (E - H_i - Z_i + i\hbar \sigma^i)^{-1} \cdot Z_i] |\phi_i\rangle, \quad (2\cdot5) \]
Our aim is to choose the distorting potentials $Z_i$ and $Z_f$ so that we get a workable $(d, p)$ theory and calculate the contribution of the breakup channel as well. Although we intend to apply our theory for $(d, p)$ or $(p, d)$ reaction at present, we present it in a form so that it can be applied to other transfer reactions as well. We define projection operators $P$ and $Q$, which commute with $H_i$, such that

$$P^2 = P \text{ and } Q = 1 - P.$$  \hspace{1cm} (2.9)

Then, following Feshbach, we can write the equations for $P |T_i^{(v)}\rangle$ and $Q |T_i^{(v)}\rangle$ and have

$$|T_i^{(v)}\rangle = [1 + G^{q0} (V_1 + V_2 + J V_c)] |T_i^{(v)}\rangle,$$  \hspace{1cm} (2.10)

where

$$G^{q0} = Q [E - Q H Q + i\epsilon]^{-1} Q.$$  \hspace{1cm} (2.11)

Next, it is to be noted that it is possible to choose the distorting potential $Z_i$ such that the second term on the right-hand side of Eq. (2.7) gives zero contribution in Eqs. (2.10) and (2.4). One such (non-trivial) choice is (see Appendix I)

$$Z_i = P M,$$  \hspace{1cm} (2.12a)

where

$$M = (V_1 + V_2 + J V_c) + (V_1 + V_2 + J V_c) G^{q0} (V_1 + V_2 + J V_c),$$  \hspace{1cm} (2.12b)

and this gives

$$P G^{q0} (V_1 + V_2 + J V_c - Z_i) |T_i^{(v)}\rangle = 0,$$  \hspace{1cm} (2.13)

$$P |T_i^{(v)}\rangle = |T_i^{(v)}\rangle = 0$$  \hspace{1cm} (2.14)

$$= [1 + G^{q0} P M P] |T_i^{(v)}\rangle + [1 + G^{q0} P M Q] Q |T_i^{(v)}\rangle.$$  \hspace{1cm} (2.15)

where

$$G^{q0} = P [E - P H P - P M P + i\epsilon]^{-1} P.$$  \hspace{1cm} (2.16)

Equation (2.15) is obtained from Eqs. (2.5) and (2.7), using the algebra discussed in Ref. 8). Using Eqs. (2.14) and (2.10) in Eq. (2.4) we get

$$T_i = T_{i1} + T_{i2},$$  \hspace{1cm} (2.17)

where

$$T_{i1} = \langle \chi_i^{(v)} | v_{sp} | P \chi_i^{(v)} \rangle,$$  \hspace{1cm} (2.18)
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\[ T_{12} = \langle \Psi_i^{(-)} | v_{np} G^{io} (V_1 + V_2 + A) | P \Psi_i^{(-)} \rangle, \quad (2.19) \]

with similar expressions for \(T_2\). These equations can be simplified further by proper choice of \(P\) and \(Q\). If, for example, we choose

\[ P = P_0 P_0, \quad (2.20a) \]

where

\[ P_0 = \langle \phi_0 | \phi_0 \rangle; \quad Q_0 = 1 - P_0 \quad (2.20b) \]

and

\[ P_0 = \langle \phi_0 | \phi_0 \rangle; \quad q_0 = 1 - P_0, \quad (2.20c) \]

then

\[ Q | \Psi_i^{(-)} \rangle = Q | \phi_i \rangle = 0, \quad (2.20d) \]

so that the last two terms on the right-hand side of Eq. (2.15) drops out. With this choice of \(P\) in Eq. (2.20), we get, from Eq. (2.15),

\[ P | \Psi_i^{(-)} \rangle = | \phi_i \Psi_i^{(+)} \rangle, \quad (2.21) \]

where \( | \Psi_i^{(+)} \rangle \) is the state vector for the elastic scattering of deuteron from target, the deuteron optical potential being given by

\[ PMP = P_0 P_0 V_{op}^{(g)} . \quad (2.22) \]

Equation (2.22) is the operator form of Eq. (2.12) of Ref. 8) (without the Coulomb potentials). The choice of \(Z_i\) and \(P\), given by Eqs. (2.12), (2.20) and (2.21) gives the usual DWBA matrix element for \((d, p)\) reaction, given by Eq. (2.18). For the comparative study made later in the paper, we write it in the form

\[ T_{11} = T^{DWBA} = g \sqrt{s} \int \tilde{\Psi}_i^{(-)*} (R) F_n^* (R) D_F (K) \tilde{\Psi}_i^{(+)} (R) d^2 R, \quad (2.23) \]

where

\[ \sqrt{s} F (r_n) = \langle \phi_0 | \phi_h \rangle, \quad (2.24) \]

\[ g = \int v_{np} (r) \phi_0 (r) d^3 r, \quad (2.25) \]

\[ D_F (k) = 1 - \frac{1}{\beta^2} \frac{1}{2} k^2 - k^2 \quad (2.26) \]

and \( F_n (r_n) \) is the orbital of bound neutron with spectroscopic factor \(S\); \( \beta \) is the range parameter of two-body potential \((p, n)\) and \( \tilde{\Psi}_i^{(-)} (R) \) is the distorted wave of elastically scattered outgoing wave of proton. The standard DWBA uses a finite cut-off radius \(R_c\) in the integration of Eq. (2.23) for good fit to experimental data, which indirectly relates it to the deuteron and proton absorption in the initial and
final channels, respectively.

As an alternative to Eq. (2.20), one may choose
\[ P = P_0, \quad Q = Q_0 \]  
(2.27)
and \( Z_i \) as in Eq. (2.12). In this case, we also get
\[ Q |\phi_i\rangle = 0 = Q |z_i^{(+)}\rangle \]  
(2.28)
and
\[ P |\Psi_i^{(+)}\rangle = P |z_i^{(+)}\rangle = |\phi_0 f_0\rangle. \]  
(2.29)
Here \( |z_i^{(+)}\rangle \) or \( |f_0\rangle \) is now generated from \( |\phi_i\rangle \) by a new optical potential PMP of Eq. (2.16), which becomes for \( P = P_0 \),
\[ P_0 M P_0 = P_0 |\phi_0\rangle (V_1 + V_2 + \Delta V_c) + (V_1 + V_2 + \Delta V_c) G^{\phi_0} \]
\times \langle V_1 + V_2 + \Delta V_c | \phi_0 \rangle \]  
(2.30a)
\[ = P_0 \hat{\delta}_0. \]  
(2.30b)
The wavefunction \( f_0 \) of Eq. (2.29) and \( \tilde{\psi} \) of (2.30) are \( f_0(r_n, r_p) \) and \( \tilde{\sigma}(r_n, r_p) \) of Ref. 8 (without the Coulomb potentials). Starting from Eq. (2.6) or (2.10) and (2.27) and using the techniques of Ref. 8), it is straightforward to obtain the equations for \( |f_0\rangle \), and its projections \( p_0|f_0\rangle \) and \( q_0|f_0\rangle \). These may be put in the form
\[ q_0|f_0\rangle = q_0 \frac{1}{E - E_0 - T_R - V_c(R) - T_{np} - v_{np} + i\epsilon} q_0 \delta p_0 |f_0\rangle, \]  
(2.31)
\[ p_0|f_0\rangle = |z_0\rangle \left[ 1 + \frac{1}{E - E_0 - \epsilon_0 - T_R - V_c(R) - V^{(s)}_{op}(R) + i\epsilon} V^{(s)}_{op}(R) \right] |\tilde{\psi}^{(+)}\rangle \]  
(2.32a)
\[ = |z_0\rangle |\tilde{\psi}^{(+)}\rangle \]  
(2.32b)
or
\[ [T_{np} + v_{np} + T_R + V_c(R) + \tilde{\sigma}] f_0(r_n, r_p) = (E - E_0)f_0(r_n, r_p), \]  
(2.32c)
where
\[ V^{(s)}_{op}(R) = \langle \tilde{\psi} | \tau | \tilde{\psi} \rangle \]  
(2.33)
and
\[ \tau = \tilde{\sigma} + \tilde{\sigma} \frac{1}{E - E_0 - T_R - V_c(R) - T_{np} - v_{np} + i\epsilon}. \]  
(2.34)
The double-bracket notation in Eq. (2.33) implies the integration only over the relative co-ordinate \( r \) of neutron-proton system. Equations (2.33) and (2.34) give Eq. (2.15) of Ref. 8), where the equality between \( V^{(s)}_{op}(R) \) of Eqs. (2.33) and (2.22) is proved. In co-ordinate representation, we may write
\[ f_0(r_n, r_p) = \gamma_0(r) |\tilde{\psi}^{(+)}(R) + B(r, R), \]  
(2.35)
where $B(r, R)$ stands for the wavefunction for the break-up part $q_0|f_0\rangle$. Since $v_{np}(r)$ is short-ranged, for the matrix element of $T_1$ of Eq. (2.17), one needs $f_0(r_n, r_p)$ for small values of $r$. Therefore, it may be a good approximation to replace $f_0(r_n, r_p)$ by its weighted average $\bar{f}(R)$,

$$\bar{f}(R) = \int \rho(r)f_0(r_n, r_p)d^3r/\int \rho(r)d^3r,$$  \hspace{1cm} (2.36)

where $\rho(r)$ is some suitable short-ranged function of $r$. In that case the integration of $r$ in Eq. (2.18) may be done using the standard approximation, and the matrix element looks like that in Eq. (2.23) with $\tilde{\xi}(R)$ replaced by $\bar{f}(R)$. Now, if $T_2$ is negligible, and $|\phi_N\rangle$ contains $|\phi_R\rangle$ only as parent, then identically $T_{12} = 0$, because of Eq. (2.27) and we have $T_{1N} = T_{11}$, which is given by Eq. (2.23) with $\zeta_{0}(r)\tilde{\xi}(R)$ essentially replaced by $f_0(r_n, r_p)$ of Eq. (2.35) (before finite range corrections are made). Since deuteron absorption is directly included, in the present case, in the break-up wavefunction $B(r, R)$ one should not need the artificial cut-off radius $R_c$ to take care of it. In other words, the use of $\tilde{v}$, instead of $V^{(0)}_{np}$ as a distorting potential of the initial state, reduces cut-off radius $R_c$ to zero, provided proton absorption has very little effect on $R_c$ and is essentially taken care of by the assumption $T_2 = 0$. Johnson et al. use the definition

$$\bar{f}(R) = \frac{1}{g}\langle k = 0|v_{np}|f_0\rangle,$$  \hspace{1cm} (2.37)

where $g$ is given by Eq. (2.25). Then one gets

$$\bar{f}(R) = \tilde{\xi}^{(+)}(R) + \bar{B}(R).$$  \hspace{1cm} (2.38)

Using the definition, Eq. (2.37), for the averaged function, the equation for $\bar{B}(R)$ from Eq. (2.31) becomes

$$\bar{B}(R) = \frac{1}{(2\pi)^3g} \int \frac{d^3q}{E - E_0 - (\hbar^2q^2/m - T_{np} - V_{np}(R)) + i\epsilon}.$$  \hspace{1cm} (2.39)

In Eq. (2.39), $|\zeta_q\rangle$ is the continuum wavefunctions of neutron-proton system, with momentum $\hbar q$ and energy $\hbar^2q^2/m$, i.e.,

$$\langle T_{np} + v_{np}|\zeta_q\rangle = \frac{\hbar^2q^2}{m}|\zeta_q\rangle,$$

$$\langle K = 0|v_{np}|\zeta_q\rangle = \langle K = 0|t_{np}|q\rangle,$$

where $t_{np}$ is the two-body scattering matrix. Since off-shell matrix elements are small, it is expected that the main contribution to the integral in Eq. (2.39) comes from low values of $q$. Hence one may use the approximation of replacing $\hbar^2q^2/m$ by an average number $\bar{\epsilon}$ in the energy denominator (as also used by Johnson), and the momentum integral in Eq. (2.39) can be performed and one gets

$$\bar{B}(R) = \frac{1}{E - E_0 - \bar{\epsilon} - T_{np} - V_{np}(R) + i\epsilon} [\bar{V}(R) - \bar{V}^{(0)}_{np}(R)]\tilde{\xi}^{(+)}(R),$$.  \hspace{1cm} (2.40)
where

$$
\mathcal{V}(\mathbf{R}) = \frac{1}{g} \langle k=0 | \psi_{\text{op}} | \chi_0 \rangle.
$$

(2.41)

Johnson’s theory is obtained by making his series of approximation in Eq. (2.39) leading to an equation for his $\mathcal{B}(\mathbf{R})$ which is very different from ours in Eq. (2.40). From Eqs. (2.32), (2.40) and (2.38), the equation for $\mathcal{f}(\mathbf{R})$ is obtained as

$$
[E - E_0 - T_R - V_c(\mathbf{R}) - \mathcal{V}] \mathcal{f}(\mathbf{R}) = \epsilon_0 \mathcal{F}^{(1)}(\mathbf{R}) + (\epsilon - \mathcal{V}(\mathbf{R})) \mathcal{B}(\mathbf{R}).
$$

(2.42)

The equation obtained by Johnson et al. for $\mathcal{f}(\mathbf{R})$ is homogeneous while our equation (2.42) for $\mathcal{f}(\mathbf{R})$ is inhomogeneous. Again, the adiabatic potential of Johnson et al., called $V_J(\mathbf{R})$ here, is obtained by making the following approximations:

$$
\tau = \tilde{\tau},
$$

(2.43)

$$
\tilde{\tau} = V_{\text{op}}(\mathbf{r}_n) + V_{\text{op}}(\mathbf{r}_p) + V_0
$$

(2.44)

in Eqs. (2.41) and (2.34), where $V_{\text{op}}(\mathbf{r}_n)$ and $V_{\text{op}}(\mathbf{r}_p)$ are the optical potentials of elastic scattering of neutron and proton from the target at half the incident energy of deuteron. It is shown in Refs. 8 and 10) that the approximation in Eq. (2.43) is probably valid, but that in Eq. (2.44) is poor, and $\tilde{\tau}$ is better represented as

$$
\tilde{\tau} = V_{\text{op}}(\mathbf{r}_n, E-T_R) + V_{\text{op}}(\mathbf{r}_p, E-T_n) + C_1,
$$

(2.45)

where $E$ is the total energy and the correction term $C_1$ is of the same order as the neutron or proton optical potential and it gives substantial contribution to the deuteron optical potential of Eq. (2.33). From general arguments, the situation is expected to be similar for $\mathcal{V}(\mathbf{R})$. That is, if $\mathcal{V}(\mathbf{R})$ is written as

$$
\mathcal{V}(\mathbf{R}) = V_J(\mathbf{R}) + V_0(\mathbf{R}),
$$

(2.46a)

$$
V_J(\mathbf{R}) = \frac{1}{g} \langle k=0 | \psi_{\text{op}} \{ V_{\text{op}}(\mathbf{r}_n) + V_{\text{op}}(\mathbf{r}_p) \} | \chi_0 \rangle
$$

(2.46b)

then one expects $V_J(\mathbf{R})$ to introduce sizable correction to $V_J(\mathbf{R})$. To compare with the adiabatic theory of Johnson et al., we rewrite Eq. (2.42), using Eq. (2.46), in the form

$$
[E - E_0 - T_R - V_c(\mathbf{R}) - V_J(\mathbf{R})] \mathcal{f}(\mathbf{R}) = V_J(\mathbf{R}) \mathcal{F}^{(1)}(\mathbf{R}) - [V_J(\mathbf{R}) + \epsilon - \mathcal{V}(\mathbf{R})] \mathcal{B}(\mathbf{R}).
$$

(2.47)

The homogeneous equation of Johnson et al. for $\mathcal{f}(\mathbf{R})$ results if the right-hand side of Eq. (2.47) is identically zero. It is very unlikely that this will happen at all energy $E$ and for all values of $\mathbf{R}$, even if $\epsilon_0 = \tilde{\epsilon}$. So one expects important corrections to the theory of Johnson et al. arising from the inhomogeneous term...
§ 3. Computational methods, results and discussion

It is noted that there are two important differences between our theory and that of Johnson et al.14 Firstly, the differential equation for \( \tilde{f}(R) \) is different, by the presence of the inhomogeneous term, the right-hand side of Eq. (2·47). Secondly, the potential \( \overline{V}(R) \) is different from the adiabatic potential of Johnson et al. It is conjectured that these extra terms would contribute significantly in stripping and pick up reactions. It is worth while verifying these conjectures in view of the fact that our theory is derived in a straightforward way, using only two of many approximations (i.e., those defining \( \tilde{f}(R) \) and \( \tilde{\epsilon} \)) used in the adiabatic theory of Johnson et al.14

Our method of computation consists of first calculating \( \tilde{\xi}^{(+)}(R) \) for a given \( V^{(d)}_{sp}(R) \) at energy \( E_q = \hbar^2 k^2 / 4m = E - E_0 - \epsilon_0 \), and then use them in Eqs. (2·40) and (2·38) to calculate \( \overline{B}(R) \) and \( \tilde{f}(R) \), respectively. The \( \tilde{\epsilon} \) of Eq. (2·40) is related to the wave number of the asymptotically outgoing wave part of \( \overline{B}(R) \) as

\[
\frac{\hbar^2}{4m} (K_e^2 - K_B^2) = \tilde{\epsilon} - \epsilon_0.
\]

The \( \tilde{\epsilon} \) is unknown as yet in our theory, but one could perhaps start with the approximation \( \tilde{\epsilon} = \epsilon_0 \), as Johnson et al.14 did and change it later if found necessary. The potential \( \overline{V}(R) \) can be computed in a number of ways, as follows:

Method I:

\( \overline{V}(R) \) can be computed directly from Eq. (2·41) when \( \tau \) is known. The \( \tau \) is obtainable from the solution of the integral equation (2·34). References 8) and 10) discuss at length such solutions of \( \tau \) towards the determination of deuteron optical potential through Eq. (2·33).

Method II:

From Eqs. (2·33) and (2·41) we have

\[
\overline{V}(R) = V^{(d)}_{sp}(R) + \frac{1}{(2\pi)^3} \int \langle k = 0 | t_{sp} | q \rangle d^3 q \langle \xi_q | \tau | \zeta_0 \rangle,
\]

which shows that \( \overline{V}(R) \) is known if the diagonal and nondiagonal matrix elements of \( \tau \) between the ground states \( |\zeta_0\rangle \) and the continuum states \( |\xi_q\rangle \) of deuteron are known. Of these, because of Eq. (2·33), the diagonal matrix elements are known from the elastic scattering data. The nondiagonal matrix elements can be obtained from the analysis of \( (d, p n) \) reactions. If \( M(q, R) = \langle \xi_q | \tau | \zeta_0 \rangle \), then (see Appendix II) the reaction amplitude \( T(d, p n) \) of \( (d, p n) \) reaction, without any approximation is given by

\[
T(d, p n) = \langle \eta_c^{(+)}(k) \xi_q | \tau | \zeta_0 \rangle \zeta_0^{(+) \star}(\gamma)
\]

(i.e., the right-hand side) of Eq. (2·47).
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\[
\langle \eta_{c}^{(-)}(k, R) | M(q, R) | \xi(\omega) \rangle, \quad (3.3b)
\]

where \( \eta_{c}^{(-)}(k, R) \) is the Coulomb distorted wave of the centre-of-mass motion. From Eqs. (3-5) and (3-2), one notes that \( M(q, R) \) of \((d,pn)\) reaction and \( V^{(d)}_{\omega}(R) \) determine \( \overline{V}(R) \).

Method III:

\( \overline{V}(R) \) can be determined indirectly by method of parametrization. For example, \( \tau \) can be parametrized and its parameters determined by requiring it to produce a simultaneous fit to \((d,d)\), \((d,p)\) or \((p,d)\) and, perhaps, \((d,pn)\) data as well, as demanded by Eqs. (2·33), (2·41) and (3·3), for each energy.

Method IV:

Finally, \( \overline{V}(R) \) could itself be put in a parametric form and its parameters determined by \((d,p)\) or \((p,d)\) data.

In the present paper we discuss the results of our calculations on \((p,d)\) and \((d,p)\) reactions using Methods I and IV and compare them with those obtained\(^{10,11} \) by using adiabatic theory of Johnson et al.\(^{11} \) Results of Methods II and III will be reported elsewhere.\(^{11} \) Experimental data used were of \((p,d)\) reaction on \(^{208}\)Pb and \(^{40}\)Ca for 22 MeV and 30.5 MeV proton, respectively and \((d,p)\) reaction on \(^{56}\)Fe for 23 MeV deuteron. Our observations from the calculation using Method I may be summarized as follows:

It was found that the approximation of Johnson et al.,\(^{11} \) i.e., \( \overline{V}(R) = V_{\tau}(R) \) and \( \xi = \epsilon_{0} \), used in our theory, does not produce the angular distribution for \((d,p)\) reaction from any state of \(^{56}\)Fe. It does not produce all the maxima and minima at smaller angles and gives too large cross-sections at backward angles. The inclusion of \( C_{1} \) of Eq. (2·45) improves the situation in forward angles, but not at backward angles. The results are much better at forward angles if larger values

\[ \text{Fig. 1. Results of Method I for } (d, p) \text{ on } ^{56}\text{Fe with } 23 \text{ MeV deuteron. The solid curve is for } C_{1}=0, \quad \tau=\epsilon_{0}=-2.2 \text{ MeV. The dashed curve is for } \tau=\epsilon_{0} \text{ but non zero } C_{1} \text{ as demanded by the fit to deuteron elastic scattering discussed in Refs. 8) and 10). The dot-dashed curve is for } \tau=10 \text{ MeV and non zero } C_{1} \text{ given by Refs. 8) and 10). Circles are experimental points.} \]
of $\varepsilon > \epsilon_0$ are used but not so at backward angles. The pattern of the result is similar for pick-up from $^{208}\text{Pb}$ and $^{40}\text{Ca}$. A typical result for $(d,p)$ reaction on $^{54}\text{Fe}$ for 23 MeV deuteron is shown in Fig. 1. This investigation with Method I was particularly confined to the form of $C$, and $\tau$ obtained under approximations used in Refs. 8) and 10) for analysis of elastic scattering of deuteron. From the quality of fit to the stripping or pick up angular distributions, it was concluded that (a) the $\tau$ of the form in Eq. (2.45), which produces elastic scattering data, does not produce $(d,p)$ or $(p,d)$ data well, (b) the approximation of Johnson et al., i.e., $\nabla(R) = V_J(R)$ and $\tilde{\epsilon} = \epsilon_0$ in our theory is very poor and that (c) finally, there are significant differences between the predictions of our theory and those of Johnson et al. This is perhaps apparent from Eq. (2.47), the right-hand side of which becomes $-V_J(R)B(R)$ under the approximation $\tilde{\epsilon} = \epsilon_0$ and $\nabla(R) = V_J(R)$ used by Johnson et al., until $B(R)$ is negligibly small, when homogeneous equation of $\tilde{f}(R)$ results. It is the non-negligible $B(R)$ that makes adiabatic theory different from DWBA and it makes our theory different from the adiabatic theory too, by its presence in the inhomogeneous term of Eq. (2.47); and perhaps, to counter its effect, one requires $\nabla(R)$ (and $\tilde{\epsilon}$) different from $V_J(R)$ (and $\epsilon_0$). If we break up $\tau$ as

$$\tau = \tau_1 + \frac{1}{(2\pi)^3} \int |\lambda_q \rangle d^3q \langle \lambda_q | \tau_2,$$

(3.4a)

then

$$\langle \lambda_q | \tau_1 \rangle_{\lambda_0} = \langle \lambda_q | \tau_1 + \tau_2 \rangle_{\lambda_0}$$

(3.4b)

and

$$\langle \lambda_0 | \tau_1 \rangle_{\lambda_0} = \langle \lambda_0 | \tau_1 \rangle_{\lambda_0}.$$  

(3.4c)

Equations (3.4) show that the data on the elastic scattering of deuteron, which uses only the diagonal matrix element of $\tau$, determine only the $\tau_1$ part of $\tau$ and not $\tau_2$. Our investigations with Method I show that for the study of $(p,d)$ or $(d,p)$ reaction, one is required to know $\tau_2$ as well. This requires more accurate solutions of the integral equation (2.34) for $\tau$ than was required in Refs. 8) and 10) for the analysis of elastic scattering of deuteron. Since such solutions (as well as results of Method II) are not readily available, direct knowledge of more accurate form of $\nabla(R)$ is not possible at present. In absence of more about the direct knowledge on $\nabla(R)$, we decided to go for indirect determination of $\nabla(R)$ with the hope of gaining more insight into our theory. We performed another calculation using method IV which determines $\nabla(R)$ indirectly. We thus took a parametric form of $\nabla(R)$, with Woods-Saxon form factor for the real part and surface derivative for the imaginary part, a phenomenological $V_{ep}^{(d)}(R)$ obtained from the table of Ref. 12), and starting with $\nabla(R) = V_J(R)$, $\tilde{\epsilon} = \epsilon_0$, the parameters of $\nabla(R)$ were varied by an automatic 6-parameter search code so as to obtain a minimum $\chi^2$ for the angular distribution of a given stripping or pick-up reaction.
Fig. 2. Angular distributions of $^{208}\text{Pb}(p, d)$ $^{207}\text{Pb}$ for various final states of $^{209}\text{Pb}$ for 22 MeV incident proton. The spectroscopic factors for the states from top are 0.91, 0.79, 0.82, 0.52, 0.50 and 0.70, respectively.

Fig. 3. Angular distributions for $^{54}\text{Fe}(d, p)$ and $^{40}\text{Ca}(p, d)$ reactions.

We first applied this procedure to get a best fit to the angular distribution of pick-up reaction from $3P_{j=2}$ state of $^{208}\text{Pb}$ for 22 MeV protons. Since $V_{sd}(R)$ obtained phenomenologically is not unique and $\chi^2$ minimum could depend on the starting value of $\tilde{\epsilon} = \epsilon_0$, the search for $V(R)$ was repeated for different values of $\tilde{\epsilon}$ and different sets of $V_{sd}(R)$ of the table in Ref. 12), till we got lowest possible $\chi^2$. The best value of $V(R)$ thus obtained was then used to compute the angular distribution of pick up reactions from other states of $^{208}\text{Pb}$, and results are displayed in Fig. 2. The results of similar calculation for $(d, p)$ and $(p, d)$ reactions from $^{54}\text{Fe}$ and $^{40}\text{Ca}$, respectively, are shown in Fig. 3. The cut-off radius is taken to be zero in all cases.

We list below the following observations made from our calculations using Method IV:

1. Effect of $V_s(R)$: Table I gives the parameters of $V(R)$ along with those of $V_{sd}(R)$ used in adiabatic theory of Johnson et al. It shows that $V(R) \neq V_s(R)$ and that the effect of $V_s(R)$ is significant, as is also shown in Fig. 4.

2. Effect of $\tilde{\epsilon}$: A typical example of the dependence of angular distribution on $\tilde{\epsilon}$ is shown in Fig. 4. It shows clearly that $\tilde{\epsilon} = \epsilon_0$ is a poor approximation in
### Table I. Parameters for the deuteron optical potential ($V_{dp}^e$). $\mathcal{V}$, $V_J$ and the proton optical potential ($V_{op}^p$).

<table>
<thead>
<tr>
<th>nucleus</th>
<th>potential</th>
<th>$V_e$ (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_e$ (fm)</th>
<th>$W_0$ (MeV)</th>
<th>$r_e'$ (fm)</th>
<th>$a_e'$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{208}$Pb</td>
<td>$V_{dp}^e$</td>
<td>97.5</td>
<td>1.16</td>
<td>0.80</td>
<td>15.8</td>
<td>1.27</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{V}$</td>
<td>79.2</td>
<td>1.096</td>
<td>0.86</td>
<td>14.0</td>
<td>1.19</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>$V_J$</td>
<td>102.0</td>
<td>1.25</td>
<td>0.682</td>
<td>13.5</td>
<td>1.25</td>
<td>0.783</td>
</tr>
<tr>
<td></td>
<td>$V_{op}^p$</td>
<td>51.8</td>
<td>1.25</td>
<td>0.65</td>
<td>10.0</td>
<td>1.25</td>
<td>0.76</td>
</tr>
<tr>
<td>$^{56}$Fe</td>
<td>$V_{dp}^e$</td>
<td>93.9</td>
<td>1.107</td>
<td>0.756</td>
<td>18.02</td>
<td>1.275</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{V}$</td>
<td>97.0</td>
<td>0.979</td>
<td>1.14</td>
<td>27.0</td>
<td>1.33</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>$V_J$</td>
<td>97.0</td>
<td>1.25</td>
<td>0.69</td>
<td>20.0</td>
<td>1.25</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>$V_{op}^p$</td>
<td>41.3</td>
<td>1.285</td>
<td>0.652</td>
<td>12.1</td>
<td>1.25</td>
<td>0.48</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>$V_{dp}^e$</td>
<td>88.0</td>
<td>1.24</td>
<td>0.82</td>
<td>12.8</td>
<td>1.23</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{V}$</td>
<td>64.2</td>
<td>1.39</td>
<td>0.73</td>
<td>11.4</td>
<td>1.21</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>$V_J$</td>
<td>90.0</td>
<td>1.28</td>
<td>0.69</td>
<td>18.6</td>
<td>1.25</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>$V_{op}^p$</td>
<td>44.0</td>
<td>1.28</td>
<td>0.65</td>
<td>11.0</td>
<td>1.25</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Fig. 4. The upper curve shows the effect of $\tilde{\epsilon}$ on $3p_{1/2}$ state for $^{208}$Pb($p, d$) reaction. For the solid curve $\tilde{\epsilon}$=8.0 MeV and for the dashed curve $\tilde{\epsilon}=\epsilon_0$. The lower curve shows the effect of $V_g$ on $1h_{9/2}$ state with $\tilde{\epsilon}$=6 MeV. The solid curve is with $\mathcal{V}$ and the dashed curve is with $V_J$ given in Table I.

Our theory. In fact, best results are obtained when $\tilde{\epsilon}$ is about equal to half the centre-of-mass energy of deuteron, in all cases. It is perhaps better that way for $\tilde{\epsilon}$, being a substitute for $\hbar^2 q^2/m$ in range of integration in Eq. (2.39), is expected to be positive (by mean value theorem) and not equal to $\epsilon_0= -2.2$ MeV, and that the results are not insensitive to $\tilde{\epsilon}$ as assumed by Johnson et al.
Table II. Various sets of $V_{ep}$ (first row) and corresponding $\tilde{V}$ (second row) with minimum $\chi^2$.

<table>
<thead>
<tr>
<th>nucleus</th>
<th>set</th>
<th>$\nu_0$ (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_0$ (fm)</th>
<th>$W_0$ (MeV)</th>
<th>$r_0'$ (fm)</th>
<th>$a_0'$ (fm)</th>
<th>$\chi^2_{\text{min}}$</th>
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</thead>
<tbody>
<tr>
<td>$^{208}\text{Pb}$</td>
<td>(3$p_{1/2}$)</td>
<td>97.5</td>
<td>1.16</td>
<td>0.80</td>
<td>15.8</td>
<td>1.27</td>
<td>0.77</td>
<td>4.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>79.2</td>
<td>1.093</td>
<td>0.86</td>
<td>14.0</td>
<td>1.19</td>
<td>1.22</td>
<td>56.0</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>79.8</td>
<td>1.336</td>
<td>0.796</td>
<td>16.6</td>
<td>1.470</td>
<td>0.598</td>
<td>28.8</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>45.5</td>
<td>1.307</td>
<td>0.333</td>
<td>23.2</td>
<td>1.379</td>
<td>0.756</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>101.5</td>
<td>1.11</td>
<td>0.92</td>
<td>11.8</td>
<td>1.39</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>94.4</td>
<td>1.03</td>
<td>1.469</td>
<td>18.2</td>
<td>1.406</td>
<td>0.67</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>81.5</td>
<td>1.32</td>
<td>0.764</td>
<td>15.2</td>
<td>1.409</td>
<td>0.662</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>82.9</td>
<td>1.442</td>
<td>0.798</td>
<td>12.0</td>
<td>1.440</td>
<td>0.37</td>
<td>42.1</td>
</tr>
<tr>
<td>$^{44}\text{Fe}$</td>
<td>(2$p_{1/2}$)</td>
<td>93.9</td>
<td>1.107</td>
<td>0.756</td>
<td>18.02</td>
<td>1.275</td>
<td>0.73</td>
<td>6.54</td>
</tr>
<tr>
<td></td>
<td></td>
<td>97.0</td>
<td>0.979</td>
<td>1.14</td>
<td>27.0</td>
<td>1.33</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>99.1</td>
<td>1.112</td>
<td>0.798</td>
<td>19.03</td>
<td>1.325</td>
<td>0.675</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>60.3</td>
<td>1.264</td>
<td>0.651</td>
<td>13.9</td>
<td>1.430</td>
<td>0.542</td>
<td></td>
</tr>
</tbody>
</table>

(3) Ambiguity of $V_{ep}^{(d)}(R)$: Table II lists four different sets of $V_{ep}^{(d)}$ tried for $^{208}\text{Pb}$ along with corresponding $\tilde{V}(R)$ and $\chi^2_{\text{min}}$ obtained and two sets for $^{44}\text{Fe}$. It confirms the earlier observation with conventional DWBA theories\(^{(10)}\) that all $V_{ep}^{(d)}(R)$ are not good enough for $(d, p)$ process. The result is not surprising in view of the fact that the elastic scattering is dominated by a few surface partial waves, and that the phase shifts for lower partial waves are ill determined by elastic scattering, giving rise to ambiguity of $V_{ep}^{(d)}$. A good $(d, p)$ theory, which uses elastic scattering wavefunctions well inside the target nucleus, may be expected to make the distinction between different optical potentials.

(4) Suppression of radial integrals: The most important result of the adiabatic theory of Johnson et al.\(^{(9)}\) is the natural suppression of the radial integrals of the stripping matrix elements for lower partial waves ($L_p$) of proton or those ($L_d$) of deuteron and that was shown to be mainly possible because the radius of the real part of $\tilde{V}(R)$ was found to be larger than that of $V_{ep}^{(d)}(R)$. In our case, however, the radius of the real part of $\tilde{V}(R)$ is less than that of $V_{ep}^{(d)}(R)$, but contrary to the expectation from arguments of adiabatic theory, we still continue to get suppression of the radial integrals for lower partial waves of proton or deuteron. The mechanism of suppression of the radial integrals in our case is different from the one suggested in Ref. 1) and it appears to be due to the following reasons. Firstly, the wave number $K_B$ of $\tilde{B}(R)$ is different from that, $K_d$, of $\xi^{(+)}(R)$, since $\tilde{e} \neq \epsilon_0$. Thus one may expect the maximum modulation of outgoing wave of the form $[A(R)e^{iKdR}+B(R)e^{iK_BR}]$ coming from $f(R)$ to take place (unless $|A|$ is not very different from $|B|$) around $R=R_i$, where

$$
\frac{1}{2}(K_d-K_B) \cdot R_i = \pi/2.
$$

(3.4) For our calculation of $^{208}\text{Pb}$, we have $K_d=1.26$ fm$^{-1}$ and $K_B=0.79$ fm$^{-1}$ as obtained from the best fit to angular distribution to the $3p_{1/2}$ state, and this gives, from Eq.
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\[ R_1 = 6.7 \text{ fm} \] which is about equal to the radius of \(^{208}\text{Pb}\). Use of local wave numbers could change the over-simplified equation \((3\cdot4)\). Curiously enough, however, Eq. \((3\cdot4)\) is still approximately valid for our calculation of \(^{56}\text{Fe}\) and \(^{40}\text{Ca}\). Secondly, the breakup wavefunction \(\tilde{B}_i(R)\) around nuclear surface \((R=R_0)\) is almost out of phase with \(\tilde{\xi}_i(R)\), for \(l^{th}\) partial waves where \(l\leq l_c = K_c R_0\) the classical impact parameter, as shown in Fig. 5. Net effect is, when \((d,p)\) matrix element,

\[ \langle \chi_p\rangle F_n \mid \varphi_{np} \rangle \approx \langle \chi_p\rangle F_n \mid \varphi_{np} \rangle \tilde{\xi}_i \]

is expanded in the partial waves of proton \((L_p)\) and deuteron \((L_d)\) then for each pair of \((L_p, L_d)\) the contribution of break-up term \(\langle \chi_p\rangle F_n \mid \varphi_{np} \rangle \tilde{B}_i\) (real and imaginary part) is opposite in sign of the corresponding terms from DWBA matrix elements A typical example is given in Table III. However, this cancellation is either absent or very poor when \(\bar{\epsilon}=\epsilon_0\) and it may provide a direct justification of the earlier version of DWBA with finite cut-off radius.

It was further noted that there is a consistency in the pattern of results \((1)\) to \((4)\) above, in all our calculation using Method IV. For example, for the sets II, III and IV of Table II, the results \((1), (2)\) and \((4)\) for \(^{208}\text{Pb}\) are repeated even though the \(\chi_{min}^2\) for these calculations are fairly large compared to the best possible one and the situation is similar for other nuclei. We are thus inclined to believe that these results are too consistent to be accidental or characteristic of

---

**Table III.** Radial integrals (in arbitrary unit) with \(\xi(M_i)\) and \(\tilde{B}_i(M_i)\) for \(^{208}\text{Pb}\) \((p,d)\) in \(1h_{11/2}\) state. \(L_p\) and \(L_d\) are the proton and the deuteron partial wave numbers, respectively.

<table>
<thead>
<tr>
<th>(L_p)</th>
<th>(L_d)</th>
<th>real part</th>
<th>imaginary part</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(M_i)</td>
<td>(M_i)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(M_i)</td>
<td>(M_i)</td>
</tr>
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<td></td>
<td></td>
<td>(M_i)</td>
<td>(M_i)</td>
</tr>
<tr>
<td>0</td>
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<td>0.059</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>-0.236</td>
<td>0.185</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>-1.06</td>
<td>0.078</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>-0.692</td>
<td>0.059</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>0.203</td>
<td>0.023</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>0.365</td>
<td>-0.004</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>0.188</td>
<td>-0.009</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>0.055</td>
<td>-0.009</td>
</tr>
<tr>
<td>8</td>
<td>13</td>
<td>-0.035</td>
<td>-0.003</td>
</tr>
<tr>
<td>9</td>
<td>14</td>
<td>-0.055</td>
<td>-0.001</td>
</tr>
</tbody>
</table>
Method IV, rather than being the characteristic of our theory and that the results (1) to (4) will remain as predictions of our theory when stripping analysis is made with $\nabla(R)$ obtained directly.

Appendix I

Using the equations of Appendix I of Ref. 8), we may write Eq. (2·13) as

$$PG^{(+)}(P + Q)(V_1 + V_2 + \mathcal{J}V_c - Z_i)|\chi_i^{(+)}\rangle = G_{\phi_0}^P \cdot P\{ (V_1 + V_2 + \mathcal{J}V_c) + (V_1 + V_2 + \mathcal{J}V_c) \cdot G^{QQ} \times (V_1 + V_2 + \mathcal{J}V_c) - Z_i - PHQG^{QQ}Z_i |\chi_i^{(+)}\rangle = 0$$

for

$$Z_i = PM.$$  

Appendix II

For deuteron breakup by a target, the amplitude of $(d, pn)$ reaction is given by. using notation of this paper,

$$T(d, pn) = \langle \mathbf{K}_p, \mathbf{K}_n, \psi_0 | \psi_{np} + V_1 + V_2 + V_c(r_p) | \mathcal{T}_i^{(+)} \rangle,$$

(II·1)

where $\hbar \mathbf{K}_p$ and $\hbar \mathbf{K}_n$ are the momenta of the outgoing proton and neutron, whose relative and centre-of-mass momenta are given by $\hbar \mathbf{k}$ and $\hbar \mathbf{K}$, respectively. We rewrite Eq. (II·1), allowing final-state interaction generated by $V_{np}$ giving $|\chi_q\rangle$ and distorting the centre-of-mass motion by $V_c(R)$ with wavefunctions $\chi_c^{(-)}(K, R)$ as

$$T(d, pn) = \langle \mathbf{K}, \chi_c^{(-)}(K) \psi_0 | V_1 + V_2 + V_c(r_p) | \mathcal{T}_i^{(+)} \rangle.$$  

(II·2)

Now, using the definitions of $P_0|\mathcal{T}_i^{(+)}\rangle$, $\bar{\psi}$ and $\tau$ of the text, we have

$$\langle \phi_0 | V_1 + V_2 + V_c(r_p) | \mathcal{T}_i^{(+)} \rangle = \bar{\psi} |P_0 \mathcal{T}_i^{(+)}\rangle = \tau |\chi_0 \hat{\xi}^{(+)}\rangle$$

(II·3)

and we get, from Eqs. (II·2) and (II·3),

$$T(d, pn) = \langle \mathbf{K}, \chi_c^{(-)}(K) | \tau |\chi_0 \hat{\xi}^{(+)}\rangle.$$  

(II·4)

References

2) See for example, N. Austern, Direct Interactions (John Wiley & Sons, 1969).
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