Nonleptonic Decays of Charmed Mesons in the Quark Model

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Nonleptonic two-body decays of charmed mesons are discussed in the quark model assuming the Okubo-Zweig-Iizuka rule. Many sum rules are derived and the predicted decay rates of the most probable modes are about $10^{11}$ sec$^{-1}$. It is also discussed that the $I=1/2$ rule in $K \to 2\pi$ is related to the $\Delta S = 1$ dominance in $SU(4)$ but charmed meson decays do not reveal the dominance of a certain representation.

§ 1. Introduction

The discoveries of new particles, $\psi$'s or $J$, have stimulated a number of theoretical investigations. Among them, there are discussions of new quantum numbers such as color and charm which are natural extensions of the $SU(3)$ symmetry. The color was introduced in order to avoid the spin-statistics problem and fractional charges of quarks$^5$ and the discrepancy between theoretical and experimental values of $\pi^0 \to 2\gamma$ decay.$^9$ The charm was first introduced for the purpose of constructing a baryon octet from integrally charged particles$^9$ and also showing the lepton-baryon symmetry.$^{11}$

In recent years the concept of the charm quantum number has been reconsidered from the viewpoint of weak interactions$^9$ and a cosmic ray event observed by Niu, Mikumo and Maeda.$^{7,8}$ (Recently a few more events are seen.$^9$) Glashow, Iliopoulos and Maiani,$^6$ whose paper is referred to as GIM in this paper, have elegantly argued that charmed quarks are needed to explain the absence of the $|S|=1$ and $|Q|=0$ transitions in the first order of weak interaction. On the basis of this model, Gaillard and Lee$^{10}$ discussed the rare decay modes of $K$ mesons. Also Gaillard, Lee and Rosner$^{11}$ have made detailed discussions on the charmed particles.

Now we should investigate the properties of charmed particles which must exist if $SU(4)$ is true, but they have not been observed.$^{12}$ They will be produced by strong interactions followed by their decays into ordinary hadrons or leptons. We discuss, in this article, the nonleptonic two-body weak decays of charmed mesons. We deduce sum rules and predict the values of their decay rates. It is naturally considered that stable hadrons should decay mainly by nonleptonic modes...
and that pseudoscalar mesons are stable. As is well known, the nonleptonic decays of strange hadrons obey the $\Delta I = 1/2$ rule; the magnitude of the $\Delta I = 1/2$ amplitudes is larger than the magnitude of the $\Delta I = 3/2$ amplitudes by a factor of twenty. However, the mechanism of enhancement of the $\Delta I = 1/2$ amplitudes has not been understood so well.\textsuperscript{13} Our explanation is based on the quark model, which was done some years ago by the two of the present authors for hyperon decays.\textsuperscript{10} At that time they were able to explain the $\Delta I = 1/2$ rule and the Lee-Sugawara triangle relation for $s$-wave amplitudes, by introducing color symmetry. According to this method, the $\Delta I = 1/2$ enhancement is related to the dominant diagrams and consequently is not directly related to a certain irreducible representation such as $8$ ($15$ or $20''$ in case of $SU(4)$). The quark diagrammatical argument is very powerful in discussing strong interactions. We expect that this method will also give a good explanation for nonleptonic decays, because the enhancement of certain processes in nonleptonic decays is essentially the problem of dynamics of strong interactions.

In § 2 the model is given. In § 3 the parameters are determined using the experimental values of $K \rightarrow 2\pi$ decay rates, the decay rates of charmed mesons are estimated and sum rules are given. The final section, § 4, is devoted to discussion.

§ 2. Model and decay amplitudes

The fundamental particles of our model are the usual quarks, $u$, $d$ and $s$, and the charmed quark, $c$, and mesons are the bound states of $(q\bar{q})$. Weak interactions are mediated by the $W$ boson, which couples to the GIM weak currents\textsuperscript{6}

$$J = \bar{u}(d\cos\theta + s\sin\theta) + \bar{c}(s\cos\theta - d\sin\theta),$$

where $\theta$ is the Cabibbo angle. We assume the Okubo-Zweig-Iizuka (OZI) rule\textsuperscript{15} even for weak interaction processes. The OZI rule implies the invalidity of the dominance of a certain irreducible representation. The validity of the OZI rule for weak interaction is questionable, but characteristic features of nonleptonic decays reflect, more or less, the effect of strong interactions, so that the OZI rule should be taken into account.

Under this assumption diagrams indicated in Fig. 1 dominate in the decays of mesons. The decay amplitudes corresponding to the diagrams in Fig. 1 are given by

![Fig. 1. The dominant diagrams.](image-url)
Nonleptonic Decays of Charmed Mesons

(a) \[ P_n^{-m} P_{n}^{-i} P_{n}^{-k} - P_{n}^{-i} P_{n}^{-m} P_{n}^{-k} \],
(b1) \[ P_{n}^{-i} P_{n}^{-j} P_{n}^{-k} - P_{n}^{-i} P_{n}^{-j} P_{n}^{-k} \],
(b2) \[ P_{n}^{-i} P_{n}^{-j} P_{n}^{-k} - P_{n}^{-i} P_{n}^{-j} P_{n}^{-k} \],

where \( i \) should belong to the parent mesons (the lower set of quark lines). These forms guarantee the \( CP \) invariance, where \( P_{n}^{-m} \) is transformed into \( P_{n}^{-m} \) under charge conjugation. Because of the structure of GIM weak currents, the decays are classified into three groups:

1. \( |\mathcal{J}| = 0, |\Delta S| = 1 \)
2. \( |\mathcal{J}| = |\Delta S| = 1 \)
3. \( |\mathcal{J}| = 1, \Delta S = 0 \).

In the cases of (1) and (3) the coupling strengths are proportional to \( \cos \theta \sin \theta \) and in the case of (2) to \( \cos^2 \theta \) or \( \sin^2 \theta \).

It seems that the (a) diagrams in Fig. 1 have no contribution, at first sight, for the same reason as the suppression mechanism of strangeness changing neutral current. However, this is not the case. The most leading terms of these self-energy-like diagrams are \( G \ln(A^2/m_0^2) \) in a renormalizable gauge theory, and they are cancelled out due to the structure of GIM weak currents. Next leading terms do not vanish since \( SU(4) \) symmetry is strongly broken in the mass term as \( \eta \lambda_3 + \xi \lambda_{15} \). By this mass breaking we can guess the strengths of the two types of transitions belonging to (a), (s→d) and (c→u), which are represented in Fig. 2. Each diagram of Fig. 2 will be proportional to the difference between contributions from two types of intermediate quark lines, so the (s→d) transition has a factor \( (-m_c + m_u) \) and the (c→u) transition, \( (m_s - m_d) \). By this argument the (c→u) transition is suppressed compared with (s→d) by one order of magnitude, because \( SU(4) \) breaking \( \xi \) is larger by this order than \( SU(3) \) breaking \( \eta \), as usually considered.

It is important to notice that the diagrams of type (a) should be dominant if we apply this method to \( K \to 2\pi \) decays. The diagrams (a) apparently obey the \( \Delta I = 1/2 \) rule but the diagrams (b2) violate the \( \Delta I = 1/2 \) rule and (b1) and (b2) are to have the same strengths.

We can now get invariant amplitudes in terms of four parameters, \( A_C, A_S, B_1 \) and \( B_2 \) corresponding to each diagram. The results are given in Table I.

Next we discuss the reduction of the number of parameters. We naturally expect that \( B_1 \) and \( B_2 \) have the same magnitude except relative phase. Therefore we set

\[ B_1 = B_2 = -B_2. \]

The minus sign is expected from the argument of nonrelativistic perturbation: the energy denominator of each diagram has opposite sign when the state between weak and strong vertices is taken as the intermediate state (Fig. 3). This is also expected if we insert tensor representation of tensor spurion and mass breaking term.
Table 1. The $D'$s and the $F'$s are assigned as $D' = c\bar{d}$, $D' = c\bar{u}$ and $F' = c\bar{s}$, following Gaillard, Lee and Rosner, and in Refs. 8) and 20) they are labelled as $D' = L^-$, $D' = L^y$ and $F' = M^-$.

<table>
<thead>
<tr>
<th>(1)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(K^+ \rightarrow \pi^+ \pi^0) = \frac{1}{\sqrt{2}} (B_2 \sin \theta \cos \theta)$</td>
<td>$(F^{+} \rightarrow K^{*+} \pi^{0}) = \frac{1}{\sqrt{2}} [B_2 + A_c] \sin \theta \cos \theta$</td>
</tr>
<tr>
<td>$(K^+ \rightarrow \pi^+ \pi^0) = -\frac{1}{\sqrt{2}} (B_2 - A_s) \sin \theta \cos \theta$</td>
<td>$(F^{+} \rightarrow K^{*+} \pi^{0}) = A_c \sin \theta \cos \theta$</td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+ \pi^0 = -\frac{1}{\sqrt{2}} [B_2 - A_s] \sin \theta \cos \theta$</td>
<td>$F^{+} \rightarrow K^{*+} \pi^{0} = -A_c \sin \theta \cos \theta$</td>
</tr>
</tbody>
</table>

$*$ These modes are less important than the others, even if they can occur, because of the smallness of $Q$-values.

Though we may set $A_s = \frac{(-m_c + m_u)}{(m_s - m_d)} \cdot A_c$ from the argument already mentioned, we do not require it by the reason that quark masses are not so well determined. Then, there remain three parameters, one of which will be only known from charmed meson data.

We are left with some more discussion on the parameters. The parameters have dimension of mass. Whether these parameters are universal or not is not determined a priori, but we may take $G = m_r^0 G_0$, in which $G$ is the representative.
of $A$'s and $B$'s and $m_I$ is the mass of the initial meson. We have taken a choice of $n=2$. The reason is mainly due to the fact that these decays occur through mass breaking and that the mass breaking of mesons is quadratic in mass. It is realized by the following consideration: If we allow derivative coupling and let them couple to tensor spurion symmetrically, then each matrix element has the form

$$G_o(m_i^2 - m_f^2) P_1 P_2 P_3.$$ 

As a consequence of this form the decay width of $P_I$ becomes

$$\Gamma(P_I \to PP) = (m_f/m_k) \cot \theta \Gamma(K^+ \to \pi^+ \pi^0)$$

$$\approx 10^{11} \text{sec}^{-1}$$

by a rough estimation. This value is compared with leptonic or semileptonic decays, whose widths estimated by Gaillard, Lee and Rosner\textsuperscript{10} are $\Gamma_{\text{lept}} = 10^9 \sim 10^{10}$ sec$^{-1}$ and $\Gamma_{\text{semilept}} \approx 10^{11}$ sec$^{-1}$. The observed lifetimes of the $X$ particles in cosmic rays, which are regarded as candidates for charmed hadrons, are of the order of $10^{12} \sim 10^{13}$ sec$^{-1}$.

We might argue the case when $G$ itself is universal; then the widths will become

$$\Gamma(P_I \to PP) = (m_f/m_k) \cot \theta \Gamma(K^+ \to \pi^+ \pi^0)$$

$$= 10^8 \text{sec}^{-1}.$$ 

When the mass dimension of $G$ is factored out by $m_f$,

$$\Gamma(P_I \to PP) = (m_f/m_k) \cot \theta \Gamma(K^+ \to \pi^+ \pi^0)$$

$$= 10^{10} \text{sec}^{-1}.$$ 

In both cases the main modes are semileptonic decays.

§ 3. Sum rules and decay rates

First we give sum rules which are easily derived from Table I. The relations among the amplitudes with the same initial masses are obtained as follows:

$$\langle F^+ \to K^0 K^+ \rangle = - \left[ \sqrt{\frac{3}{2}} \langle F^+ \to K^+ \eta \rangle + \frac{1}{\sqrt{2}} \langle F^+ \to K^+ \pi^0 \rangle \right] \cot \theta,$$

$$\sqrt{2} \langle F^+ \to K^0 \pi^+ \rangle - 3 \langle F^+ \to K^+ \eta^0 \rangle - \sqrt{3} \langle F^+ \to K^+ \eta \rangle = 0,$$

$$(D^+ \to K^0 \pi^+) \tan \theta = \sqrt{2} \langle D^+ \to \pi^+ \pi^0 \rangle$$

$$= \sqrt{\frac{2}{3}} \left[ \sqrt{2} \langle D^+ \to K^0 K^+ \rangle - \sqrt{3} \langle D^+ \to \pi^+ \pi^+ \rangle \right],$$

$$(D^0 \to K^- \pi^+) \tan \theta = \frac{1}{2\sqrt{2}} \left[ (D^0 \to K^+ K^-) - (D^0 \to \pi^+ \pi^-) \right],$$

$$\sqrt{6} \langle D^0 \to \pi^0 \eta \rangle - 3 \langle D^0 \to \eta \pi^0 \rangle - (D^0 \to \pi^0 \pi^0) = 0.$$
\[
2\sqrt{2} (D^+ \to K^+ \bar{K}^0) = (D_i^0 \to K^+ K^-) + (D_i^0 \to \pi^+ \pi^-) \\
= \frac{3}{\sqrt{2}} [(D_i^0 \to \pi^+ \pi^-) + (D_i^0 \to \eta \eta)], \\
\sqrt{2} (D_i^0 \to K_L^0 \pi^0) = \sqrt{6} (D_i^0 \to K_L^0 \eta) = -\sqrt{2} (D_i^0 \to K_S^0 \pi^0) \cos 2\theta \\
= -\sqrt{6} (D_i^0 \to K_L^0 \eta) \cos 2\theta \\
= -\frac{\cos 2\theta}{2 \sin \theta \cos \theta} [(D^0 \to K^- \pi^+) - (D^+ \to \bar{K}^0 \pi^+)] \\
= -\frac{\cos 2\theta}{2 \sin \theta \cos \theta} \left[ -\frac{1}{2} (D_i^0 \to \pi^+ \pi^-) + \sqrt{3} (D_i^0 \to \pi^0 \eta) \right].
\]

These are obtained from Table I by eliminating \(B_1, B_2\) and \(A_c\). If we make use of \(B_i = -B_i\), then we have more relations, for example

\[
(D^0 \to K^- \pi^-) + (D^+ \to \bar{K}^0 \pi^+) = 0.
\]

To relate the amplitudes with different initial masses, we assume \(G = (m_1)^2 G_0\). By using this we can obtain another relation connecting \(F^{-}\) and \(D\)-meson decays through \(A_c\), and also get several of decay rates through \(B, F\) and \(D\) decays are related as

\[
(F^{-} \to \bar{K}^0 \pi^+) = (m_F/m_D)^2 (D^+ \to K^+ \bar{K}^0).
\]

In order to estimate the decay rates we need the explicit value of \(B\) which is deduced from experimental data of \(\Gamma (K \to 2\pi)\). The comparison must be done with the invariant amplitudes \(\mathcal{M}\), which are defined by using phase volume factor \(\phi, \quad \Gamma (I \to ab) = \phi_{I \to ab} |\mathcal{M}|^2, \phi_{I \to ab} = \frac{8\pi m_I^2}{p}, \)

where \(p\) is the momentum of daughter mesons in the center-of-mass system. Let us set \(B\) as real and positive, then we get\(^{30}\)

\[
B(K) = m_K^2 B_0 = \frac{\phi_{K^{-} \to \pi^-}}{\sin \theta \cos \theta} (5.84 \times 10^3) \text{ (GeV/sec)}^{1/2}, \]

\[
A_S(K) = m_K^2 A_0 = \frac{\phi_{K^{-} \to \pi^-}}{\sin \theta \cos \theta} e^{i\chi} (6.56 \times 10^3) \text{ (GeV/sec)}^{1/2}, \]

\[
\chi = 53.4^\circ.
\]

The decay rates are represented in Table II. \(\Gamma_0\), used in it, is defined as

\[
\Gamma_0 (I \to ab) = 2 \frac{\phi_{K^{-} \to \pi^-}}{\phi_{I \to ab}} \left( \frac{m_I}{m_K} \right)^4 \Gamma (K^+ \to \pi^+ \pi^-),
\]

and its dependence upon \(m_I\) is shown in Fig. 4. As an example the numerical values corresponding to \(m_F = 2.2\text{ GeV}/c^2\) and \(m_D = 2.1\text{ GeV}/c^2\) are also shown in Table II.
Table II. The decay widths of $D$ and $F$ mesons. $\Gamma_0$ is defined in the text. There are also numerical values, for example, in the case of $m_F=2.2$ GeV and $m_D=2.1$ GeV. The $m_f$ dependence of $\Gamma_0$ is shown in Fig. 4.

<table>
<thead>
<tr>
<th>$\Gamma$(mode)</th>
<th>Width expressed by $\Gamma_0$</th>
<th>Numerical value in the case of $m_F=2.2$ GeV/c$^2$ and $m_D=2.1$ GeV/c$^2$ ($\text{sec}^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma(F^+ \rightarrow K^0 K^-)$</td>
<td>$\Gamma_0 \cot \theta$</td>
<td>5.2 x $10^9$</td>
</tr>
<tr>
<td>$\Gamma(F^+ \rightarrow K_s K^+)$</td>
<td>$\Gamma_0 \tan \theta$</td>
<td>1.98 x $10^9$</td>
</tr>
<tr>
<td>$\Gamma(F \rightarrow KK)$</td>
<td>$\Gamma_0$</td>
<td>3.2 x $10^9$</td>
</tr>
<tr>
<td>$\Gamma(D^+ \rightarrow K^+ \pi^0)$</td>
<td>$\Gamma_0 \cot \theta \frac{1}{\sin^2 \theta}$</td>
<td>1.07 x $10^{11}$</td>
</tr>
<tr>
<td>$\Gamma(D^0 \rightarrow K^- \pi^+$)</td>
<td>$\Gamma_0 \tan \theta \frac{\cos^2 \theta - \sin^2 \theta}{\sin^2 \theta \cos \theta}$</td>
<td>9.5 x $10^{11}$</td>
</tr>
<tr>
<td>$\Gamma(D^+ \rightarrow K^+ \pi^-)$</td>
<td>$\Gamma_0 \tan \theta$</td>
<td>1.81 x $10^9$</td>
</tr>
<tr>
<td>$\Gamma(D^0 \rightarrow K^+ \pi^+)$</td>
<td>$\Gamma_0 (D \rightarrow K \pi)$</td>
<td>2.9 x $10^9$</td>
</tr>
<tr>
<td>$\Gamma(D_s^0 \rightarrow K^0 \eta)$</td>
<td>$\frac{2}{3} \Gamma_0 \frac{1}{\sin^2 \theta}$</td>
<td>3.3 x $10^{10}$</td>
</tr>
<tr>
<td>$\Gamma(D_s^+ \rightarrow K^+ \eta)$</td>
<td>$\frac{2}{3} \Gamma_0 \frac{\cos^2 \theta - \sin^2 \theta}{\sin^2 \theta \cos \theta}$</td>
<td>2.9 x $10^{10}$</td>
</tr>
</tbody>
</table>

We have argued that $A_c$ has comparable order of magnitude with $B$. It, however, appears necessarily with $\sin \theta \cos \theta$, then this parameter $A_c$ is not relevant to see the most probable decays that are proportional to 1 or $\cos^2 \theta$.

\section{Discussion}

There is a striking distinction between our method and group theoretical arguments of the dominance of a certain irreducible representation. Altarelli, Cabibbo and Maiani\cite{27} have pointed out that GIM current structure contains only 84 and
but not 15, so that the enhanced representation corresponding to 8 enhancement in SU(3) must be 20". Diagrammatically, 20" is realized by adding another terms related to Fig. 5, with equal weight but opposite sign, to type (b) diagrams. They are omitted in our model because of the OZI rule.

Fig. 5. Omitted diagrams, which are necessary if 20" is dominant.

If unitary symmetry is exact, however, $P\to PP$ decays cannot occur by CP conservation, so the symmetry breaking should be incorporated in order for the decays to occur. Then 20" dominance is not necessarily true. Indeed, Iwasaki has proposed a method of perturbation in symmetry breaking and realized effective 15 dominance. Therefore in spite of Altarelli et al.'s remark there might remain significance to consider 15 dominance. The 15 dominance is expressed by type (a) diagrams. However, in our model 15 and 20" equally contribute and further 15 is involved, because of symmetry breaking. Moreover the $I=1/2$ rule in $K\to 2\pi$ decays belong to 15 representation in SU(4), but charmed particle decays have not a certain dominant representation but certain dominant diagrams. Which scheme is the best should be tested by future experiments.

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