The design of miniature wideband seismometers

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Summary. Conventional seismometers employ masses of several kilogrammes suspended with periods of several seconds, but it is possible to achieve the same detection capability with much smaller masses suspended at shorter periods. Such instruments are valuable for borehole applications or where many instruments must be rapidly set up.

The problems of the design of miniature wideband force-feedback seismometers are discussed and two such instruments are described. Both instruments use a capacitive displacement transducer to detect the relative motion of a mass of about 0.05 kg suspended with a natural period near 1 s. A force-feedback system maintains the mass stationary with respect to the instrument frame and the instruments have a response defined by feedback from DC to 10 Hz. A single miniature instrument can thus provide data over the whole of the seismic range.

Details are given of the experimental difficulties encountered and of a comparison of the instruments with conventional seismometers.

1 Introduction

A seismometer can most simply be considered to be a transducer between input acceleration $\ddot{x}_i$ and mass position $x_r$ relative to the instrument frame. The transfer function for a mass $M$, suspended by a spring of compliance $C$, with natural undamped angular frequency $\omega_0$ and damping ratio $\xi$ is

$$x_r = \frac{1}{\ddot{x}_i \frac{1}{s^2 + 2\xi\omega_0s + \omega_0^2}}$$

where $s$ is the Laplace operator,

$$\omega_0^2 = \frac{1}{MC}, \quad \xi = \frac{1}{2Q}$$
and $Q$ is the quality factor of the suspension. The device behaves as an accelerometer for applied accelerations at angular frequencies $\omega < \omega_0$, and as a displacement meter for $\omega > \omega_0$.

The fundamental limit to the detection of earth motion by a seismometer is set by the Brownian motion of the suspended mass. It can be shown (Usher 1973) that the noise-equivalent acceleration $(\dot{x})_{ne}$ in a bandwidth $\Delta f$ is given by

$$ (\dot{x})_{ne}^2 = \frac{4RkT\Delta f}{M^2} = \frac{8\pi kT\Delta f}{MT_0Q} $$

where $R$ is the viscous damping resistance and $T_0$ the natural period. It is most important to note that $(\dot{x})_{ne}$ does not directly involve the natural period of the suspension, depending only on mass and damping.

The range of frequencies of ground motion of most interest in seismology is usually considered to be from 0.01 to about 10 Hz. Conventional long-period seismometers employ masses of the order 10 kg at natural periods near 20 s; conventional short-period instruments have masses of about 1 kg and natural periods near 1 s. Both instruments normally employ velocity transducers (electromagnetic types) and operate near the peak of the resonance curve, though the response may be shaped by subsequent filtering.

A wideband instrument must operate over the whole of the above range, preferably producing an output flat to velocity or acceleration. This can be achieved most conveniently by the use of negative force-feedback, in which the mass is maintained in position relative to the supports by a feedback force proportional to the relative displacement of the mass. Such feedback increases the natural frequency by a factor equal to the square root of the loop gain. The response to acceleration is flat from DC nearly to the new resonant frequency, being determined by the force-transducer in the feedback path (provided the loop gain is sufficient). The effective damping ratio or step-response can be determined by the loop parameters, but does not affect the Brownian noise level.

The use of an AC-excited displacement transducer, instead of an electromagnetic transducer (as in a conventional instrument), greatly reduces the amplifier noise by removing the problem of $1/f$ noise at low frequencies. Good DC amplifiers may have a noise figure of at least 10 at 1 Hz, increasing almost linearly with decreasing frequency, whereas an AC amplifier can have a noise figure close to unity in its operating range.

2 Design of miniature seismometers

2.1 Spectrum of Earth motion and choice of mass

Fig. 1 (after Fix, 1972) shows an acceleration power density spectrum of minimum Earth motion in the period range from 0.1 to 100 s. Also included are Brownian noise-equivalent accelerations for various natural periods and $Q$ factors and a theoretical curve for a widely-used long-period seismometer system (Geotech S11 with a mass of 10 kg and natural period 20 s, with a standard preamplifier unit, after Melton 1976). In order to be able to detect the minimum observable noise over the above period range, the mass size must be chosen such that the Brownian noise-equivalent acceleration is sufficiently small, and it can be seen that a value of $3 \times 10^{-10}$ ms$^{-2}$/Hz$^{1/2}$ is satisfactory. To achieve this Brownian noise-equivalent acceleration the required relation is $M_0TQ \sim 1$ so that for a mass of 0.1 kg, $Q = 10$ at $T_0 = 1$ s, $Q = 100$ at $T_0 = 0.1$ s or $Q = 1$ at 10 s would all be satisfactory. A mass of only 0.01 kg would require $Q \sim 100$ and $T_0 \sim 1$ s or $Q \sim 10$ and $T_0 \sim 10$ s.
2.2 SIGNAL AND NOISE LEVELS

The transducer/amplifier noise must clearly be made less than the equivalent Brownian noise and this can be done by increasing the signal (increasing the period of the suspension and the responsivity of the transducer) and minimizing the noise.

Linear electrical displacement transducers have a responsivity \( r \) of the form \( r = V_e/d \), where \( V_e \) is the excitation voltage and \( d \) the linear range. For a given \( V_e \), \( r \) can only be increased by decreasing the range. Capacitive variable-separation transducers are very satisfactory in this respect, having much higher responsivity than say an LVDT, which is essentially a larger range device. In addition, they are basically noiseless and have been widely used for precise measurement (Jones & Richards 1973). A responsivity of 3000 V/m can be achieved easily with a plate separation of 0.3 mm and excitation of 1 V rms. Higher responsivities can be achieved by increasing \( V_e \) or decreasing \( d \), but the practical problems (mechanical precision, adjustment, air damping and electrostatic forces) become more severe, and the above value appears to be an optimum for this application.

A noise-equivalent circuit of a capacitive transducer feeding a charge amplifier is shown in Fig. 2. The arrangement is a Blumlein bridge, and has the particular advantage that stray capacitances do not affect the balance point since they appear in parallel with the amplifier input. Moreover, the low input impedance of a charge amplifier means that the signal level is not attenuated by stray capacitances and that pick-up problems are minimized. It can be
shown that the series noise-equivalent resistance \( R_n \) is given by

\[
R_n = \frac{R_{nv}}{\left( \frac{C_s + C_e}{C_s} \right)^2 + \frac{1}{\omega^2 C_s^2 R_{e}^2}} + \frac{1}{\omega^2 C_s^2} \left( \frac{1}{R_{ni}} + \frac{1}{R_{e}} \right)
\]

where \( C_s \) is the transducer capacitance, \( C_e \) the parallel stray capacitance, \( R_e \) the effective parallel resistance (bias and strays), and \( R_{nv} \) and \( R_{ni} \) are the noise equivalent resistances of the active device. Choosing \( R_{nv} = 1 \, \text{k} \Omega \) and \( R_{ni} = 10 \, \text{M} \Omega \) for an FET input device, \( C_s \approx C_e = 20 \, \text{pF} \) and \( R_e = 5 \, \text{M} \Omega \), \( R_n \) has a value of about 4 k\( \Omega \) at the optimum excitation frequency of about 100 kHz. The corresponding noise-equivalent acceleration is shown in Fig. 1, curve B.

At long periods the noise-equivalent input acceleration is

\[
(\dot{x})_{ne}^2 = \frac{4kTR_n \Delta f}{(1/\omega_0^2)^2 r^2}
\]

and this must be made less than or equal to the Brownian noise-equivalent input acceleration. Using the Brownian limit chosen above \((3 \times 10^{-10} \, \text{ms}^{-2}/(\text{Hz})^{1/2})\), we find that a natural frequency of less than 1.8 Hz is required, so that a mass of say 0.1 kg suspended with a \( Q \) of 20 and a natural period of about 0.5 s should be satisfactory.

2.3 Practical Considerations

The above analysis is intended as a guide to the magnitudes of mass, period, \( Q \), responsivity etc. rather than a demonstration that a precise combination of parameters must be used. A number of assumptions have had to be made, such as the Brownian noise limit required, responsivity and amplifier noise levels etc. However, it is clear that if we take a mass of 0.1 kg as indicating a miniature seismometer, the natural period will have to be of the order 1 s and the \( Q \) factor of the order 10.
In order to obtain a response basically flat to acceleration over the seismic range, it is possible to use a natural period of about 1 s with a loop gain of 100, giving a closed-loop response from DC to 10 Hz (with proportional control). Alternatively, by using a suitable feedback network one can obtain virtually any desired response, provided the loop gain is sufficiently high over the range of interest to make the response truly determined by feedback. For example, a response flat to acceleration from DC to, say, 1 Hz with a response flat to velocity from 1 to 10 Hz can easily be obtained with a natural period of 1 s. The desired response for a practical application will be determined by filters outside the loop in any case, but it is preferable that the response be accurately defined on closed-loop. The first alternative simplifies the external filters whereas the second eases the constraints on natural period.

In order to obtain a natural period of say 0.5 s or more, one can use either a simply-supported arrangement or a period-lengthening mechanism (La Coste, swinging-gate etc.). Taking 10 cm as the maximum linear dimension of a 'miniature' instrument, a pendulum of this length (or a spring with this static deflection) will have a period of about 0.6 s, or just within the requirement. Amplifier/transducer noise requirements can be eased by increasing the natural period, but only at the expense of the complication of a period-lengthening mechanism. It is not easy to make miniature high Q suspensions of long period, due to the restoring force of the pivots and the precision required in adjustment of mass position, and the advantage of increased DC gain is very evenly balanced by the increased mechanical complexity.

The main practical problems in miniature seismometers arise from long-term creep, thermal and pressure effects in the mechanical system, and air movements. A change in mass position due to long-term creep is not important in a feedback instrument, provided its range is not exceeded. The effect is similarly not important in an open-loop velocity-transducer instrument, provided it does not hit the end stops. However, since the stress in the spring of a miniature instrument may be as high as that in a conventional instrument, careful mechanical design and choice of spring material are very important. We have not noted any problems due to creep, even after several months of continuous operation.

Thermal effects due to ambient temperature changes are more serious, but the larger temperature fluctuations have a period well outside the seismic range of interest. Mechanical systems must clearly be designed kinematically and as symmetrically as possible to reduce thermal gradients, but provided the electronics are not overdriven the effect is relatively unimportant. We have experienced little difficulty due to thermal effects, and have mostly operated our instruments in a vault having daily variations of about ±2°C but with no additional thermal control.

Pressure changes may be very serious, since they may include the seismic range of frequencies. Miniature instruments are normally enclosed in an evacuated jacket for thermal and pressure control and to obtain a suitably high Q factor. A given change in mass position is of course much more serious with a short natural period than with a long natural period (in the ratio of periods squared), but the effect is offset to some extent by the smaller mass and stresses in a miniature instrument. Comparing a conventional 10 kg/20 s instrument with a 0.1 kg/1 s device, the mass ratio is similar to that of the periods squared, so pressure effects may be comparable. A miniature instrument with a longer period would have advantages in terms of the effect of mass movement, but would be more susceptible to changes in effective length due to pressure distortions. A rigid base-plate and evacuated jacket are essential in a miniature instrument; conventional instruments normally have a sealed jacket, though it is not usually evacuated.

The effect of air movements is important in both conventional and miniature
seismometers. A given air current may be assumed to produce a force on the mass proportional to the area of the mass and an equivalent acceleration inversely proportional to the mass. For the example above, assuming that the effective area is proportional to \(M^{2/3}\), the ratio of effective accelerations will be about 5:1. In practice, however, the mass of a miniature device is usually in the form of a paddle, making the ratio much larger (say 50:1) and evacuation is essential to reduce the effect to a tolerable level. It is inconvenient to reduce the pressure to less than about one torr, but we have found this to be satisfactory.

It can be seen from the above that miniature seismometers require careful mechanical design because very small relative displacements must be detected, but that the main problems are often eased simply because a smaller mass (and correspondingly smaller stresses) are involved. Close thermal control and a rigid evacuated jacket are clearly essential if the detection capability of a miniature device is to be achieved in practice, but although some of the advantages of overall size and weight are thus lost, the instrument can still be considerably smaller and more robust than conventional instruments.

3 Experimental seismometers

3.1 Horizontal-component instrument

A miniature horizontal-component instrument is shown in Fig. 3 and has been described in detail by Usher, Buckner & Burch (1977). It employs a mass of about 0.04 kg in an inverted pendulum arrangement of length 5 cm with supporting strips of \(N\)-span \(D\), giving a natural period of 1 s and a \(Q\) factor of 20. The simple spring strips shown were used, rather than cross-spring pivots, because of simplicity of mounting and availability. The capacitive transducer had a responsivity of \(10^4\) V/m making the amplifier noise \((\xi)_{ne} \sim 5 \times 10^{-11}\) ms\(^{-2}\) per \(\sqrt{\text{Hz}}\) less than the theoretical Brownian noise \((3 \times 10^{-10}\) ms\(^{-2}\) per \(\sqrt{\text{Hz}}\)). The feedback was arranged to make the response flat to acceleration from DC to 0.35 Hz, and flat to velocity from 0.35 Hz to the unity loop-gain frequency of 100 Hz.

The instrument was enclosed in a thermally controlled cylinder and in an evacuated jacket (at pressure 1 torr) making the overall dimensions 20 cm high \(\times\) 12 cm diameter. It was compared with a standard long-period seismometer (Geotech S12 with type 610 pre-amplifier) for several months in the Blacknest Seismological Centre’s vault at Wolverton, Berkshire. Comparisons were made in various frequency bands, though the most useful was a long-period narrow-band response centred on 20 s (LPNB). The coherence observed in this band was initially poor, but was much improved by the use of an invar base-plate. Fig. 4 shows a typical comparison of the LPNB outputs for a distant small event.

The rms difference between the two instruments was generally close to the theoretical Brownian noise level, but occasional long-period fluctuations of three or four times the theoretical level occurred. These are thought to be due to residual thermal and pressure effects on the mechanical system of the miniature instrument.

3.2 Vertical-component instrument

A miniature vertical-component instrument is shown in Fig. 5. It employs a mass of 0.025 kg with a boom length of 5 cm, and employs cross-spring pivots and a coiled supporting spring in a configuration suggested by Dr P. L. Willmore (private communication). In order to obtain a natural period of about 0.6 s the spring extension must be about 10 cm; considerable space can be saved by using the method shown, which does not suffer from the same defect as helical springs in La Coste suspensions as movement perpendicular to the spring axis does not affect mass position to a first order. The same arrangement can be used
Figure 3. Photograph of a miniature horizontal-component instrument. A mass of about 0.04 kg is employed in an inverted pendulum arrangement, with a natural period of about 1 s.
in any orientation by adjusting the spring tension, so that it could be operated as a horizontal-component instrument if required. It proved to be difficult to obtain a longer natural period than 0.6 s as the spring tends to buckle if 'wound-up' more than one complete turn.

The electronic details are similar to those for the horizontal-component instrument, though the feedback has been arranged to produce a response flat to acceleration from DC to the closed-loop resonant frequency at 20 Hz, with a loop gain of 100 at 1 Hz.
The instrument was enclosed in the same type of evacuated jacket as the horizontal-component instrument, and has been compared at Wolverton with a standard S 11 vertical-component seismometer. The results suggest that the theoretical noise level is correct but that serious thermal and mechanical effects are present. The prototype instrument is being modified to improve its behaviour—at present it is not thermally controlled and employs a simple mild-steel helical spring.

4 Conclusions

The principles of design of miniature wideband seismometers have been discussed and it has been shown that it is feasible to produce a device employing a mass of less than 0.1 kg with a feedback-controlled response from DC to 10 Hz or more.

Practical difficulties with such devices arise from pressure and thermal effects in the mechanical system, and from air currents, and can be at least partly eliminated by careful design, thermal control and evacuation. It appears that the smallest mass that can be used in practice is largely determined by these effects, and that a mass of about 0.1 kg may be preferable to those used in the instruments described. We have obtained results comparing very favourably with the Geotech instrument in the LPNB range with a different vertical-component instrument with a mass of about this value, which is at present being developed and tested in collaboration with AWRE.

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References