Violation of $s$-Channel Helicity Conservation
and Composite Structure of Hadrons

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It is shown that the $s$-channel helicity conserving quark-quark scattering amplitudes lead to the $s$-channel helicity non-conserving $\pi N$ scattering amplitudes and give a definite value to the ratio: $F^{+}/(F^{+}/\sqrt{-t/s}) = -2/3$.

§ 1. Introduction

$s$-channel helicity conservation (SHC) for Pomeron exchanging amplitude was discovered empirically for $p p \rightarrow p_{t} p$ reaction, and was suggested to hold also in the case of $\pi N$ elastic scattering. Later analysis on $\pi N$ scattering amplitudes, however, showed that the ratio, $|F^{+}|/(|F^{+}|\sqrt{-t/s})$, which may be considered as a barometer of the violation of SHC for $I_{t}=0$ exchanging amplitudes, was found to be small but significantly different from zero and its energy dependence was rather weak. As an origin of this non-vanishing $F^{+}$, contributions from $P'$ trajectory are usually assumed but this possibility seems quite puzzling, since the experimental analysis shows that the relative phase between helicity flip and non-flip amplitudes is very close to $180^\circ$.

In this paper we would like to show the possibility that the violation of SHC in $\pi N$ scattering mainly comes from the composite structure of hadrons. It is shown that the $s$-channel helicity conserving quark-quark scattering amplitudes inevitably lead to the violation of SHC in $\pi N$ scattering and that the correct ratio between the helicity flip and non-flip amplitudes including their relative phase can be obtained.

In § 2, the general formalism for the construction of $\pi N$ scattering amplitudes from quark-quark scattering amplitudes is presented and comparison with experimental results is made. In the last section, we discuss the consequences of our results and further developments.

§ 2. Calculation of scattering amplitudes

As a composite model of hadrons we adopt the quark model, i.e., mesons are composed of a quark and an anti-quark and baryons are composed of three quarks. We also consider quark-quark scattering as scattering of two spin one-half particles.

The quark-quark scattering amplitude $t_{qq}(k_{i}, k_{f}, \sigma_{i}, \sigma_{f})$ can be expressed in
a general form,

\[ t_{q, q'}(k_i, k_f, \sigma_i, \sigma_f) = a(k_i, k_f) + ib(k_i, k_f) (\sigma_i + \sigma_f) \cdot \hat{n} \]

\[ + c(k_i, k_f) (\sigma_i \cdot \hat{n}) (\sigma_f \cdot \hat{n}) + \frac{d(k_i, k_f)}{2} (\sigma_i \cdot \hat{K}) (\sigma_f \cdot \hat{K}) \]

where

\[ q = k_i - k_f, \quad \hat{q} = q/|q|, \]

\[ K = k_i + k_f, \quad \hat{K} = K/|K|, \]

\[ \hat{n} = \hat{q} \times \hat{K} \]

in which \( k_i \) and \( k_f \) are the momenta of incident and scattered particles in the center-of-mass system respectively; \( \sigma_i \) and \( \sigma_f \) are the Pauli spin operators for the particles.

In a previous paper\(^5\) we have postulated that the eigenphase shifts corresponding to the four scattering spin states are identical with each other for the same values of the total angular momentum and then obtained the following relations.

\[ a = i s \int_0^\infty \tilde{\rho}_c J_0(\sqrt{-t} b) b db, \quad (2.2a) \]

\[ b = - i s \int_0^\infty \tilde{\rho}_c J_1(\sqrt{-t} b) b db, \quad (2.2b) \]

\[ c = i s \int_0^\infty \tilde{\rho}_c \left[ J_0(\sqrt{-t} b) - J_2(\sqrt{-t} b) \right] b db, \quad (2.2c) \]

\[ d = i s \int_0^\infty \tilde{\rho}_c \left[ J_0(\sqrt{-t} b) + J_2(\sqrt{-t} b) \right] b db, \quad (2.2d) \]

\[ e = 0 \quad (2.2e) \]

in which

\[ \tilde{\rho}_c = 1 - \exp \{ i \chi_c(b) \} \cos^2 \chi_c(b), \quad (2.3a) \]

\[ \tilde{\rho}_s = \exp \{ i \chi_s(b) \} \sin \chi_s(b) \cos \chi_s(b), \quad (2.3b) \]

\[ \tilde{\rho}_n = \exp \{ i \chi_n(b) \} \sin^2 \chi_n(b), \quad (2.3c) \]

where \( s \) and \( -t \) are the total energy of the system squared and the transferred momentum squared in the center-of-mass system, and \( \chi_c \) and \( \chi_s \) are the two kinds of eikonals.

Considering that \( \pi N \) scattering is scattering of spin zero and one-half particles, we can write the scattering amplitude \( F_{\pi N} \) in a general form,

\[ F_{\pi N}(s, t) = f(s, t) + i \sigma \cdot \hat{n} g(s, t), \quad (2.4) \]

where \( \sigma \) is the Pauli spin operator for nucleons.
Violation of s-Channel Helicity Conservation

Since we are dealing only with Pomeron exchanging amplitudes, the quark-quark scattering amplitudes can be expressed through one set of eikonal \( \tilde{\rho}_c, \tilde{\rho}_s \) and \( \tilde{\rho}_R \). Therefore we also obtain \( \pi N \) scattering amplitude \( F^\alpha_{\pi N} \) in terms of these two eikonal, i.e., in terms of the profile functions \( \tilde{\rho}_c, \tilde{\rho}_s \) and \( \tilde{\rho}_R \) according to the recipe developed in the previous paper. Thus,

\[
\begin{align*}
  f^\alpha(s, t) &= i s \int_0^\infty \tilde{f}^\alpha(s, b) J_0(\sqrt{-t} b) b db, \\
  g^\alpha(s, t) &= i s \int_0^\infty \tilde{g}^\alpha(s, b) J_1(\sqrt{-t} b) b db,
\end{align*}
\]  

where

\[
\begin{align*}
  \tilde{f}^\alpha(s, b) &= 1 - (1 - \tilde{\rho}_c + \tilde{\rho}_R)^2(1 - \tilde{\rho}_c - \tilde{\rho}_R), \\
  \tilde{g}^\alpha(s, b) &= 2(1 - \tilde{\rho}_c + \tilde{\rho}_R)^2 \tilde{\rho}_s.
\end{align*}
\]

Now we postulate that SHC holds in quark-quark scattering, which requires the relation

\[
b = \sqrt{-t/s} \quad (2.7)
\]

for \( |t| \ll s \), or equivalently in the form of the profile functions,

\[
\tilde{\rho}_c(s, b) = -\sqrt{\frac{1}{s}} \frac{\partial \tilde{\rho}_c(s, b)}{\partial b}. \quad (2.8)
\]

From Eqs. (2.6) and (2.8) we get

\[
\tilde{g}^\alpha(s, b) = -\frac{1}{3} \frac{\partial \tilde{f}^\alpha(s, b)}{\partial b}, \quad (2.9)
\]

i.e.,

\[
g^\alpha(s, t) = \frac{1}{3} \sqrt{\frac{-t}{s}} f^\alpha(s, t), \quad (2.10)
\]

Neglecting the quantity of order \((-t/s)\) we can calculate the s-channel helicity flip and non-flip amplitudes for \( \pi N \) scattering as follows:

\[
F^\alpha_{\pi N} = f^\alpha(s, t), \quad (2.11a)
\]

\[
F^\alpha_{\pi N} = -\sqrt{\frac{-t}{s}} f^\alpha(s, t) + g^\alpha(s, t) = -\frac{2}{3} \sqrt{\frac{-t}{s}} f^\alpha(s, t). \quad (2.11b)
\]

From Eqs. (2.11a) and (2.11b) we have a very simple relation,

\[
F^\alpha_{\pi N} / \left( F^\alpha_{\pi N} \sqrt{\frac{-t}{s}} \right) = -\frac{2}{3}. \quad (2.12)
\]

Comparisons with existing data are shown in Fig. 1.
§ 3. Discussion

It is very interesting to note that the agreements with experimental data seem to indicate the quark behaves as a “real” spin one-half Dirac particle not only as a scattering center nor a carrier of quantum numbers. And we note that the relation (2·12) also holds for the case of single quark-quark scattering, i.e., the effects of multiple scattering do not alter the relation as we have derived in the preceding section.

As for the ordinary Reggeon exchanging amplitudes the situations are rather troublesome. This is partly because the multiple scattering effects or the absorption effects are not negligible in this case.

It is easy to see that from our derivation of the relation (2·12) we can expect the violation of SHC even in the case of $\gamma p \rightarrow \phi p$, if the measurements on target proton’s polarization are made. And this would be a crucial test for our model.

References