Double-Layer Model Analysis of Elastic Scattering of Complex Nuclei

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Heavy ion elastic scattering is analyzed in terms of the double-layer model which gives emphasis on the separation of the interaction region between complex nuclei into two parts: a surface region described by an optical potential and an absorptive interior region which is replaced by a boundary condition at the boundary of the two regions $R'$. In this model, lower partial waves can be modified by the boundary condition while higher ones are determined mainly by the optical potential in the surface region. A backward rise with a prominent oscillation in the angular distribution is well reproduced due to the effective contribution of the lower partial waves. The inner limit of the surface region $R'$ can be well determined and the logarithmic derivative of an incoming wave $-iK R'$ is confirmed to be an appropriate boundary condition.

§ 1. Introduction

A backward rise with a prominent oscillation is a characteristic feature in the angular distribution of heavy ion elastic scattering above the Coulomb barrier. It is difficult to reproduce such feature in the framework of the conventional optical model (OM). The OM analysis of the forward angular distribution in heavy ion scattering, however, gave the evidence that only the potential edge is important and scattering is mostly determined by the tail of the optical potential as can be seen in the work of Igo for elastic scattering of alpha particles. On the contrary, it is well known that there exist ambiguities in the optical potential in the small distance. It has been mentioned that when two complex nuclei overlap appreciably, one can no longer consider the interaction potential to be describable as merely a function of the separation of their centers.

From the above facts, it is reasonable to assume that the optical potential in the surface region is established as an appropriate description of the interaction between complex nuclei. On the other hand, since lower partial waves have a fairly large amplitude inside the nucleus, they are sensitive to the feature of the interaction in the interior region.

The discrepancy between OM predictions and the observed angular distributions at backward angles suggests, therefore, that the treatment of the lower partial waves is insufficient in OM. Since the scattering amplitudes of the higher partial waves are assumed to be well defined by the surface potential, the aim of this paper is to investigate the interior interaction through the behaviour of the
lower partial wave amplitudes determined from the experimental backward angular distribution.

In order to clarify the relation between the behavior of the lower partial waves and the backward angle data, the double-layer model (DLM) analysis is carried out. In DLM, the interaction region is separated into a surface region described by an optical potential and an absorptive interior one. The interaction in the interior region is replaced by an appropriate set of boundary condition at the boundary of the two regions. Only the scattering amplitudes of the lower partial waves are affected significantly by the boundary condition. The backward angle data determine these scattering amplitudes and thus the boundary condition.

Through the present analysis, the validity of the incoming wave boundary condition is confirmed and the inner limit of the surface region is defined in the various scattering systems. That is to say, the range of the interaction in the interior region is determined.

Many attempts have been made to interpret heavy ion scattering. On the basis of the core exchange model, the method of a linear combination of nuclear orbitals and the coupled channel method (CCM) have given satisfactory fits to the backward angular distribution. In these models, the information from the backward angle data is incorporated in the exchange integrals or kernels in the coupled equations. In DLM, the relation between the backward angle data and the interaction in the interior region is revealed through the behavior of the lower partial waves.

In the next section, the necessity of a treatment different from OM for the lower partial waves is discussed. The prescription of DLM is outlined in § 3 and the results of the DLM analysis are shown in § 4. The concluding remarks and discussion are summarized in the last section.

### § 2. The importance of lower partial wave amplitudes

In the case of elastic scattering between non-identical heavy ions above the Coulomb barrier, the discrepancy between experimental angular distributions and conventional OM predictions is observed generally at $\theta_{\text{c.m.}} \geq 90^\circ$. We intend to explain the discrepancy by means of a proper treatment of the lower partial waves.

Except for the particular scattering system like $^{16}\text{O}+^{12}\text{C}$ consisting of tightly bound nuclei, a general characteristic of the experimental angular distributions at large angles lies in a prominent oscillation between $10^{-1}$ and $10 \text{ mb}/\text{st}$. These features are illustrated in Fig. 1 for the data of some typical scattering systems. The angular distribution of the scattering system like $^{16}\text{O}+^{12}\text{C}$ shows a sharp diffractive backward rise. To reproduce such backward rise, an even-odd difference in the higher partial waves is considered to be necessary. Since, however, we are interested in the behavior of the lower partial waves, we restrict ourselves to angular distributions with the same trend as those of the data shown in Fig. 1.
The role of the lower partial waves is remarkable for the general trend of the angular distribution shown in Fig. 1. In scattering of $^{16}$O by $^{18}$O at $E_{\text{lab}}=32$ MeV, the effect due to an additional constant $s$-wave amplitude is examined in Fig. 2. When an artificial $s$-wave amplitude is added, a rise in the angular distribution happens at the large angles. In the actual case, several additional lower partial wave scattering amplitudes are required for the fit to the backward angle data. This implies that for heavy ion scattering it is important to introduce a proper interaction in the interior region which is different from the complex potential in OM.
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§ 3. The outline of the DLM analysis

In DLM, the emphasis is laid on the remarkable effects on the backward angular distribution induced by the interaction in the interior region. A projectile is supposed to be decaying there almost completely due to the complicated interaction. This interaction is taken into account implicitly by setting up an appropriate boundary condition for the radial part of the relative wave function between colliding nuclei at \( r = R' \). \( R' \) is assumed to be much less than the contact distance. In the region where \( r > R' \), it is also assumed in DLM that an optical potential description of the interaction is appropriate.

Although we have no definite information in the interior region at the present
time, it is possible to set phenomenologically a suitable boundary condition which is considered to represent the interaction in the interior region. In the present analysis, the logarithmic derivative of an incoming wave $-iKR'$ is used as the boundary condition at $R'$ for every partial wave, where $K$ is the wave number of the incoming wave at $R'$. The choice of this boundary condition is made from the consideration of the continuum theory and the situation that the incident heavy ion loses its identity almost completely in the overlap region.

A similar incoming wave boundary condition (IWB) has been used to obtain elastic $\alpha$-Ni and d-Cu scattering cross sections by Rawitcher. In that work, he mentions that the difference of the phase shift between OM and IWB is remarkable for the lower partial waves, but he does not emphasize the role of the lower partial waves in the cross section. On the contrary, DLM pays attention to the relation between the lower partial waves and the backward angular distribution.

Let $l_{\text{DLM}}$ be defined as a maximum orbital angular momentum among the partial waves affected significantly by the boundary condition at $R'$. It is clear that $l_{\text{DLM}}$ becomes small as $R'$ approaches to zero. The scattering amplitudes of the partial waves lower than $l_{\text{DLM}}$ play the important role in giving the backward rise of the angular distribution. The validity and systematics of this boundary condition should be examined by analyzing the experimental data.

The scattering amplitude can be obtained by solving the Schrödinger equation numerically in the surface region $R' \leq r \leq R_M$, where $R_M$ is a matching point. The starting value of the wave function is given by the boundary condition. At $R_M$ the continuity condition is imposed upon the solved wave function and the Coulomb one. Throughout the present analysis, use is made of the four-parameter Woods-Saxon type potential with shallow depth. To obtain the best fit to the experimental data, $R'$ is searched in the surface region with an interval of 0.1 fm and $K$ is varied from 0.5 fm$^{-1}$ to 2.0 fm$^{-1}$ with a step of 0.1 fm$^{-1}$.

§ 4. Results of the DLM analysis

In order to determine the parameters of the boundary condition, the backward data in the angular distribution are necessary. Furthermore, the data over a wide range of incident energies and in the various scattering systems are required for investigating the systematics of the parameters. In this paper, the scattering data of the following systems are analyzed: $^{16}\text{O} + ^{18}\text{O} (E_{\text{lab}} = 24 \sim 60 \text{ MeV}), ^{16}\text{O} + ^{28}\text{Mg} (E_{\text{lab}} = 30 \sim 55 \text{ MeV}), ^{6}\text{Li} + ^{18}\text{O} (E_{\text{lab}} = 29.8 \text{ MeV}), ^{6}\text{Li} + ^{40}\text{Ca} (E_{\text{lab}} = 30 \text{ MeV}), ^{16}\text{O} + ^{19}\text{F} (E_{\text{lab}} = 32 \text{ MeV})$ and $^{16}\text{O} + ^{18}\text{O} (E_{\text{lab}} = 33 \text{ MeV})$.

4.1. Scattering of $^{16}\text{O}$ by $^{16}\text{O}$

At first, the parameters $R'$ and $K$ are determined to reproduce the respective angular distributions at $E_{\text{cm}} = 16.9, 19.8, 21.1, 23.7, 26.4$ and $29.0 \text{ MeV}$. The angular distributions calculated from DLM are shown in Fig. 3(a) with OM-limit.
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Fig. 3(b). The absolute value of the scattering amplitude $\eta$. These graphs correspond to the DLM and the OM-limit angular distributions of the $^{16}$O+$^{16}$O elastic scattering at $E_{\text{c.m.}} = 19.8, 21.1, 23.7, 26.4$ and 29.0 MeV, which are shown in Fig. 3(a). The points calculated from DLM are joined by straight lines and the amplitudes in the OM-limit are depicted by dotted lines. The difference between DLM and OM-limit is remarkable in the lower $l$.

Fig. 3(a). Scattering of $^{16}$O by $^{16}$O. The solid line shows the DLM prediction and the dotted line is the OM-limit curve.

curves, where the OM-limit curve is calculated from DLM with $R' = 0.1$ fm and is almost the same as OM calculation. The parameters of the optical potential used in the surface region and the value of $R'$ and $K$ are listed in Table I. The geometrical factors of the optical potential $r_0$ and $a$ are the same as the ones used in Ref. 12), but the depth of the potential is slightly different. As can be seen from Fig. 3(a), the backward rise in DLM (at $\theta_{\text{c.m.}}>100^\circ$) is prominent in comparison with the OM-limit curve. Figure 3(b) shows that these effects are produced by the lower partial wave scattering amplitudes calculated from DLM. Al-
Table I. Optical potential parameters used in the surface region and parameters of the boundary condition. The optical potential is the conventional Woods-Saxon type and is defined as \((V+iW) [1+\exp((r-R_0)/a)]^{-1}\) for \(r \geq R'\), where \(R_0 = \rho_0(A_\gamma^{1/3}+A_\gamma^{1/3})\). \(A_\gamma\) and \(A_\gamma\) are the mass of target and incident nucleus respectively. The ratio of \(R'\) to \(R_0\) is also listed for the estimation of the surface nature. The parameter \(r_\gamma\) for the Coulomb potential is 1.4 fm throughout the analysis.

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<th>(V(\text{MeV}))</th>
<th>(W(\text{MeV}))</th>
<th>(r_\gamma(\text{fm}))</th>
<th>(a(\text{fm}))</th>
<th>(R'_{\text{odd}}(\text{fm}))</th>
<th>(K(\text{fm}^{-1}))</th>
<th>(R_0(\text{fm}))</th>
<th>(R'/R_0)</th>
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<td>19.0</td>
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<td>6.1</td>
<td>1.7</td>
<td>7.14</td>
<td>0.85</td>
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though there are no available data at \(\theta_{\text{c.m.}} > 110^\circ\) for the incident energies \(E_{\text{c.m.}} \geq 21.1\) MeV, the DLM calculation is carried out by taking into account the feature around \(\theta_{\text{c.m.}} = 100^\circ\) and the general trend of the angular distribution discussed in § 2. In the result, the average values of \(R'\) and \(K\) are determined to be 4.9 fm and 0.7 fm\(^{-1}\) respectively and will be used later for the DLM calculation of the excitation function. This value of \(R'\) is 71\% of \(R_0\) where the strength of the potential becomes one half.

The DLM analysis for the data near the Coulomb barrier is shown in Fig. 4. On the data at \(E_{\text{c.m.}} = 12.7\) MeV which is the vicinity of the Coulomb barrier, the different values of \(R'\) are required for even and odd partial waves to reproduce the backward oscillation and are written as \(R'_{\text{even}}\) and \(R'_{\text{odd}}\) in Table I respectively. It is clear that at the respective boundaries the potential for odd partial waves is deeper than that for even ones in this case. This situation produces similar effect on the even-odd difference introduced by the exchange integral in the molecular two-state approximation.\(^{17}\) In DLM, the even-odd effect can be taken into account by using the respective value of \(R'\) for the even and the odd partial waves without introducing a parity dependent potential. This effect is necessary to reproduce a prominent oscillation for the data in the vicinity of the Coulomb barrier.

For comparison, the calculated result in terms of CCM\(^9\) is depicted in Fig. 4.
Fig. 4. Scattering of $^{16}$O by $^{18}$O with the incident energy near the Coulomb barrier. The best fit in DLM is shown by the solid line and the result of CCM by the dotted line.

by the dotted line. Both DLM and CCM give similar predictions up to $\theta_{\text{c.m.}}=150^\circ$ which fit well to the data. DLM, however, cannot produce a sharp diffractive backward rise beyond $\theta_{\text{c.m.}}=160^\circ$ as in CCM.

The effect of the boundary condition on the scattering amplitude $\gamma_i$ is remarkable and illustrated in Figs. 5 and 6. In DLM, since the resonance feature disappears for the partial waves lower than $l_{\text{DLM}}$, the difference of the argument of $\gamma_i$ and $\gamma_{i+1}$ is small and each point can be connected smoothly as shown in the figures. On the contrary, $\gamma_i$ in OM represents a jagged appearance as in Fig. 5. This is similar to the fact discussed in IWB calculation. Furthermore, $|\gamma_i|$ of DLM is slightly larger than that of OM for $l<7$.

As shown in Fig. 6, if the value of $K$ decreases, $|\gamma_i|$ becomes large for the lower $l$ due to the large reflection at the boundary $R'$. However, it is to be noted that when the value of $K$ increases beyond 1.5 fm$^{-1}$, $|\gamma_i|$ also becomes large for the lower $l$. Thus, the parameter $K$ controls the effect of absorption of the lower partial waves. On the other hand, the argument of $\gamma_i$ for the lower partial
Fig. 5. Comparison of the partial wave scattering amplitude in DLM (points) with that in OM (open circles) for scattering of $^{16}$O by $^{18}$O.

Fig. 6. The parameter dependence of the partial wave scattering amplitude in DLM for scattering of $^{16}$O by $^{18}$O.
wave is shifted as a whole according to the variation of $R'$.

To see the sensitivity of the angular distributions on the variation of $R'$ and $K$, the DLM calculation is carried out by using the parameters varied slightly around the best fit ones and is shown in Fig. 7. As the phase of the scattering function of the lower partial waves is shifted as a whole by the variation of $R'$, the change is brought about in the phase of the oscillation only in the backward angular distribution as shown in the lower part of Fig. 7. Although a similar phase of the oscillation appears in the variation of $R'$, $R'$ can be determined in connection with $K$ from the backward angle data. As can be seen from the upper part of Fig. 7, the backward rise is controlled by the parameter $K$ but the phase of the angular distribution remains unchanged. $K$ is the parameter which determines the absolute value of the cross section at the backward angles.

The systematic nature of $R'$ and $K$ for the wide range of the incident energies is examined by the analysis of the excitation function at $\theta_{c.m.}=70^\circ$, $80^\circ$, $90^\circ$ and $100^\circ$. The best fits are shown in Fig. 8, where use is made of $R'=4.9$ fm but the parameter $K$ is assumed to have an energy dependence and is determined in connection with $W$. The energy dependence of $W$ and $K$ is illustrated by the
curves (b) and (c) in Fig. 9 for the best fit. The parameter $K$ is not necessarily a function of $E_{\text{c.m.}}$ and expected to be almost constant beyond $E_{\text{c.m.}} = 35$ MeV. The well depth $W$, however, has to depend quadratically on $E_{\text{c.m.}}$. If we use the curve (a) in Fig. 9 for $R'$, almost the same results as the ones shown in Fig. 8 can be obtained by setting $K$ to be $0.8 \text{ fm}^{-1}$ over the energy range. It is found that $K$ and $R'$ have a mild dependence on $E_{\text{c.m.}}$ above 15 MeV.

4-2. Scattering of $^{16}\text{O}$ by $^{26}\text{Mg}$

Figure 10 shows the DLM prediction of the angular distribution at $E_{\text{lab}} = 45$ and 50 MeV. The excitation function in the energy range between 30 and 55 MeV in the laboratory system is analyzed using the parameters adopted in the calculation of the angular distribution and the results are shown in Fig. 11. The parameters of the optical potential are determined to reproduce the forward angle data. The same argument as in 4-1 can be applied to this case. In Fig. 10, the calculated cross section at backward angles is larger by two orders of magnitude.
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Fig. 10. Scattering of \(^{16}\)O by \(^{25}\)Mg. The parameters used in the calculation are listed in Table I.

than the OM-limit curve. The DLM prediction shows the general trend of the angular distribution discussed in § 2. In Fig. 11, \(W\) has a linear dependence on \(E_{\text{lab}}\), as the energy range of the data is limited. The data beyond \(E_{\text{lab}} = 45\) MeV are well reproduced by the parameters similar to the ones used in the scattering of \(^{16}\)O by \(^{30}\)O while the oscillation below 45 MeV is averaged out.

4-3. Scattering of \(^{6}\)Li by \(^{16}\)O and \(^{40}\)Ca

It has been reported that a six-parameter optical model calculation is necessary to obtain a good fit for the angular distribution of the scattering of \(^{6}\)Li by \(^{16}\)O and \(^{40}\)Ca at \(E_{\text{lab}} = 30\) MeV.\(^{10}\) An important feature of this optical potential is a deep real well depth and a long range of the imaginary part compared with the real one. In this paper, the DLM analysis with four-parameter optical potential in the surface region is made. As DLM has two parameters for the boundary condition, the number of the parameters used in the calculation is six and the fits are
good as shown in Fig. 12. Although the effect due to a deep real potential on the scattering amplitude is not clarified, a similar effect is induced by the prescription of DLM. The optical potential used in this calculation is the same as the four-parameter potential in Ref. 14 except for the well depth of the imaginary part.

4-4. Scattering of $^{16}$O by $^{19}$F and $^{19}$F by $^{16}$O

On the data of the scattering of $^{16}$O by $^{19}$F at 32 MeV in the lab system, the analysis in terms of the core-exchange model$^{19}$ and the parity dependent optical potential model$^{20}$ has been reported. Both models reproduce the features of the backward rise fairly well. The result from DLM is shown in Fig. 13 by the solid line and for comparison the prediction of the Kyoto group$^{21}$ is depicted by the dotted line. DLM reproduces the data very well beyond $\theta_{\text{c.m.}}=100^\circ$ both in the phase of the oscillation and the peak-to-valley ratio. The parameters used in the DLM calculation are listed in Table I. The optical potential is determined so as to reproduce the forward angle data. A large value of $K$ is required to fit the

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Fig. 12. Scattering of $^6$Li by $^{16}$O and $^{40}$Ca. The parameters used in the calculations are listed in Table I.

Fig. 13. Scattering of $^{16}$O by $^{19}$F and $^{19}$F by $^{16}$O. The parameters used in the calculations are listed in Table I. The weak imaginary depth is required in general near the Coulomb barrier.
data which is a common feature near the Coulomb barrier. Thus the reflection of the lower partial wave from the vicinity of the boundary $R'$ is considered to be large and important for the backward rise.

Figure 13 also shows the DLM calculation for scattering of $^{19}$F by $^{18}$O at $E_{\text{lab}} = 33 \text{ MeV}$ and the calculation by means of the molecular two-state approximation is depicted by the dotted line for comparison. DLM gives good agreement with the data while the molecular two-state approximation in general becomes inadequate at higher energies.

Finally, we notice that $R'$ corresponds to the inner limit of the peripheral region of the collision process discussed by Satchler. The outer limit of that region has been estimated by the distance of the closest approach of the classical orbit $D_c$. In Table II, the value of $D_c$ corresponding to the orbit $l_{1/2}$ is listed up for the scattering systems in this work, where $l_{1/2}$ is defined as the angular momentum which satisfies $|\gamma_1|=1/2$. As can be read from the Table, the width of the peripheral region is estimated to be 2~3 fm. This region is considered to be a direct coupling region with other channels.

Table II. The width of the peripheral region $\delta$. $l_{1/2}$ is defined for which $|\gamma_1|=1/2$ and the distance of the closest approach of the classical orbit $D_c$ is calculated from $D_c=n(1+cosec(\theta/2))/k, \cot(\theta/2)=l_{1/2}/n$, where $n=mZ_1 Z_2 e^2/(\hbar k)$, $m=m_1 m_2/(m_1+m_2)$ and $\theta$ is the scattering angle of the orbit $l_{1/2}$. $k$ is a wave number in c.m. and the suffixes $I$ and $T$ imply "incident" and "target" respectively. $R_0$ and $R'$ have the same meaning as in Table I.

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<th>$E_{\text{lab}}$ (MeV)</th>
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<th>$R_0$ (fm)</th>
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<td>6.94</td>
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<td>1.14</td>
<td>2.91</td>
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<tr>
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<td>1.21</td>
<td>3.55</td>
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<td>0.72</td>
<td>1.20</td>
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<td>12.6</td>
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<td>5.23</td>
<td>0.80</td>
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§ 5. Summary and concluding remarks

In the present analysis of heavy ion elastic scattering, DLM is adopted to effect the modification of the lower partial waves which are necessary for the backward angle data. The logarithmic derivative of an incoming wave $-iKR'$ is
assumed as the boundary condition at $R'$ for every partial wave. This choice of the boundary condition is based on the physical consideration that the incident nucleus loses its identity almost completely in the interior region $r < R'$ and so the amplitude of the outgoing wave to the entrance channel is very small at $R'$. In the surface region $r \geq R'$, however, a shallow Woods-Saxon type optical potential is adopted as an appropriate description of the interaction. In spite of the simple prescription, DLM can reproduce the data at the backward angle very well in the various scattering systems above the Coulomb barrier and is found to be a useful model for searching the interaction in the interior region.

The inner limit of the surface region $R'$ is determined to be $0.76 R_0$ on average in the scattering systems listed in Table I. This means that the optical potential inside of $0.76 R_0$ should be reconsidered and the interaction in the interior is effective for the feature of elastic scattering cross sections at backward angles. The lower partial wave amplitudes calculated from DLM with the incoming wave boundary condition are found to be essential for the backward rise in the cross section of heavy ion scattering with which this paper is concerned. In DLM, the sudden change in the interaction potential at $R'$ is also important to the reflection of the lower partial waves. It is also to be noted that the width of the peripheral region obtained in the present analysis is slightly larger than the estimation by Satchler ($\sim 2$ fm).

The parameter $K$ is determined to be $0.7 \sim 0.8$ fm$^{-1}$ in the various scattering systems and over the wide range of the incident energies except in the vicinity of the Coulomb barrier. When the incident energy approaches the Coulomb barrier, the large value of $K$ is required in order to reflect strongly the lower partial waves. Whereas the difference between the local wave number $k$ at $R'$ and the value of $K$ is different for each $l$ due to the assumption that $K$ is independent of $l$, these situations are considered to produce the effective reflection of the lower partial waves.

Furthermore, as can be seen from the analysis of the excitation function, the parameters $R'$ and $K$ are not so sensitive to the incident energy and do not depend so severely on the scattering systems. Thus the systematic treatment of the data is possible. A theoretical estimation of $R'$ and $K$ is an interesting problem but in the present stage it is one of the future problems which should be solved in connection with the reaction phenomena other than elastic scattering.

Acknowledgements

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Double-Layer Model Analysis

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