From rupture to structure: the physical character of flocs

D.H. Bache
Department of Civil Engineering, University of Strathclyde, Glasgow, G4 ONG, UK

Abstract The paper focuses on the physical character of flocs. This is developed through analysis of the impact of hydrodynamic stress on floc size. Theory is developed on the basis of an energy criterion which balances the turbulent kinetic energy against the energy expenditure associated with rupture. For turbulence, the kinetic energy per unit volume is modelled by the velocity scale \((v\varepsilon)^{1/4}\) and the \(d/\eta\) ratio in which \(v\), \(\varepsilon\), \(\eta\) refer to the kinematic viscosity, the rate of energy dissipation per unit mass and the Kolmogorov length respectively. The distance scale, \(d\), is equivalent to the maximum floc size. In its most rudimentary form, floc structure is based on the model \(S \propto k\phi/d^3\) in which \(k\) is the number of bonds broken and \(\phi\) the potential energy expenditure per bond broken. With appropriate development, this transforms to \(S = S_0 (d/d_o)^{D-3}\) in which \(d_o\) is the primary particle size, \(D\) – the fractal dimension, and \(S_0\) – a scaling factor controlling the mechanical strength. From the energy criterion, analytical expressions are derived for \(d\) in the form \(d = \gamma \varepsilon^{-m/2}\) in which \(\gamma\) and \(m\) are constants. Beside the proposal of a model for \(S\), a valuable advance is the development of rupture theory for the continuous domain of \(d/\eta\) consistent with the Kolmogorov description of turbulence. Theory is compared with a number of published data sets in which there exists knowledge of parameters \(\gamma\), \(m\) and \(D\). The paper demonstrates how the model can be used as an analytical tool for dissecting the factors such as raw water quality or floc aids which control \(S_o\).

Keywords Flocs; fractal; Kolmogorov; strength; structure

Introduction

Particles in the natural environment, as well as those encountered in the controlled environment of a water treatment plant often exist in aggregated form. The character of a floc depends not only on the nature of the raw materials from which it is formed, but also on a complex sequence of factors which come into play during its formation. The physical character of flocs is reflected in their size, density and strength. Such properties have a marked influence on the efficiency of solid–liquid separation processes. Flocs behave as fractal structures, wherein the solids mass \(m \propto d^D\), in which \(d\) is the size scale (say diameter) and \(D\) the fractal dimension (1 \(\leq D \leq 3\)). For example, the integrated solids volume concentration may be expressed by

\[
\phi = A'(d/d_o)^{D-3}
\]

in which \(A'\) is a packing factor and \(d_o\) is the diameter of the primary particle from which the flocs are formed. Similarly, the effective density is characterised by the relationship

\[
\rho_e = A'(\rho_s - \rho_w)(d/d_o)^{D-3}
\]

in which \(\rho_s\) and \(\rho_w\) refer to the density of primary particles and water respectively. The specification of strength is uncertain both in terms of representation and evaluation. Apart from the recent emergence of micro-mechanical techniques for measuring strength e.g. Yeung and Pelton (1996), most estimates of strength stem from studies of rupture promoted by hydrodynamic stress. In its simplest terms, rupture occurs when the applied forces exceed the binding forces. Hence by evaluating the prevailing rupture forces, one may determine the strength. For laminar conditions, a good example of this approach is provided in Blaser...
In turbulent flows, it is commonly observed that the maximum floc size behaves in accord with the empirical relationship

\[ d_{\text{max}} = C G^{-m} \]  

in which \( G \) is the average velocity gradient. The index \( m \) lies in the range 0 and 2 and parameter \( C \) is interpreted as a strength constant. Strength is reflected in two ways. In the first instance it is reflected in the magnitude of \( C \) i.e. the larger the value of \( C \) for a fixed \( G \), the stronger the floc. Second, it is implicit in the \( d_{\text{max}}-G \) dependence wherein \( d_{\text{max}} \) is reduced as \( G \) increases. There have been various attempts to explain the form of Eq. (3) e.g. Parker et al. (1972), but none has been satisfactory. Two principal factors have to be tackled. The first is to provide a model to describe floc strength. The second is to specify the rupture forces.

Addressing the structural factors, Sonntag and Russell (1987) suggested that the elastic properties (shear and bulk modulus) might comply with a scaling of the form \( \sigma = \sigma_0 \phi r \), in which \( \sigma \) refers to a strength modulus (units, Pa) with \( \sigma_0 \) as a scaling factor and \( r \) a coefficient. With \( \phi \) defined by Eq. (1) and implicit in Eq. (2), it suggests that strength and density are intimately tied – a feature recognised in Tambo and Hozumi (1979).

Concerning the rupture forces, all analyses tend to employ the logic described in Levich (1962). This makes use of Kolmogorov’s description of the fine structure of turbulence. In this theory (Kolmogorov, 1941), a critical scaling parameter is the Kolmogorov length defined by

\[ \eta = (\nu^3/\varepsilon)^{1/4} \]  

in which \( \nu \) is the kinematic viscosity and \( \varepsilon \) the local rate of energy dissipation per unit mass. The theory describes the properties of turbulence in the viscous domain \((d << \eta)\) and in the inertial domain \((L >> d >> \eta)\) in which \( L \) is a macroscale. It is from this base that the hydrodynamic rupture forces can be defined. Data in Table 1 shows that the breakage of the largest flocs occurs in the range \( 0.9 < d/\eta < 14 \), a domain in which breakage theory is acutely lacking.

To circumvent this problem and other difficulties, Bache et al. (1999b) examined the rupture process in energy terms. The Kolmogorov framework permits the specification of the turbulent kinetic energy existing at different length scales. For example at the length scale or spacing, \( d \), corresponding to the floc size, the term \(<\Delta u(d)^2>\) scales the turbulent kinetic energy. The symbols \(<...>\) mean a locally defined average value. When a floc is ruptured, it may be envisaged that a number of bonds \( k \) are stretched and broken, and that work is expended in overcoming the various frictional components during distortion. To simplify analysis, it was assumed that \( \phi \) units of energy are associated with each bond during the rupture process; thus the product \( k\phi \) may be regarded as a measure of the amount of potential energy to be overcome. With these concepts, Bache et al. (1999b) proposed a criterion for rupture based on an energy balance of the form

\[ \frac{1}{2} \rho_w <\Delta u(d)^2> \sim S \]  

where \( S = 6k\phi / \pi d^3 \)  

The left hand side of criterion (5) specifies the locally available turbulent kinetic energy per unit volume. The term \( S \) defines the energy expenditure \( (k\phi) \) within the floc volume \( (=\pi d^3/6) \). The dimensions of criterion (5) are equivalent to units of pressure.
However, it is more appropriate to interpret the criterion in energy terms, i.e. energy per unit volume.

In this paper the foundation work described in Bache et al. (1999b) is elaborated. It tackles the problem of specifying rupture in the continuous domain of $d/\eta$ and develops a model of floc structure. It also illustrates the use of the framework as a diagnostic tool.

**Turbulence description**

Referring to criterion (5), an immediate task is to define $\langle \Delta u(l)^2 \rangle$. Kolmogorov hypothesised that the small scale motions of turbulence (defined by the velocity difference $\Delta u(l)$ over the distance $l$) were uncorrelated with the large scale and that their statistics e.g. $\langle (\Delta u)^2 \rangle$ must be universal, provided these motions are defined in terms of relative velocity. The statistics of the relative velocity are defined by structure functions i.e.

$$G_p(l) = \langle \Delta u(l)^p \rangle = \langle (u(x + l) - u(x))^p \rangle$$

where $p$ represents the order of the structure function. The Kolmogorov theory requires that

$$G_p(l) = \nu_K \beta_p \langle \eta \rangle / l$$

in which $\beta_p$ is a universal function and $\nu_K$ is the Kolmogorov velocity scale defined by $\nu_K = \langle \varepsilon \rangle^{1/4}$. For the case under consideration ($p = 2$), the theory yields

$$\beta_2 = C_2 (l/\langle \eta \rangle)^{2/3} \text{ when } L > > l >> \eta$$

$$\beta_2 = C'_2 (l/\langle \eta \rangle)^2 \text{ when } l << \eta$$

The terms $C_2$ and $C'_2$ are universal constants. From this brief resume, it is seen that $G_2(l)$ is known at the extremes of $l/\eta$, the quest being to define $\beta_2$ in the intervening domain. In Batchelor (1947), it was argued that $\beta_2$ must be an even function and could be represented by a series approximation of the form

$$\beta_2 = a l^2/\langle \eta \rangle^2 + b l^4/\langle \eta \rangle^4$$

with $a \equiv C'_2 = 1/15$ and $b = -2.8 \times 10^{-4}$. Batchelor stated that the approximation shown by Eq. (11) holds “at least as far as the point of zero curvature”, which occurs at $l/\langle \eta \rangle = 6.3$. It is of interest to note that form shown by Eq. (10) can be used without serious loss of accuracy (within 5%) over the extended range $0 < l/\eta < 3.5$. In Batchelor (1951) the following empirical expression was used:

$$\beta_2(x) = \frac{x^2 / 15}{[1 + (15C_2)^{-3/2} x^2]^{2/3}}$$

in which $x = l/\langle \eta \rangle$ and $C_2 = 2.0$. In contrast to Eqs (9)–(11), Eq. (12) has no theoretical basis beyond behaving appropriately at the limits $x << 1$ and $x >> 1$. From this resume it is apparent that the definition of $\beta_2$ can be extended to the range of interest.

**Structural model**

Here the task is to define the right hand side of criterion (5). In Bache et al. (1999b) it was assumed that the number of bonds broken ($k$) was scaled by the number of particles ($n_o$) i.e.
\[ k = \beta n_o \]  

with \( \beta \) a coefficient of proportionality, and \( n_o = A'(d/d_o)^D \). From this base Eq. (6) becomes

\[ S = S_o (d/d_o)^{D-3} \]  

where \( S_o = 6\beta \varphi A'/\pi d_o^3 \).

Eq. (14) represents a particular case of a guessed form \( S = S'_o (d/d_o)^{(D-3)} \) used in Sonntag and Russell (1987). Eq. (15) provides insight into the factors which control the strength scale.

**Interaction with turbulence**

While Eqs (8)–(12) enable the definition of the left hand side of Eq. (11), it is advantageous to represent the \( \beta_2 \) in the form of a power law, i.e.

\[ \beta_2 (x) = Q x^q \]  

with \( Q(x) \) and \( q(x) \) as fitting coefficients. In the case of Eq (11), parameters \( q \) and \( Q \) are defined by

\[ q = \frac{d(\ln \beta_2)}{d(\ln x)} = \frac{d(\ln \beta_2)}{dx} \left( \frac{d(\ln x)}{dx} \right) \equiv \frac{2a + 4bx^2}{a + bx^2} \]  

\[ Q = ax^{2-q} + bx^{4-q} \]  

When using Eq. (12) to represent the \( \beta_2 \) function, the following definitions of \( Q \) and \( q \) apply

\[ q = \frac{d(\ln \beta_2)}{d(\ln x)} \equiv 2 - \frac{4\xi^2}{3(1 + \xi^2)} \quad \text{with} \quad \xi = (15C_2)^{-3/2} \]  

\[ Q = \frac{1}{15} \frac{x^{2-q}}{(1 + \xi^2)^{2/3}} \]  

By making use of Eqs (8), (14)–(16), criterion (5) becomes

\[ \frac{1}{2} \rho_o (\varepsilon \eta)^{1/2} Q \left( \frac{d}{d_0} \right)^q = S'_o \left( \frac{d}{d_0} \right)^{D-3} \]  

For convenience of expression (rather than physical significance), the symbol “~” is dropped and is treated as an equality. The advantage of this form is that the criterion can be used for all \( d/\eta > \) satisfying the Kolmogorov hypothesis – provided parameters \( Q \) and \( q \) are locally defined. When the range of \( d/\eta > \) is not too large (see Table 1) \( q \) and \( Q \) may be treated as quasi-constants within the range, because \( q \) and \( Q \) are only slowly varying functions of \( d/\eta > \). In the range \( 0 < d/\eta < 3.5, Q = 1/15 \) and \( q = 2. \) In the lower range \( 0 < d/\eta \leq 6.3 \) parameters \( q \) and \( Q \) are defined by Eqs (17) and (18) respectively. In the upper range, say \( d/\eta > 6.3 \) parameters \( q \) and \( Q \) may be estimated using Eqs (19)–(20) respectively. Eq. (22) behaves as

\[ d = \gamma \varepsilon^{-m/2} \]  

\[ \text{(23)} \]
Parameter $m$ stems from Eq. (3) and recognizes that $\langle \varepsilon \rangle \propto \varepsilon$ within mixing vessels (see Shinnar, 1961). The term $\bar{\varepsilon} = \nu G^2$ defines the volume average value of $\varepsilon$ within the mixing vessel. The link between Eq. (3) and Eq. (23) can be established by defining

$$\langle \varepsilon \rangle = f \bar{\varepsilon} \quad (26)$$

where $f$ is a proportionality coefficient dependent on the location within in a mixing vessel. Hence

$$\gamma = C f^{m/2} \nu^{m/2}. \quad (27)$$

### Comparison with data

#### Foundation data

While there have been a multitude of studies in which $m$ has been measured, and others in which $D$ has been determined, there are relatively few studies in which both $m$ and $D$ have been assessed for the same suspension. Table 1 provides a summary of such information for cases of floc rupture in turbulent flow. Information is drawn from: Tambo and Hozumi (1979) (“TH” set); Bache et al. (2000) (“BRMJ” set) and Bache et al. (1999b) (“BRMM” set). Coefficient $A'$ was calculated using Eq. (2) from size–density plots cited in the above using appropriate values of $\rho_s$ and $d_o$. Parameters $\gamma$ and $m$ were gained via $d_{\max} - G$ data. The $d/<\eta>$ range reflects the cited conditions.

The TH set refers to a clay suspension coagulated with alum. The “ALT ratio” is cited for identification purposes. When using Eq. (2) to calculate $A'$, the values $\rho_s = 2,650 \text{ kg m}^{-3}$, $d_o = 3.5 \mu m$ were used. Tambo and Hozumi estimated $\langle \varepsilon \rangle = 0.15 \bar{\varepsilon}$, enabling the calculation of $<\eta>$. The BRMJ set refers to flocs sampled from the flocculators of a series of water works receiving low-turbidity coloured waters. Water quality and coagulation data is summarised in Bache et al. (2000). Most of the waters were coagulated with aluminium sulphate. The BRMM set refers to coagulation of a humic acid (HA) suspension using alum. In the calculation of $A'$, the values $\rho_s = 2,000 \text{ kg m}^{-3}$ and $d_o = 8 \text{ nm}$ were used for both the BRJM and BRJM sets. In each of these sets $d_{\max}$ was set by the 95 percentile in the volume size distribution. In the BMJ set flocs were subject to shear in a standard 2 L Phipps and Bird square “jar test” beaker. The spatial structure of turbulence in this regime was described in Stanley and Smith (1995). By using the two zone turbulence model discussed in Okamoto et al. (1981), it was estimated that $<\varepsilon> / \bar{\varepsilon} = f = 0.5$ for inclusion in Eq. (23). In the case of the BRMM set, flocs were subject to shear in an oscillatory multigrid mixer in which the turbulence was fairly homogeneous in terms of its spatial distribution (Bache and Rasool, 2001). On this basis $<\varepsilon> = \bar{\varepsilon}$ i.e. $f = 1$ in Eqs (26)–(27).

#### Specification of $m$ and $S_o$

The evaluation of $m$ via Eq. (25) requires knowledge of $q$ and $D$. Parameter $q$ was calculated using either Eq. (17) or (19) using the mid-range value for the $d/<\eta>$ data shown in Table 1. The agreement between columns 8 and 9 is particularly good for the TH set. In the remaining data there is a tendency for the theory to underestimate the experimental value of
For all data (including the TH set) this represents an average discrepancy of \(-0.04\). The evaluation of \(S_0\) (shown in column 10) was based on the formula

\[
S_0 = \frac{1}{2} \frac{Q \rho_w d_o^{D-3} \gamma^{q+3-D} \varepsilon^{1/2+q/4-mq/2-m(3-D)/2}}{v^{3q/4-1/2}}
\]

(28)

Parameters \(\gamma\) and \(m/2\) refer to the experimentally determined values. \(S_0\) can only be independent of \(<\varepsilon>\) if its index value = 0 (i.e. when Eq. (25) is valid). To get around the problems posed by experimental scatter, it was found appropriate to use the median value of \(<\varepsilon>\) within the individual data sets and to use this to evaluate \(q\) and \(Q\) (treated as constants over the range of \(<\varepsilon>\)). In essence this calibrates the theory at the mid-point of the observed range of \(\ln <\varepsilon>\) or \(\ln G\). It is apparent that \(S_0\) varies by several orders of magnitude. Eq. (15) shows \(S_0 \propto A'\). Thus the final column of Table 1 corresponds to \(6\beta \phi/\pi d_0^3\). For the humic flocs the median value of \(S_0/A'\sim 1.5\) for cases of no polymer (see Table 1) and ~ 17 when the floc aid is present. Similar features are evident in the TH set, but with smaller values of \(S_0/A'\). Corresponding values of the energy term \(\beta \phi\) are shown in Table 2. It is apparent that the \(\beta \phi\) values are higher in the aluminium–clay flocs compared with the aluminino-humic flocs. This indicates that stronger bonds exist in the former, possibly because of electrostatic bridging (see Bache et al., 1999a). Similar trends are evident when a polymer is present, the increase in \(\beta \phi\) indicating the existence of stronger bonds.

**Yeung–Pelton set**

Yeung and Pelton (1996) used a micromechanical technique to pull apart single flocs in order to measure their strength. Flocs were formed by aggregating aqueous precipitated calcium carbonate (PCC) sols with two water-soluble polymers. These were labelled as “A

<table>
<thead>
<tr>
<th>Code</th>
<th>Identifiers</th>
<th>Floc aid</th>
<th>(A')</th>
<th>(D)</th>
<th>(\gamma)</th>
<th>(d/\varepsilon)</th>
<th>(m/2) exp</th>
<th>(m/2) Eq. 26</th>
<th>(S_0)</th>
<th>(S_0/A')</th>
</tr>
</thead>
<tbody>
<tr>
<td>TH set</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>0.02 [ALT]</td>
<td>3.32</td>
<td>1.93</td>
<td>1.91E-05</td>
<td>1.1–1.3</td>
<td>0.33</td>
<td>0.33</td>
<td>7.48E-02</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>0.02 ,, Poly</td>
<td>3.32</td>
<td>1.93</td>
<td>9.92E-05</td>
<td>4.7–5.2</td>
<td>0.31</td>
<td>0.33</td>
<td>1.17E+01</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>0.1 ,,</td>
<td>10.4</td>
<td>1.60</td>
<td>1.91E-05</td>
<td>1.1–1.3</td>
<td>0.33</td>
<td>0.29</td>
<td>3.57E-01</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>0.03 [ALT] [Si]</td>
<td>4.57</td>
<td>1.85</td>
<td>2.78E-05</td>
<td>1.4–2.1</td>
<td>0.33</td>
<td>0.32</td>
<td>4.20E-01</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>BRMJ set</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N Neilston</td>
<td>2.15E+06</td>
<td>1.17</td>
<td>6.93E-05</td>
<td>2.6–3.9</td>
<td>0.31</td>
<td>0.26</td>
<td>7.56E+06</td>
<td>3.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P Picketlaw</td>
<td>6.80E+05</td>
<td>1.21</td>
<td>8.75E-05</td>
<td>2.8–3.1</td>
<td>0.26</td>
<td>0.26</td>
<td>3.79E+06</td>
<td>5.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S Southmhs.</td>
<td>5.85E+06</td>
<td>1.07</td>
<td>6.73E-05</td>
<td>2.4–3.1</td>
<td>0.29</td>
<td>0.25</td>
<td>1.23E+07</td>
<td>2.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G Garshake</td>
<td>3.38E+06</td>
<td>1.10</td>
<td>2.42E-05</td>
<td>0.9–1.5</td>
<td>0.32</td>
<td>0.25</td>
<td>3.87E+05</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O Overton</td>
<td>6.57E+06</td>
<td>1.14</td>
<td>9.59E-05</td>
<td>3.7–6.3</td>
<td>0.32</td>
<td>0.26</td>
<td>5.12E+07</td>
<td>7.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M Muidykes</td>
<td>2.23E+07</td>
<td>1.00</td>
<td>2.41E-04</td>
<td>5.6–7.0</td>
<td>0.22</td>
<td>0.25</td>
<td>6.62E+08</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C Camphill</td>
<td>2.83E+06</td>
<td>1.18</td>
<td>4.81E-04</td>
<td>9.5–13.3</td>
<td>0.21</td>
<td>0.26</td>
<td>6.15E+08</td>
<td>217</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B Burncrooks</td>
<td>2.17E+06</td>
<td>1.23</td>
<td>8.65E-05</td>
<td>4.0–10.5</td>
<td>0.38</td>
<td>0.27</td>
<td>5.33E+07</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W Wybersley [Ca]</td>
<td>7.47E+06</td>
<td>1.08</td>
<td>3.24E-05</td>
<td>1.6–6.1</td>
<td>0.40</td>
<td>0.25</td>
<td>1.80E+07</td>
<td>2.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H Huntington</td>
<td>3.33E+06</td>
<td>1.15</td>
<td>1.06E-04</td>
<td>3.8–5.1</td>
<td>0.29</td>
<td>0.26</td>
<td>3.14E+07</td>
<td>9.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BRMM set</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HA</td>
<td>4.17E+05</td>
<td>1.24</td>
<td>6.37E-05</td>
<td>1.7–1.8</td>
<td>0.22</td>
<td>0.27</td>
<td>4.55E+05</td>
<td>1.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Dimensions of \(\gamma\) are consistent with Eq. (24) with \(d\) and \(\varepsilon\) in SI units

**Table 2** Estimates of energy scale factor for different floc types and conditioning

<table>
<thead>
<tr>
<th>Floc type</th>
<th>Without polymer</th>
<th>With polymer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\beta \phi(D))</td>
<td>(\beta \phi(D))</td>
</tr>
<tr>
<td>Alumino-humic</td>
<td>(4 \times 10^{-25})</td>
<td>(70 \times 10^{-25})</td>
</tr>
<tr>
<td>Aluminium-kaolin</td>
<td>(3 \times 10^{-19})</td>
<td>(140 \times 10^{-19})</td>
</tr>
</tbody>
</table>
flocs” and “B flocs” according to the polymer which was in use. Boller and Blaser (1998) transformed this data into the equivalent tensile strength (σ) by dividing the rupture force by the cross-sectional area. Plots of this form are shown in Figure 1, emphasising the dependence of tensile strength on size.

Discussion

The framework which has been developed emanates from criterion (5). Its value must be judged by its capacity to represent the floc rupture–turbulence interaction. The left hand side of the criterion is embedded in turbulence theory, the axioms of which are well established. The details which have been added to extend its range of applicability are well-founded and adequate. Greatest difficulty arises in the definition the right hand side of the criterion, namely the strength model. The particular form arises because the strength is tied to the solids volume concentration (φ), this aspect underpinning Sonntag and Russel’s description of strength and structure.

It is the form of Eq. (6) which leads to the observed variation $d = CG^{-m/2}$. The “strength constant” C depends on $S_o$. Eq. (25) provides a theoretical estimate of the index term m. The comparison between the measured and cited values of the index $m/2$ shown in Table 1 is encouraging. There is a lot of experimental scatter and one must recognise that the fractal dimension is affected by mixing Jung et al., 1996). The form $S = S_o(d/d_o)^{D-3}$ with $1 \leq D \leq 3$ shows that S diminishes with increasing size, compelling evidence being provided in Figure 1.

In contrast to Sonntag and Russel’s analysis, Eq. (15) provides detail of the factors which may influence the scaling term $S_o$. Amongst these, the energy term $\beta \phi$ (see Table 2) appears to distinguish between the differing floc types in terms of water composition, and also responds to the conditioning by a polymer. With greater knowledge, this could be a valuable tool for describing floc character. Parameter $S_o$ also scales the notional strength.
As an example: a 200 µm alumino-humic floc without polymer is adequately represented by $S_0 = 3 \times 10^6 \text{ N m}^{-2}$, $d/d_0 = 25,000$ and $D = 1.1$. Substitution into Eq. (14) yields $S = 0.013 \text{ N m}^{-2}$. These are weaker than the ferric–silica flocs reported in Blaser (2000), but of similar magnitude of strength.

Overall it appears that the framework described in the paper provides a reasonable description of the limiting floc size in turbulent flow. Useful progress has been made on the specification of floc strength. The analytical scheme has many shortcomings, such as being confined to ideal suspensions with monosized primary particles. It does not address the subtle changes in fractal structure with size and shear. Neither does it attempt to examine the rupture process in statistical terms. Nevertheless, it provides a straightforward and physically based modelling approach which can be developed further. The framework can also be used as a tool for diagnosing the effects of water type and the influence of conditioning on the strength and character of the resultant floc. Strength impinges on the behaviour of flocs in measuring instruments, treatment systems and the natural environment and it is desirable to gain greater knowledge of this fundamental property.

Conclusions
- An analytical framework stemming from an energy-based criterion describes the limiting floc size in a turbulent flow.
- Floc strength may be characterized by a model of the form $S = S_0 (d/d_0)^{D-3}$ in which $S_0$ behaves as a scaling factor which is independent of the strain rate.
- Universal functions are identified for describing the second order structure function $G_2(l)$ throughout the $l/h$ domain.
- It is demonstrated that a floc aid increases the magnitude of the potential energy to be overcome during rupture through its influence on the energy scaling factor $\beta\phi$.

References