Optimization model for the Aguas Group drinking water production and main distribution network

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Abstract A model based on mixed-integer network optimization is developed and applied to the Aguas Group drinking water production and main distribution system. The four sanitary companies owned by the group, which supply nearly all Santiago, Chile, with drinking water, possess an intricate network of hydrological sources, water treatment plants, wells, pipelines and elevation plants, providing profuse alternatives to supply their clients. The Production and Main Distribution Optimization Model (MOPYT) searches for the global optimal provision scheme from an operational costs standpoint, specifically electricity, chemical inputs and extra labour expenses. The model provides weekly benchmarks for the diverse productive quarters. It has also been used for budgetary exercises planned for water demand forecasts. MOPYT has been particularly beneficial for generating consensus among complementary operational areas such as production and main distribution, achieving global costs efficiency.

Keywords Costs; distribution; model; network; optimization

Introduction
Finding the minimum total operative cost to drinking water production and main distribution represents a task difficult to grasp when the alternatives for supplying a population are multiple. A clear example is the entangled provision scheme for clients belonging to Aguas Andinas, Aguas Cordillera, Aguas Los Dominicos and Aguas Manquehue. There the options for supplying a single area with drinking water are numerous, beginning with its treatment carried out by water plants or wells, followed by its main distribution through several gravitational pipelines or elevations, in order to satisfy the diversity of clients. It is certain that a well equipped infrastructure with abundant interconnections may assure the supply of this vital element; nevertheless, it can also cause headaches when looking for the cheapest alternative to produce and distribute water.

There is no doubt that the vast operative experience has been allowed to come across ideal local alternatives for water production and supply. Nevertheless, the concern for a better alternative shall always arise. Among the many goals of the Aguas Andinas Operations Control Area is the task of precisely solving this mystery by means of an optimization model based on mixed-integer linear programming whose principal virtue is finding a global optimal among the span of existing production and main distribution possibilities given a known hydrological and water tank demand scenario.

The Production and Main Distribution Optimization Model (MOPYT) is a multifunctional operative tool developed within the Aguas Group’s operations environment. Its target is the proposal of weekly and monthly operative benchmarks, which minimize the total variable operative costs of drinking water production and main distribution, specifically chemical inputs and electricity, for all four sanitary companies of the Aguas Group. Basically, it resolves a water supply and demand problem given hydrological, operational and economical restraints by looking for the minimum global operational cost and satisfying a specified demand. Its engine is the commercial optimization software Premium Solver Platform® (Frontline Systems, 2003), adequate for large-scale problems with mixed variables.
Based on transport network models, the structure of MOPYT admits all types of productive quarters represented by means of nodes, such as water treatment plants, wells, water elevations plants, supply tanks and dams, including their operative limitations, for example, design capacity and operating time. Additionally, water conduits subject to flow capacity, such as pipelines and open canals, are symbolized by means of arcs, which converge to and disperse from these nodes, thus allowing the formulation of a mass balance for each one.

Other roles given to MOPYT have been the evaluation of projects, motivated by inquiries on possible economies and operational efficiency improvements as the result of the formulation of new projects, which can generate new alternatives of water treatment and supply. MOPYT has also been key for budgetary estimation, providing optimal production and main distribution volumes for demand projections.

**Variables**

MOPYT optimizes a mixed-integer problem, that is, it finds the most favorable solution to a linear problem with both linear and integer variables under certain conditions or constraints. The linear variables correspond to water flow through conduits, and pump operating time of wells and elevation plants. In other words, each water conduit or arc is related to a flow variable while each pump is associated with a time variable. On the other hand, binary variables constitute the integer part of MOPYT, that is to say, null or unitary values which indicate the operative state of these conduits and pumps, within others. The value 1 indicates that the element is operable, whereas the value 0 denounces the contrary. Experience has revealed that when implementing functions and restrictions associated with binary variables it is possible to transform non-linear problems to linear, thus reducing computational effort and assuring convergence to a feasible solution. Nevertheless, this practice shall not always guarantee convergence to a global optimal solution, reason why MOPYT often predetermines the value of the binary variables and reduces the original mixed-integer problem to a linear one, for which there exists algorithms that do assure a global optimal convergence.

**Objective function and operational costs involved**

**Objective function**

MOPYT minimizes the total operative cost of drinking water treatment and main distribution among the four sanitary companies owned by the Aguas Group. The objective function contemplates direct variable costs related solely to this process, that is, chemical inputs and electricity. In effect, the objective function can be expressed by:

$$C_T = C_{ChIn} + C_E + C_{ExL} \ [\$/\text{day}]$$

where $C_T [\$/\text{day}]$ corresponds to the average total daily cost, $C_{ChIn} [\$/\text{day}]$ is the total cost of the chemical inputs and $C_E [\$/\text{day}]$ is the total electricity cost. As an exception, extra labour costs $C_{ExL} [\$/\text{day}]$ are included under certain circumstances, such as, during the simultaneous exploit of the water plants Lo Gallo, Vitacura and Padre Hurtado, owing to the employ of the same personnel to operate the latter two plants. In other words, the extra labour cost is directly related to the operative state of the water conduits that exit these plants and is set off when the sum of these operative states equal three.

**Chemical inputs cost**

The chemical inputs cost is represented via a linear function containing water production volume in plants and wells and the unit cost of chemical inputs associated with each
productive quarter, denoted using the expression:

\[ C_{ChIn} = 86.4 \times c_{ChIn} \times Q \quad \text{[\$/day]} \]  

(2)

where \( C_{ChIn} \) [\$/day] is the average total chemical inputs cost, \( c_{ChIn} \) [\$/m³] is the related chemical inputs unit cost and \( Q \) [l/s] the average water production. In order to suitably estimate well average water production, pump average operating time must be considered, using the expression:

\[ Q_{pump} = \frac{t}{24} \times Q_{max} \quad \text{[l/s]} \]  

(3)

where \( t \) [hr/day] is the average operating time of the pump and \( Q_{max} \) [l/s] is its effective productive capacity.

Pump electricity cost
The total pump electricity cost is estimated using a function of the high tension electrical tariff, plus the power and energy consumed by a pump, denoted by the expression:

\[ C_E = C_F + C_{EE} + C_P \quad \text{[\$/day]} \]  

(4)

where \( C_E \) [\$/day] is the average total electricity cost, \( C_F \) [\$/day] is a flat tariff, \( C_{EE} \) [\$/day] is the energy cost, which depends on the operating time of the pump and its power requirements, and \( C_P \) [\$/day] is the power cost, which varies depending on the time of day due to the existence of a different tariff for low and high electricity demand periods.

Extra labor cost
Extra labor costs are calculated using the expression

\[ C_{ExL} = C_{Exl} \times x \quad \text{[\$/day]} \]  

(5)

where \( C_{Exl} \) [\$/day] is the total extra labor cost. \( x \) is a binary variable which indicates whether this cost should be triggered, for example, if the water treatment plants Lo Gallo, Vitacura and Padre Hurtado require to be operated simultaneously.

Constraints
The inclusion of constraints in a water network optimization problem allows the setting of feasible limits on the solution from a numerical, physical and operational standpoint. Intuitive concepts such as an all-positive solution must be interpreted using mathematical conditions imposed on the different values the optimized variables may adopt. The constraints used in MOPYT are mainly associated with physical supply and demand conditions, which simulate the operative reality of the drinking water treatment and main distribution process. Additionally, the model handles strategic operational restrictions, such as reservoir security levels, in order to tune in midterm exploitation goals. On the other hand, MOPYT also employs constraints with only numerical purposes, linked to integer variables, which relieves the effort of the optimization algorithm. The convenience of implementing constraints coupled to integer variables in a network problem resides on the faculty to simulate real life operative conditions that would be troublesome or even impassable by means of restrictions with only continuous variables.
Pipeline and pump capacity

Every water conduit and pump has associated an effective flow or production capacity interpreted by MOPYT as an upper and lower bound for each variable, according to the expression

\[ 0 \leq Q \leq Q_{\text{max}} \quad [\text{l/s}] \] (6)

where \( Q \quad [\text{l/s}] \) corresponds to the water flow through a conduit or pump and \( Q_{\text{max}} \quad [\text{l/s}] \) is its effective capacity.

Pump operational time limit

The total operating time of a pump is subject to the duration of the daily electrical demand peak period, set from 18:00 to 23:00 in Chile, which has associated a superior power tariff generally five times higher than the regular price. However, this situation only exists from May until September and is left without effect for the rest of the year. Therefore, total pump operating time will not be able to surpass this limit without incurring in higher electrical costs during peak demand. Therefore, MOPYT imposes a lower and upper bound for total pump operating time, given by the expression

\[ 0 \leq t \leq t_{\text{max}} \quad [\text{h/day}] \] (7)

where \( t \quad [\text{h/day}] \) corresponds to the daily operating time of the pump and \( t_{\text{max}} \quad [\text{h/day}] \) the maximum operating time allowed in the day, equal to 19 hours.

Node mass balance

Entrance and exit water flow through each node must obey a mass conservation law, constituting a forced condition for all points represented by nodes such as water tanks, reservoirs, water treatment plants, water elevation plants, valves, etc. This condition is employed in MOPYT by means of the constraint

\[ Q_{\text{in}} - (Q_{\text{out}} + Q_{\text{loss}}) = \frac{1}{3.6} \times \frac{V}{t} \quad [\text{l/s}] \] (8)

where \( Q_{\text{in}} \quad [\text{l/s}] \) corresponds to the total entrance flow, \( Q_{\text{out}} \quad [\text{l/s}] \) is the total exit flow, \( Q_{\text{loss}} \quad [\text{l/s}] \) represents total operative losses or infiltrations, \( V \quad [\text{m}^3] \) is the stored volume and \( t \quad [\text{h}] \) is the average storage time. However, MOPYT considers the fourth term of the previous expression only for tanks and dams, and uses the simplification:

\[ Q_{\text{in}} - (Q_{\text{out}} + Q_{\text{loss}}) = 0 \quad [\text{l/s}] \] (9)

for nodes without storage faculties and which are only meant to be a simple connection between an exit and entrance flow. The previous expression suitably simulates the behavior of water treatment plants, water elevation plants and valves.

It is important to point out that MOPYT does not recognize instant variations in demand but simulates an average situation, therefore given that tank regulation volume usually supplies weekly or monthly average demand, then instant variations of water tank levels are irrelevant within the analysis. Thus, all volume entered in a tank will be drained by the end of the period of analysis, leaving the final storage volume equal to the initial starting point. In other words, MOPYT is based on closed water tank cycles rather than differential variations of demand in time.
Hydrological supply

Water stream hydrological scenarios are considered as supply constraints in MOPYT in order to satisfy demand. Availability in surface water sources imposes an upper bound for drinking water production in all plants while deficits are supplied by wells. MOPYT replicates hydrological situations in all related water intakes and streams using a study based on statistical analysis, which provides tables with net average monthly water flow, related return period or probability of exceedance and legal water rights (DICTUC, 2004).

MOPYT optimises water distribution along streams by efficiently allocating this scarce resource in all water treatment plant catchments. A clear example is the management of the five different intakes or canals belonging to the Aguas Group along the Mapocho river: Punta de Águilas, El Bollo, Lo Gallo, La Dehesa, and San Enrique. However, in order to calculate total water availability in a source, it is previously necessary to determine existing water flow in each water inlet and subtract infiltrations, according to the expression:

\[ q_{\text{source}} = \sum q_{\text{inlet}} \left( 1 - \frac{i_{\text{inlet}}}{l_{\text{inlet}}} \right) \quad [\text{l/s}] \]

where \( q_{\text{source}} \) [l/s] corresponds to the total water flow available in a stream, \( q_{\text{inlet}} \) [l/s] the water rights for an intake and \( i_{\text{inlet}} \) the associated infiltration rate (DICTUC, 2004).

Water demand satisfaction

MOPYT uses clusters of clients with similar water consumption behavior in order to carry out demand balances. These clusters are interpreted as nodes subject to water flow conservation constraints where exit flow is preset according to a specified demand, denoted by the expression

\[ Q_{\text{in}} - Q_{\text{out}} = D \quad [\text{l/s}] \]

where \( Q_{\text{in}} \) [l/s] corresponds to the total entrance water flow, \( Q_{\text{out}} \) [l/s] is the total exit flow and \( D \) [l/s] is the average demand of the consumption area including network losses.

Average water demand estimation

Average demand constitutes an essential boundary condition for MOPYT and generally is not straightforward to assess when water consumption behavior noticeably varies in time and from one client to another, openly visible between Aguas Manquehue and Aguas Andinas, mainly due to socioeconomic reasons. Additionally, day-to-day demand is virtually impossible to determine when actual water consumption is read once a month and for billing purposes only. A simple yet effective approach has been implemented in MOPYT so to overcome these difficulties when estimating weekly average water demand for clusters of clients. Since total water production in plants and wells is registered daily, this piece of information is employed in order to estimate demand during shorter periods. In addition, billing information from previous months is used to distribute production among the clusters of clients using the weight of the monthly water volume consumed by each cluster in the total demand. As a result, a weekly demand that includes network losses is allocated to each cluster as follows:

\[ D_{\text{cluster}} = \frac{1}{n} \times \frac{V_{\text{cluster}}}{V_{\text{total}}} \times P_{\text{total}} \quad [\text{l/s}] \]

where \( D_{\text{cluster}} \) [l/s] corresponds to the cluster’s average demand for the specified period, \( n \) is the number of days in the month, \( V_{\text{cluster}} \) [m³/month] is the cluster’s previous monthly...
water consumption, $V_{\text{total}}$ [m³/month] is the total monthly volume consumption and $P_{\text{total}}$ [l/s] is the average total production during the analysed period. A similar but less accurate treatment may be used with weights based on number of clients in each cluster, however dissimilarities in demand behavior among clients will not be incorporated in the results.

**Field experience**

MOPYT has provided for over a year weekly production benchmarks for water treatment plants and wells, analysed every Monday among complementary operations teams of the company in order to jointly program efficient production levels for the week. These regular meetings have enabled a valuable chance to raise discussion regarding how far off is the actual operation from optimal economical standards, verify what occurred
the previous week which originated differences and what can be done to achieve proposed goals. Figures 1 and 2 illustrate weekly follow-up of average total superficial and underground water production, contrasting operational statistics with MOPYT proposals. Discrepancies are analysed in detail for each water treatment plant and groups of wells, identifying operational events, and detecting deficiencies both in the actual operation and the model. As a result, exploitation strategies and opportunities are brought up during meetings, enabling fine-tuning of the real operation to minimal costs practices.

Several improvements have surfaced from this dialogue, modifying former operational schemes and revealing global operational efficiency opportunities. In addition, MOPYT has been a helpful tool for evaluating operational costs associated to infrastructure projects, such as new pipeline connections, water treatment plant development, demand growth, etc.

**Conclusions**

MOPYT has become a consolidated tool within the operations environment of the Aguas Group, merging antecedents, information, costs and operative guidelines from different operative areas into a common language and discussion platform. Discrepancies among the production and main distribution divisions of the company regarding best operational schemes have slowly been diluted, setting aside local optimal alternatives and giving way to consensus. Consequently, it has been possible to unify criteria, establishing among the operations team an overall view of the many productive quarters in order to achieve global economical efficiency.

**References**
