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Discussion

Two-Dimensional Grained Composites of Extreme Rigidity[1]

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1 Introduction and Some Notation

The recent article of Vigdergauz (1994) shows that periodically placed inclusions of the proper size and shape can generate an elastically extremal composite in the setting of two-dimensional plate theory. The derivation of the microstructure is correct and quite interesting. But the paper goes on, in Section 4, to "evaluate" the effective Hooke's law associated with this composite.

I wish to point out in this note that Section 4 of Vigdergauz' paper contains an irremediable error. Therefore the problem of evaluating this composite's Hooke's law remains at present open.

Throughout this note I will use the notations of S. Vigdergauz. I will refer to specific equations in Vigdergauz (1994) by prefixing them with the letter V. The equation numbers without prefixes refer to the formulas in this note.

2 Periodicity

From the double periodicity (henceforth referred to as periodicity) of the tensor M, using formulas (V1) and the existence of a *single-valued* potential ϕ_2 it follows that ϕ_2 is quasi-periodic, while $\bar{z}\phi_2'' + \psi_2'$ is periodic. Therefore, for periods w_k, $k = 1, 2$ ($w_1 = 1$, $w_2 = i$) we have

$$\overline{w_k}\phi_2''(z) + \psi_2'(z + w_k) = \psi_2'(z).$$

Integrating we obtain

$$\psi_2(z + w_k) - \psi_2(z) = \rho_k - \overline{w_k}\phi_2'(z), \qquad (1)$$

where ρ_k, $k = 1, 2$ are some complex constants of integration. From (1) we obtain

$$\psi_2(z - 1) - \psi_2(z) = \rho_1 - \phi_2'(z),$$
$$\psi_2(z - i) - \psi_2(z) = \rho_2 - i\phi_2'(z). \qquad (2)$$

Obviously, formulas (2) are inconsistent with formulas (V3). From now on we will "replace" the incorrect formulas (V3) with formulas (2).

3 Properties of $\Omega_0(z)$

The function $\Omega_0(z)$ is defined as the potential ψ_2 for "equal-strength load" associated to principal average moments P_0 and Q_0. This function is quasi-periodic. Therefore Ω_0 can be written as a sum of a periodic function and a linear function:

$$\Omega_0(z) = F_0(z) + q_1 z + q_2 \bar{z}, \qquad (3)$$

where q_1 and q_2 are some complex constants and $F_0(z)$ is periodic (but not analytic!). Integrating the formula (V11) over the boundary of the inclusion and using the relation (3) we obtain

$$2ic_1 d_0 = \oint_{\Gamma_0} \Omega_0(t)dt = \oint_{K_{00}} \Omega_0(t)dt = 2iq_2.$$

So

$$\Omega_0(z) = F_0(z) + q_1(z) + c_1 d_0 \bar{z},$$

or

$$\Omega_0(z + w_k) = \Omega_0(z) + q_1 w_k + c_1 d_0 \overline{w_k}. \qquad (4)$$

4 Formulas (V18) Lead to Contradiction

Let us assume the ansatz (V18). By (V18) and (4) we obtain

$$\psi_2(z + w_k) - \psi_2(z) = C_k - d_0^{-1}\big((B_0 + q_1)w_k + c_1 d_0 \overline{w_k}\big)\phi_2'(z), \qquad (5)$$

where C_k are some complex constants (their exact values are unimportant). Comparing (5) with (1) we get

$$d_0^{-1}\big((B_0 + q_1)w_k + c_1 d_0 \overline{w_k}\big) = \overline{w_k},$$

whence letting $w_1 = 1$ and $w_2 = i$ we obtain

$$B_0 + q_1 = c_2 d_0,$$
$$B_0 + q_1 = -c_2 d_0. \qquad (6)$$

Since $d_0 \neq 0$ the formulas (6) cannot possibly be true. Thus the ansatz (V18) leads to a contradiction. The same calculation shows that the assumption of a constant field in the inclusion for bending moments other than "equal-strength" moments always leads to a contradiction.

Author's Closure[3]

I fully agree with the comments made by Dr. Y. Grabovsky. The representation of the function $\psi_2(z)$ for an arbitrary loading given in Eq. (18) (Section (4) is inconsistent with its translation properties.

However, the basic idea of writing the potentials $\varphi_j(z)$, $\psi_j(z)$, $j = 1, 2$ in terms of the characteristic function $\Omega_0(z)$ seems correct to me. I believe that the error can be corrected by additional mathematical manipulations. I am presently pursuing this problem.

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