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## DISCUSSION

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The most difficult problem in making calculations of flow fields with separated regions is the determination of the pressure in the separated region or at the point of separation. In this paper, the pressure coefficient at the separation is fixed completely by the arbitrary selection of the wall velocity far upstream from the step. In fact, the pressure coefficient at the separation point is exactly  $[U_{w0}/U_{\infty}]^2$  and hence the solution to the flow field is a sensitive function of the selection of the initial wall velocity,  $U_{w0}$ . (The authors' discussion of examples A and B suggests that they have overlooked this point.) Because the forward separation pressure coefficient is important in fixing the details of the flow field, the present approach in which this quantity must be specified does not add to our understanding of the problem.

The authors imply that potential flow theory cannot give a good estimate of the pressure field upstream of the step. This implication is not correct. Gran,<sup>3</sup> Stevenson,<sup>4</sup> and I found that excellent agreement between theory and calculations could be achieved when experimental pressure coefficients for the separated zones in front of and on top of the plate were included in a potential flow computation which allows for both separated regions. Our study also showed that the separated region on top of the step

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<sup>3</sup> Gran, Robert, "Step-Induced Separation of a Turbulent Boundary Layer. I. Experiments in Supersonic Flow, II. A Model for Incompressible Flow in a Channel," PhD thesis, California Institute of Technology, 1970.

<sup>4</sup> Stevenson, R. G., "Experimental Study of a Flow of a Subsonic and Turbulent Boundary Layer over a Forward-Facing Step," Engineer's thesis, California Institute of Technology, 1969.

formed an essential part of an accurate calculation of the upstream pressure field.

### Authors' Closure

Professor Zukoski's remarks are appreciated. He is correct in pointing out that the pressure coefficient at the point of separation is determined by the value of the initial upstream wall velocity. This occurrence is implied by our discussion under the section, "Boundary Layer and Separation." In the bend-flow analysis of Robertson and Huang [9], however, the separation occurrence was more sensitive to the shape of the upstream profile than to the wall velocity. The initial wall region profile as well as the initial wall velocity in the step analysis was not chosen without guidelines. The initial wall region profile for the two cases considered was selected in a consistent manner to reflect the vorticity at the inner edge of the outer profile. Our goal was to test the hypothesis that the flow near the step is determined primarily by the shape of the outer region of the initial profile. We obtained reasonable results without specifying partial answers a priori; of course, we could have used experimentally determined pressure coefficients to evaluate the initial upstream wall velocity. It is ironic that the discussor criticizes our approach because of the apparent need for having to specify the pressure coefficient at separation when in the next paragraph he states that he and his students obtained good results with potential flow theory using experimentally determined pressure coefficients for the separated regions.

The authors did not mean to imply that potential flow theory in general could not yield adequate results for some aspects of the flow. Straightforward potential flow theory does not yield valid results near the step as clearly indicated in Fig. 4. The model given by Lighthill [1] with a single free streamline does not give good comparisons where his results are comparable to ours. However, Barrows [3], with his extension of Lighthill's model, obtained satisfactory results for the pressure distribution and separation position for laminar flow. And the discussor points out that potential flow with free streamlines can be made to give satisfactory results if the pressure coefficients for both separated regions were fixed at their experimental values. Despite these considerations, potential flow analysis with free streamlines has serious defects. First, each attempt to improve the model, such as reattachment of the separation streamline to the step face, results in an additional unknown parameter. Second, the use of free streamlines introduces singular points which give rise to discontinuous pressure gradients. Finally, the constancy of velocity on free streamlines bounding separated regions is unrealistic. These facts led us to seek an alternative approach.