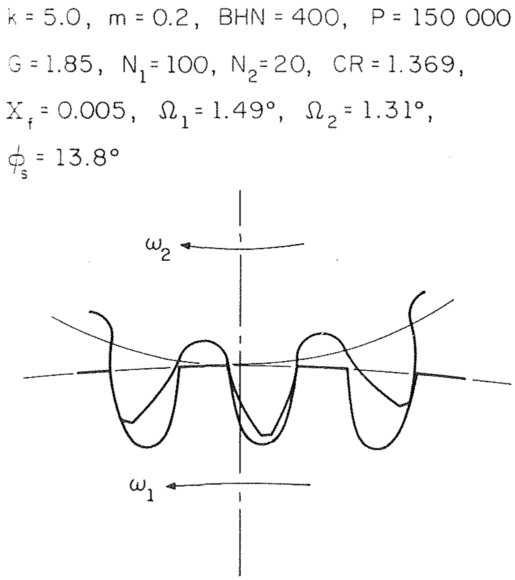

Fig. 12 Synthesized gear

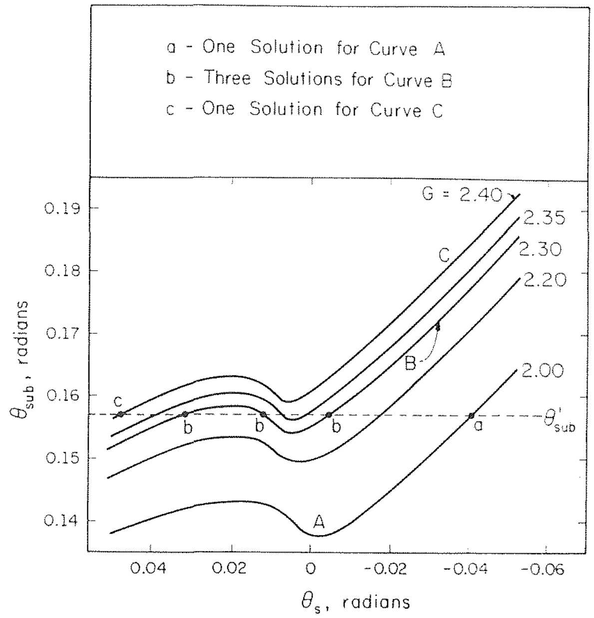

Fig. 13 Typical family of solution curves

the Hertz stress capacity. (Ω_1 is found by an iterative scheme such that $\sigma_{\max_1} = \sigma_{\max}(\Omega_1)$ using equations (20) through (28). $\sigma_{\max}$ is determined for any Ω_1 by finding the point of maximum tensile stress along the tooth profile. Ω_2 is found using equation (29).)

5 Calculate $(\theta_{sub})_{actual}$ using equation (19).

6 Repeat Steps 2 through 5 incrementing θ_s until a curve such as shown in Fig. 13 is established. From this curve solutions are found such that $\theta_{sub}' = (\theta_{sub})_{actual}$.

Using the preceding iterative scheme, a compatible solution is found where the bending, shear, and Hertz stress capacities are equal. The aforementioned iterative solution was repeated for many different values of the boundary condition X_f until curves such as shown in Fig. 6 were established. Choosing in each case the point where contact ratio was maximized, the results of Figs. 7, 8, and 9 were established.

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DISCUSSION

W. Coleman[7]

The authors are to be congratulated upon their fine presentation and intriguing method for optimizing the design of a spur gear tooth. Based on their assumptions the analysis appears to show some very interesting conclusions. It is obvious from this paper that it is not necessary to go to the Wildhaber-Novikov circular arc tooth profile to obtain some of the advantages associated with the same.

There are several points which should receive further consideration before any final conclusions may be drawn. First, the authors begin by saying that "the capacity based on scoring is generally compatible with the surface and bending capacities so that the overall capacity of the gear is not lowered . . . ." Toward the end of the paper, the above statement is justified by the use of the PV criterion for scoring analysis. Since most gear researchers use the Blok[8,9] hypothesis for scoring analysis at present, it would be interesting to know whether the same conclusions would be drawn if the latter method of analysis were applied, especially since the sliding velocity in the proposed gears will be greater than with involute gears. Also, one wonders whether the assumption that the torque will be halved when two pairs of teeth share the load, is a realistic one on actual spur gears.

Second, with teeth having concave tooth profiles on the generating profile, there is considerable danger of undercut unless the pressure angle is high or unless the numbers of teeth are large. Both of the latter conditions tend to increase manufacturing costs because of the relatively more delicate tools required for their production. There is also the added difficulty of producing the tools with the required profile shape. It would be interesting to know how the relative cost of producing these gears would compare with the cost of using carburized gears or the additional expense of using slightly larger conventional involute designs.

Third, it is noted that the authors chose to use the fillet radius of curvature at the section being analyzed, rather than at the

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point where the radius is a minimum. It would be interesting to know why this was done, when fatigue failures in actual gear teeth usually do initiate at the point in the fillet where the radius is a minimum. Also, why was the Dolan and Broghamer formula for stress concentration factor for $14^1/_2$-deg pressure angle gears used, when the authors were primarily analyzing gears with 20-deg or higher pressure angles?

There are undoubtedly applications where the method of analysis outlined in this paper can be used to advantage. Where weight and space are at a premium, the gear designer should consider a further study of this procedure.

M. J. French[10]

I think the authors should have mentioned that the idea and the method of synthesizing gear profiles of any desired (and possible) variation of Hertz stress capacity through the mesh were given in my paper (their reference [5] I also think them a little perverse not to have followed my coordinate system, which gives much more simple forms. For example, instead of having the authors' equations (1) and (15) to integrate simultaneously, in my version it is only necessary to integrate the single equation

$$R = \frac{1}{1+k}\left(y + x(2p + 1 - k) + \frac{x^2}{y}\,[p^2 + (1-k)p - k]\right)$$

where R is the specified relative radius of curvature at contact, $p = \frac{dy}{dx}$, x is the distance from the pitchpoint and y is $\sin\phi$ in the authors' notation.

The authors are to be congratulated, however, on having made a serious attempt at an optimization with respect to bending strength and on having considered the effects of variation of hardness, matters which I glossed over and ignored, respectively. I also applaud their achievement in delimiting boundary conditions which lead to cusps and undercutting.

With regard to their conclusions, they are much more optimistic than I was about noninvolute spur gears, and the reason for this is that they have underestimated the involute, as no doubt many contributors will point out, by insisting that it be badly designed, i.e., standard. When the comparison with good involutes is made the curves for G (Fig. 8) will show much less variation with k and very much lower values, e.g., B.H.N. 410, C.R. 1.4, $k = 3$ will give G about 1.2–1.3. By departing from the constant g principle of design adopted by the authors (which is not opt'mal) it is possible to hoist G to about 1.5, as in my paper. The authors do consider the possibility of reducing $1/g$ in the regions of two pair contact, but dismiss it, presumably because they think they have a larger advantage over the involute than, in fact, they have. I should perhaps point out that the superiority of well-designed over standard involutes is at least as great in practice as in theory, reference [12].[11]

Because of the variation of pressure angle, I think it is essential for all but very low speed noninvolute gearing to be helical, but it is quite realistic to design spurs and then twist them. It is not realistic to ignore what is known of gear tooth action, which is that load must be transferred gradually to and from gear teeth. The theory and finding of Harris, reference [13], on this point have been amply borne out by subsequent work. It is upon this view of gear tooth action, experience as a designer and developer of aero-engine gearing, and the important contribution of Davies of Rolls-Royce Ltd., reference [14] that I based my design principle, which was as follows: Where $R\left(\text{the authors' } \frac{1}{g}\cos\phi\right)$ is the relative radius of curvature at the point of contact, R should have a rising portion, a nearly level portion, and a falling portion to suit the natural loading cycle. Subject to this restriction, maximize

$$\int_S^F R\,dx$$

where S, F denote the start and finish of contact and x is the distance from the pitch point.

While the authors are right to point to bending strength as the difficulty with my more extreme forms, their simple beam treatment is scarcely adequate in this field. In effect they consider thin slices of the gears, ignoring support from neighboring sections. It is plain this approach must break down for short path of contact gears because it gives the bending strength under a point load as zero. A minor point is that a uniform distribution of shear stress across a tooth section, as assumed by the authors, does not meet the equilibrium condition, since there can be no complementary shear on the free surface.

I believe that hard parallel-axis gears of noninvolute form with long paths of contact (my type A) cannot show an advantage in capacity over *well-designed* involutes of more than about 50 percent. This is not enough to warrant work on them for commercial purposes unless some critical application of great importance arises. I believe that hard short path of contact and point contact gears may be able to show advantages in capacity of 100–140 percent but this is still unlikely to prove enough. The advantages of such "high-conformity" gears are clearly greater with softer materials, so an "ecological niche" might arise for them there, but I do not think it likely.

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[11] Numbers in brackets designate Additional References at end of discussion.

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Authors' Closure

Before replying to the detailed remarks, we would like to make it clear that the intention of this paper is to present a comprehensive mathematical approach to the problem of noninvolute spur gear capacity maximization. Although it is possible to solve the problem using fewer simplifying assumptions and more accurate mathematical descriptions than used herein, it is thought that these refinements have their place in future work on the subject, and it is sincerely hoped that more refined analyses will follow.

In reply to Mr. Coleman, we agree that the implication of this paper is that it is not necessary to use the Wildhaber-Novikov circular arc tooth to obtain a large capacity advantage. We think that a helical version of the proposed gear will have a capacity approaching that of the Wildhaber-Novikov, but will have the additional advantage of conjugate action in the transverse plane and therefore smoother meshing action.

Based on the Blok hypothesis (Mr. Coleman's footnotes 8, 9), the scoring capacity of the noninvolute as described herein does not look as promising as the relative capacity based on pitting and bending. However, based on the results of Fig. 8 it would not be expected that the noninvolute would be competitive with the involute in the high hardness range where scoring might become a problem, and should scoring become a limitation even at lower hardnesses, then the g function can be appropriately modified to reduce the oil film temperature where scoring is critical. This would of course result in a small decrease in overall useable capacity.

The assumption of the torque being halved during the double tooth pair contact phase is of course an ideal simplifying assumption. To accurately compute the load sharing ratio, it is necessary to decide on profile modification and to calculate the stiffness of the teeth. (Stiffness has been calculated for one case and found to be slightly lower than the involute.) To include the actual load sharing ratio as a part of the synthesis-optimization problem solution is possible, but it would make solutions much more complex.

Two typical cases of the proposed gear teeth have been generated numerically using a conjugate rack form with a polynomial tip form, and no significant undercutting problems were discovered. We agree that it would be interesting to know how the total cost of producing the proposed gears would compare to case hardened involutes; however, test stand results must first be obtained to assess the true capacity advantage of the noninvolute.

Concerning the method used for evaluating the stress concentration factor, the Dolan and Broghamer formula was used for lack of a better but simple formula. Whether the formula for $14^{1}/_{2}$ deg or 20 deg was used mattered very little since there were no experimental data available at the time the choice was made to suggest which form might be more accurate. The local radius of curvature was used for the tooth forms on gear 1 because the point of maximum stress occurs at a significant distance away from the point of minimum radius; for the involute these points are close together such that maximum stress would be more dependent on the minimum radius than for the case of the noninvolute. Subsequent photoelastic tests showed that the capacity determined using the Dolan and Broghamer equation was 10–20 percent larger than it should have been. This was considered to be a remarkably small error due to the great dissimilarity between the involute and proposed noninvolute forms.

In reply to Mr. French, as we indicated in the conclusions section of the paper, he was the first to synthesize tooth profiles of any desired capacity. Furthermore, since the synthesis portion of this paper was in fact completed early in 1967 as part of the research leading up to the Thesis (reference [11]) during which time we were not aware of Mr. French's work (accepted for publication in 1966), we felt perfectly correct in presenting the equations as our own as indeed they are.

It should be pointed out that while Mr. French's equation is of a simpler form and requires only one integration to obtain the path of contact, a second integration is required to obtain intrinsic profile coordinates, and a third integration is required to obtain Cartesian coordinates. Cartesian profile coordinates are obtained directly from the authors' equations by the integration of two simultaneous equations.

The difference in the two approaches is in reality one of coordinate systems, and our choice was made because it was desired to directly obtain tooth geometry in the rotating coordinate system because of its relationship to bending strength and other important parameters.

Concerning the standard of comparison used for the evaluation of the noninvolute relative load capacity, the 20 deg standard involute spur gear was chosen simply because it is a widely accepted, well understood, and readily evaluated gear. Comparison could have been made to perhaps a 28 deg involute (reference [4]), and the comparative results would not have looked as promising. But the increased pitting resistance capacity of the 28 deg involute of the cited reference compared to a 20 deg standard involute was not predictable from Hertz stress formulas, and the authors are led to wonder whether or not the increased radius of curvature that caused the larger than predicted capacity for the involute might not indeed affect the capacity of the noninvolute in the same favorable manner.

The g = const. assumption was made for the sake of simplification. We agree with Mr. French that the parameter g should be altered in the region of double tooth pair contact near the pitch point according his recommendations, and a small capacity increase would be attainable. However, the same modification in the double tooth pair contact region which is most distant from the pitch point would drastically reduce scoring capacity, and as pointed out earlier in this discussion, it might be desirable from a scoring consideration to actually increase the capacity in this region.

These authors' treatment of bending strength is idealized and applies only to spur gears with line contact as is the case at hand. There was no intention of predicting point contact system capacity with this model. Shear stress across the tooth tip was used as a constraint for lack of a better but simple model. A better model should be developed for this constraint because it significantly influences the bending strength on tooth 1, and it is possible that the top land of tooth 2 could be made advantageously smaller.

We think that final judgement on the proposed noninvolute system cannot be made until (*a*) an approach similar to that used herein for spur gears is used to maximize the capacity of noninvolute helical gears and establish the theoretical relative capacity of this more practical form, and (*b*) the proposed noninvolute has been evaluated on the test stand.