

Buckling of Composite Conical Shells⁴

M. Baruch⁵. In his calculations Liyong Tong assumed that the stiffness coefficients of a composite conical shell are constants. That is, the coefficients A_{11} , A_{22} , A_{12} , A_{66} , B_{11} , B_{22} , D_{11} , D_{22} , D_{12} , and D_{66} are independent of the coordinates of the conical shell.

In a recent paper Baruch, Arbocz, and Zhang (1994) showed that due to the geometry of the conical surface, in connection with the filament winding process necessary to build the laminated conical shell, constant stiffness coefficients can never be achieved. The stiffness coefficients of a laminated conical shell vary always and are defined functions of the coordinates of the shell. It is also shown that by proper filament winding process these functions, although still strong, can be made to be functions of the longitudinal coordinate only.

Hence the calculations performed by Liyong Tong are for some imaginary composite conical shell which cannot be practically realized.

Reference

Baruch, M., Arbocz, J., and Zhang, G. Q., 1994, "Laminated Conical Shells—Considerations for the Variations of the Stiffness Coefficients," *Proceedings of the 35th AIAA/ASME/ASCE/AHS/ASC S.S.D.M. Conference*, Hilton Head, SC, pp. 2505–2516 (also Report LR-671, Apr. 1992, TU Delft, The Netherlands).

Author's Closure⁶

When a laminated conical shell is fabricated using the filament winding process, the stiffness coefficients are functions of the coordinates of the shell. This is because the thickness and fiber orientation of each ply vary with the coordinates. It is difficult, if not impossible, to include these factors in the analytical solution. A special case for filament wound conical shells was investigated by Tong et al. (1991) who assumed that the stiffness coefficients are functions of the longitudinal coordinate only. In addition to the filament winding process, hand lay-up is another possible manufacturing process for laminated conical shells. In this process, each ply laid up on the mandrel consists of a number of splices. Evidently, for each ply the thickness is constant while the ply angle varies. Hence, assuming that the stiffness coefficients be constants is an approximation of the real laminated conical shells made using both processes. The assumption may be more appropriate for the laminated conical shells fabricated using the hand lay-up process than these using the filament winding process.

Reference

Tong, L., Tabarrok, B., Xiong, Y., and Wang, T. K., 1991, "Buckling Analysis of Axially Compressed Composite Conical Shells With Varying Stiffness," *Proceedings of the First Canadian International Composite Conference and Exhibition*, Montreal, Canada, Sept. 4–6, pp. 536–544.

⁴By Liyong Tong and published in the Sept. 1994 issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 61, pp. 718–719.

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Swerve During Three-Dimensional Impact of Rough Rigid Bodies⁷

G. P. Mac Sithigh⁸. Professor Stronge's paper is a welcome addition to the all-too-sparse literature on its subject. Indeed, the only other modern paper which treats a three-dimensional impact as an evolving process would appear to be Keller's (1986). I, myself, consider the problem in a forthcoming paper (1994). In light of my results, some points in Prof. Stronge's paper seem to me to warrant clarification.

Let $\mathbf{i}_1, \mathbf{i}_2$ be an orthonormal basis for the tangent plane at the impact site, let τ be the cumulative normal impulse, and let $\mathbf{v}_t := v_t[\cos \theta \mathbf{i}_1 + \sin \theta \mathbf{i}_2]$ be the relative slip velocity. Let μ be the coefficient of friction at the impact site. Define μ_* , and θ_* , such that the (rescaled) friction force required to maintain a state of stick is $\mathbf{f} := -\mu_*[\cos \theta_* \mathbf{i}_1 + \sin \theta_* \mathbf{i}_2]$. Then Keller's equations of evolution for \mathbf{v}_t may be written as

$$\frac{dv_t}{d\tau} = g(\mu, \theta), \quad (1)$$

$$v_t \frac{d\theta}{d\tau} = h(\mu, \theta), \quad (2)$$

for certain functions $g(\cdot)$ and $h(\cdot)$.

Clearly, μ_* and θ_* must satisfy $g(\mu_*, \theta_*) = h(\mu_*, \theta_*) = 0$. Moreover, swerve-free motions can occur only for directions θ such that $h(\mu, \theta) = 0$. I show that, depending on the values of certain parameters, $h(\mu, \cdot)$ has either two or four zeros. In the two-root case, these two values of θ will not in general differ by π , and neither of them will coincide with θ_* . However, in problems whose impact-geometry is sufficiently symmetrical, θ_* and $\theta_* + \pi$ will be roots of $h(\mu, \cdot)$ for all values of μ . In this symmetrical situation there will, in general, be a range of μ -values for which the four-root case obtains.

In this discussion of the possibility that slip resumes after momentarily ceasing, Professor Stronge seems to assume that if this happens, the new direction of slip must be (in the present notation) θ_* . In fact, the correct condition for this to occur is that $h(\mu, \cdot)$ should have a zero, θ_1 , say, with $g(\mu, \theta_1) > 0$. Provided $\mu < \mu_*$, it turns out that there exists exactly one such θ_1 , and it is the new slip direction. In the symmetrical case, $\theta_1 = \theta_*$. Thus, Professor Stronge's assertion is true for sufficiently symmetrical impact geometries, but is not true in the generality claimed.

Professor Stronge does not discuss the four-root case.

Finally, an analysis of this type tacitly assumes that normal relative velocity is a monotone function of τ . There exists a critical value of μ , in excess of which this ceases to be so. This critical value depends only on the inertial properties of the colliding bodies, and on the geometry of impact. For a particular example, I find typical values for it to be in the range 1.6–3.0. While these are very high, such values of μ are to be found in the tribological literature. Thus, a model failure of this sort is a real, if remote, possibility.

References

Keller, J. B., 1986, "Impact with Friction," ASME JOURNAL OF APPLIED MECHANICS, Vol. 53, pp. 1–4.
Mac Sithigh, G. P., 1994, "Three-Dimensional Rigid-Body Impact with Friction," submitted for publication.

⁷By W. J. Stronge and published in the Sept. 1994 issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 61, pp. 605–611.

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