True, there is the constitutive relation (or assumption, or idealization) of "ideal" constraints which make the expressions for Q_a easy to get in terms of certain contributions (the "applied" forces) to the vector **F**. But, should the constraints be non-ideal, so that there is sliding (kinetic) friction; $F = \mu N$, where N is the normal force, and μ the coefficient of kinetic friction, then the Lagrange equations, $(d/dt)(\partial T/\partial \dot{q}^a) - (\partial T/\partial q^a) = Q_a$, are still valid and can, for example, help us calculate the sliding friction contribution to Q_a if we know the motion.

In this sense I still think it is wrong to say that there is a "Principle" that must be invoked to get Lagrange's equations. A constitutive relation is not a "principle" but a mathematical model of forces which sometimes holds, sometimes not. A thorough recent study of derivations of Lagrange's equations by Casey (1994) takes the same point of view as the present author as regards these matters.

3 No comment.

4 No comment.

5 My choice of wording may have been unfortunate here. Papastavridis' statement is correct.

Reference

Casey, J., 1994, "Geometrical Derivation of Lagrange's Equations for a System of Particles," *American Journal of Physics*, Vol. 62, pp. 836–847.

Analytical Solution of Hill's Equation[13]

R. T. Shield[14]. The author attempts to solve a nonlinear second-order p.d.e. for a function $u(x, y)$ by introducing a function $\phi(x, y)$. Equations (3) introduce ϕ by defining the derivatives ϕ_x, ϕ_y in terms of the unknown function u. However, in order for ϕ to exist, u must satisfy a compatibility equation obtained from (3) by setting $\partial\phi_x/\partial y$ equal to $\partial\phi_y/\partial x$, giving an additional nonlinear second-order p.d.e. for u which will not hold in general. Thus the method of solution is invalid.

[13]By Y. Pala and published in the Dec. 1994 issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 61, pp. 1000–1001.

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Author's Closure[15]

One of the most difficult things in engineering discipline is to give an analytical solution to nonlinear differential equations encountered in science. However, on the other hand, the simplest thing is to criticize pre-established methods or solutions. I think Professor Shield prefers the second one. After the equation of motion is obtained, the establishment of a solution method becomes a pure mathematical problem. From the mathematical point of view, such transformations as those given by Eqs. (3) in the paper can well be introduced on the condition that they satisfy the equation of equilibrium and the boundary conditions. If there had been a compatibility equation as Professor Shield claimed but not showed, that would surely be Eq. (5), which is a different form of the equation of equilibrium. As can be followed from the references, the comparison of results of the present method with the numerical ones also show that the predictions of the theory are in excellent agreement with those given by numerical methods, without leaving any suspicion on the method. A second paper which is in the pipeline considers nonregular boundaries and also proves that the method gives excellent results and such claims cannot be regarded.

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