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upon static hole errors for low and moderate Reynolds numbers Re.

It is interesting to compare the authors' results with those of previous investigators who examined more or less the same problem. Thus Thom and Apelt [1][3] assumed a Poiseuille-type flow over their cavity of aspect ratio 2:1, and Re = 5. While these early results may not be particularly accurate, they are at variance with the results of the authors, and some discussion of this would seem valuable. A particular point in case would perhaps be the surprising degree of symmetry retained at Re = 100 in the authors' results and the shape of the separating streamline. Burggraf [2] apparently does not find such a degree of symmetry in his case (which, however, assumed a different type of shear flow).

An interesting comparison could also be made with the various models which impose a rate of shear at the cavity upper ("free") surface [3–6] and with the recent treatment of the authors' case by O'Brien [7].

## References


1 Thom, A., and Apelt, C. J., "The Pressure in a Two-Dimensional Static Hole at Low Reynolds Numbers," Aerospace Research Council Report and Memoranda No. 3090, H.M. Stationary Office, London, 1958, 13 pp.

2 Burggraf, O. R., "A Model of Separated Flow in Rectangular Cavities at High Reynolds Numbers," *Proceedings of the 1965 Heat Transfer and Fluid Mechanics Institute*, A. F. Charwat, ed., Stanford University Press 1965, pp. 190–229.

3 Greenspan, D., "Numerical Solution of a Class of Nonsteady Cavity Flow Problems," BIT, Vol. 8, 1968, pp. 287–294.

4 Ratkowsky, D. A., and Rotem, Zeëv, "Viscous Flow in a Rectangular Cut Out," *The Physics of Fluids*, Vol. 11, 1968, pp. 2761–2763.

5 Greenspan, D., "Numerical Studies of Prototype Cavity Flow Problems," *The Computer Journal*, Vol. 12, 1969, pp. 89–94.

6 Rotem, Zeëv, "Viscous Flow in Angle Sectors," to be published, 1969.

7 O'Brien, V., "Vortices in Viscous Shear Flows Over Wall Cavities," *Bulletin of the American Physical Society*, Division of Fluid Dynamics Meeting, Series 11, Vol. 14, 1969, p. 1088.


## Authors' Closure

In comparing our results with those of other investigators it should be recalled that we considered a Couette flow over the cavity and that the Reynolds number is based on the velocity of the moving wall rather than the velocity at the free surface.

Thom and Apelt [8][4] consider a Poiseuille flow over the cavity and obtain a dividing streamline that appears to start at the convex corners but dips lower than our dividing streamline. O'Brien [9, 10] has solved the same problem for both Couette and Poiseuille flow. She comes to the conclusion that "Poiseuille flows over cavities uniformly produce lower dividing streamlines than Couette flows for the same geometry."

The discusser correctly observes that in our case there is a large degree of symmetry at $N_{\mathrm{Re}} = 100$ while Burggraf [11] does not find such symmetry in his cavity flow with a scrapping lid at the same $N_{\mathrm{Re}}$. The discrepancy is readily resolved if we recall that Burggraf's Reynolds number is based on the velocity of the upper cavity surface. Since in our problem the velocity of the cavity surface is approximately four percent of the velocity of the channel wall our Reynolds number of 100 corresponds to Burggraf's $N_{\mathrm{Re}}$ of approximately 8 for which case his structure would also be symmetric.

The shear and the velocity at the cavity upper ("free") surface is not constant in our problem (which models a two-dimensional static hole). Therefore, no quantitative comparison can be made between the present problem and investigations that consider constant shear [12] and constant velocity [11–14] at the free

Table 1

| | $\psi_{\max}$ | | $h/AR$ | |
|---|---|---|---|---|
| $AR$ | O'Brien | Present | O'Brien | Present |
| 1.0 | 0.0102 | 0.0121 | 0.933 | 0.962 |
| 2.0 | 0.0105 | 0.0125 | 0.9665 | 0.9875 |

surface. Qualitatively, we see the similar flow phenomenon in both problems.

O'Brien [9] should be complimented for her thorough and careful investigation of vortices in viscous shear flows over wall cavities. She considers an identical geometry to ours and obtained solutions for $N_{\mathrm{Re}} = 0$. We can compare her results [15] with our calculations at $N_{\mathrm{Re}} = 1$.

1 In both cases the dividing streamline originates at the convex corners and is concave. The lowest height ($h/AR$) of O'Brien's dividing streamline is lower than ours; see Table 1 of this Closure.

2 While the flow structure is similar in both cases the actual values differ considerably. The values of $\psi_{\max}$ for two geometries are shown in Table 1 of this Closure.

It should be pointed out that for the scrapping lid problem at $N_{\mathrm{Re}} = 0$ our results agree within 4.5 percent with those of O'Brien. The larger discrepancy in the cavity problem is most likely due to computational difficulties associated with the singular nature of the convex corners.

## References


8 Thom, A., and Apelt, C. J., "The Pressure in a Two-Dimensional Static Hole at Low Reynolds Numbers," Aero. Research Council, Report and Memoranda No. 3090, H.M. Stationary Office, London, 1958, 13 pp.

9 O'Brien, V., "Vortices in Viscous Shear Flows Over Wall Cavities," *Bulletin of the American Physical Society*, Division of Fluid Dynamics Meeting, Series 11, Vol. 14, 1969, p. 1088.

10 O'Brien, V., "Poiseuille Flows With Boundary Disturbances," The Johns Hopkins University, Applied Physics Laboratory, Technical Memorandum TG-814, Mar. 1966.

11 Burggraf, O. R., "A Model of Separated Flow in Rectangular Cavities at High Reynolds Numbers," *Proceedings of the 1965 Heat Transfer and Fluid Mechanics Institute*, Charwat, A. F., ed., Stanford University Press, 1965, pp. 190–229.

12 Ratkowsky, D. A., and Rotem, Z., "Viscous Flow in a Rectangular Cut Out," *The Physics of Fluids*, Vol. 11, 1968, pp. 2761–2763.

13 Greenspan, D., "Numerical Solution of Class of Nonsteady Cavity Flow Problems," BIT, Vol. 8, 1968, pp. 287–294.

14 Greenspan, D., "Numerical Studies of Prototype Cavity Flow Problems," *The Computer Journal*, Vol. 12, 1969, pp. 89–94.

15 O'Brien, V., private communication.


# Buckling of Composite and Homogeneous Isotropic Cylindrical Shells Under Axial and Radial Loading[1]

**G. WEMPNER.**[2] The authors have managed to obtain relatively simple buckling modes which fulfill eight different boundary conditions by imposing two simultaneous equations involving the circumferential wavelength and the critical load (or axial length). Simultaneous solutions of the polynomial and transcendental equations are obtained numerically by a procedure of trial and correction. A difficult problem has been treated in a thorough manner and useful numerical results have been given. However, several questions arise.

[3] Numbers in brackets designate References at end of Discussion.

[4] Numbers in brackets designate References at end of Closure.

[1] By M. M. Lei and Shun Cheng, published in the December, 1969, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 36, TRANS. ASME, Vol. 91, Series E, pp. 791–798.

[2] Research Institute, University of Alabama, Huntsville, Ala.

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The approximation, equation (4), of the strain field retains the quadratic terms in the thickness coordinate $(z/a)$. Since the authors give no explanation, the reviewer poses the following questions: Why were these equations truncated at the quadratic terms? What effect have these terms upon the particular problems considered in the paper?

The equilibrium equations (9) are based upon the assumption that the prebuckled state involves only membrane forces. Some bending is produced, depending upon the edge conditions. Here, too, the influence of such bending is more pronounced in thicker shells. Perhaps, the authors could speculate on the influence of prior bending.

In their introductory remarks the authors indicate that some earlier investigators did not fulfill all boundary conditions while others employed a different theory for the aelotropic shell. Unfortunately, no comparison of results is provided. This reviewer feels that it would be interesting and reassuring to see additional comparisons with previous results, if only for the simplest cases.

The analysis presented is essentially a classical approach to the determination of the critical load. Actual buckling of thin shells may occur at much smaller loads. This occurs because the critical state is unstable and imperfections reduce the critical load. The problem has been studied by Koiter [1],[3] Almroth [2], Kempner [3], and others. In view of this, the engineer needs additional advice about the applicability of the present results; specifically, how much discrepancy is anticipated between the actual buckling loads and the calculated values?

## References


1 Koiter, W. T., "The Effect of Axisymmetric Imperfections on the Buckling of Cylindrical Shells Under Axial Compression," *Proceedings, Koninklijke Nederlandsche Akademie van Wetenschappen*, Amsterdam, Series B, Vol. 66, 1963, pp. 265–279.

2 Almroth, B. O., "Postbuckling Behavior of Axially Compressed Circular Cylinders," *AIAA Journal*, Vol. 1, 1963, p. 630.

3 Kempner, J., "Postbuckling Behavior of Axially Compressed Circular Cylindrical Shells," *Journal of Aerospace Sciences*, Vol. 21, 1954, pp. 329.


## Authors' Closure

The authors would like to thank Professor Wempner for his thoughtful comments and constructive remarks, and welcome this opportunity to reply to the pertinent questions raised, responding in the respective order of the questions:

1 Equation (4) in the paper is obtained from Flügge's exact strain-displacement relationships

$$e_x = u_{,x} - zw_{,xx}$$

$$e_\theta = \frac{v_{,\theta}}{a} - \frac{z}{a}\frac{w_{,\theta\theta}}{a+z} + \frac{w}{a+z}$$

$$e_{x\theta} = \frac{u_{,\theta}}{a+z} + \frac{a+z}{a}v_{,x} - \left(\frac{z}{a} + \frac{z}{a+z}\right)w_{,x\theta}$$

by expanding $\frac{1}{a+z}$ in a power series and neglecting terms of order $\left(\frac{z}{a}\right)^3$ or higher in comparison with unity. Flügge's strain-displacement relationships are considered to be exact in the sense that, except for the basic geometrical hypothesis that a straight-line element normal to the shell middle surface remains straight, invariant in length, and normal to the middle surface after deformation. No other approximations or simplifications are employed.

Since the shell is assumed to be thin, the term $\left(\frac{z}{a}\right)^3$ and other higher-order terms in equation (4) are small in comparison with the terms $\left(\frac{z}{a}\right)$ and $\left(\frac{z}{a}\right)^2$ that are retained in equation (4). As has been pointed out by various authors, the lack of the term $\left(\frac{z}{a}\right)^2$ will result in an asymmetrical structure of the equations of equilibrium such as Love's equations, which contradicts the Betti's theorem of reciprocity. Moreover, the terms $\left(\frac{z}{a}\right)^2$ in equation (4) yield quantities, due to the curvature of the shell, whose order is the same as that of the basic moment terms which have the coefficient $\frac{h^2}{12R^2}$.

Exact strain-displacement relationships could be used without neglecting these higher-order terms. However, this would increase somewhat the complexity of calculations and, as can be expected, will effect the final solution of the problem only slightly. Thus the fundamental governing equations used in this paper are considered accurate. On the other hand, it is well known that in order to obtain Donnell's equations further approximations are required. By using the more accurate equations of the paper, the authors found that the minimum critical axial compression for a simply supported shell with boundary conditions SS1[4] is as low as 79 percent of the minimum critical axial compression for a shell with classical boundary conditions SS3. Hoff investigated the same problem using the boundary conditions of the present paper and reference [4];[5] however, he used Donnell's equations. A comparison of results is given in answer to question 3 below.

2 In the paper, it is not intended to investigate the influence of bending, which may be produced in the prebuckled state, on the critical stress. The authors do believe, however, that it is important to publish their findings that the linear equations of shell stability do have solutions for which the buckling stress of an axially compressed, homogeneous, isotropic, circular cylindrical shell is only 43 percent of the classical value.

3 The elastic stability limit for axially compressed circular cylindrical shells was determined many years ago by several earlier investigators such as Southwell and Timoshenko. They considered only one set of simple-support conditions along the circular edges of thin-walled cylindrical shells. These classical end conditions are represented by the requirement that the radial displacement, the axial moment resultant, circumferential displacement, and the axial membrane stress resultant be zero along the circular edges. The critical stress for the classical theory is

$$\sigma_{cl} = \frac{E}{[3(1-\nu^2)]^{1/2}}\frac{h}{a}$$

where $E$, $h$, and $a$ represent Young's modulus, the shell thickness, and the radius of the shell middle surface, respectively. Four permissible combinations of boundary conditions for simply supported shells and four permissible combinations of boundary conditions for clamped shells are treated in the authors' paper. It was found that details of the boundary conditions have strong influence on the buckling loads of thin shells. Actually, in 1963, Cheng and Ho [4, 5] have already considered these permissible combinations of boundary conditions, other than the classical ones, and showed that they exert a strong influence on the critical stress. The problem of homogeneous, isotropic, cylindrical shells under axial compression, treated as a special case in our paper, has been investigated by Hoff [6] in 1965. Using Donnell's small deflection equations, he obtained the critical value of the uniformly distributed axial normal stress which is one-half the classical critical value. In the authors' paper it was found that under the simply supported boundary conditions SS1 (i.e., the vanishing of the radial displacement, the axial moment resultant, the axial membrane stress resultant, and the membrane

[3] Numbers in brackets designate References at end of Discussion.

[4] SS1, SS3 refer to various permissible cases of simply supported boundary conditions discussed in the authors' paper.

[5] Numbers in brackets designate References at end of Closure.



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shear stress resultant) the minimum critical axial compressive stress for a homogeneous, isotropic, circular cylindrical shell is 43 percent of the classical critical stress.

4 It was not the purpose of the paper to study the many interesting results obtained by various research investigators with the aid of nonlinear stability theories. It is known that, based on the Koiter's general theory of initial postbuckling behavior [7, 8], axially compressed cylindrical shells are highly sensitive to imperfections in the sense that imperfections which are small, relative to the shell thickness, will result in a large reduction of the buckling load [9–12]. Accurate investigations of the buckling load require the knowledge of the initial imperfections of the unloaded shell, but, in general, such information is not available beforehand to the engineers, primarily because of the difficulties involved in measuring shell imperfections. However, information as to the relative imperfection sensitivity of various types of shells may be provided from Koiter's theory, thus establishing the validity of the theoretical analysis of the perfect shell configuration. Hence, it is not unexpected that the experimental buckling load of axially compressed cylindrical shells are lower than the theoretical results. The critical stress obtained by the authors for a homogeneous, isotropic, cylindrical shell is much closer to the actual test value compared to the critical stress given previously by the classical theory. If one accepts, as is generally accepted, that the actual buckling load for an axially compressed cylindrical shell is approximately one-third the critical load based on the classical theory, then our result of only 43 percent of the classical critical load is much closer to the actual test value than that of previous investigations.

## References

4 Cheng, S., and Ho, B. P. C., "Stability of Heterogeneous Aeolotropic Cylindrical Shells Under Combined Loading," *AIAA Journal*, Vol. 1, No. 4, Apr. 1963, pp. 892–898.

5 Ho, B. P. C., and Cheng, S., "Some Problems in Stability of Heterogeneous Aeolotropic Cylindrical Shells Under Combined Loading," *AIAA Journal*, Vol. 1, No. 7, July 1963, pp. 1603–1607.

6 Hoff, N. J., "Low Buckling Stresses of Axially Compressed Circular Cylindrical Shells of Finite Length," Journal of Applied Mechanics, Vol. 32, No. 3, Trans. ASME, Vol. 87, Series E, Sept. 1965, pp. 533–541.

7 Koiter, W. T., "Over de Stabititeit van het Elastisch Evenwicht," thesis, Delft, H. J. Paris, Amsterdam.

8 Koiter, W. T., "Elastic Stability and Postbuckling Behavior," *Proceedings, Symposium on Nonlinear Problems*, ed., Langer, R. E., University of Wisconsin Press, 1963, p. 257.

9 Budiansky, B., and Hutchinson, J. W., "Dynamic Buckling of Imperfection Sensitive Structures," *Proceedings of the 11th International Congress of Applied Mechanics*, Springer, 1964.

10 Hutchinson, J. W., "Initial Postbuckling Behavior of Toroidal Shell Segments," *International Journal of Solids and Structures*, Vol. 3, Jan. 1967, pp. 97–115.

11 Horton, W. H., and Durham, S. C., "Imperfection, a Main Contributor to Scatter in Experimental Values of Buckling Load," *International Journal of Solids and Structures*, Vol. 1, 1965, pp. 59–72.

12 Wu, M. T., and Cheng, S., "Nonlinear Asymmetric Buckling of Truncated Spherical Shells," to be published in the Journal of Applied Mechanics.

# Axial Impact of Short Cylindrical Bars[1]

**H. F. YUAN.**[2] Using Hamilton's principle, the authors treated the problem of two identical bars impacting each other axially with equal but opposite velocities. Second-order corrections due to Love were considered. Assuming zero displacement at the impact end and the initial condition of constant velocity, explicit expressions for displacement, strain, and impact force were obtained.

The authors noted, however, that experimental results indicate a more gradual loading than that predicted by theory because of the presence of air trapped between colliding surfaces that also have microscopic deviation from perfect flatness. Therefore, from a practical standpoint, the zero displacement boundary condition at the impact end is not fulfilled. The actual problem then is not different from that of a bar subject to an impact loading at one end that starts off from zero. For this case, the approach of the paper does not seem to apply. Such a problem can, however, be conveniently handled by first assuming a series solution for the displacement as

$$u(x, t) = \phi_0(t) + \sum_{i=1}^{\infty} \phi_i(t) \cos \frac{i\pi x}{l} \tag{1}$$

By using the virtual displacement technique, Timoshenko[3] arrived at the following classical solution:

$$u(x, t) = \frac{t^2 P(t)}{2Al\gamma} - \frac{2}{\pi\gamma A c_0} \sum_{i=1}^{\infty} \frac{(-1)^i}{i} \cos \frac{i\pi x}{l} \times \int_0^t P(z) \sin \left[\frac{i\pi c_0}{l}(t - z)\right] dz \tag{2}$$

where $P(t)$ = impact load.

The writer noted that by using Lagrange's equations

$$\frac{d}{dt}\frac{\partial L}{\partial \phi_i} - \frac{\partial L}{\partial \phi_i} = Q_i \tag{3}$$

with

$$\begin{aligned} L &= T - U \\ T &= \text{total kinetic energy} \\ U &= \text{total elastic potential energy} \\ Q_i &= P(t)(-1)^i = \text{virtual force} \end{aligned}$$

equation (2) can be derived in a more straightforward way.

When Love's radial inertia term is accounted the equation can be shown as

$$u(x, t) = \frac{1}{M} \sum_{i=1}^{\infty} (-1)^i \cos \frac{i\pi x}{l} \cdot \frac{1}{\eta_i{}^2\Omega_i} \int_0^t P(z) \sin \Omega_i(t - z) dz + \frac{t^2 P(t)}{2Al\gamma} \tag{4}$$

with

$$\begin{aligned} M &= \gamma A l/2 \\ p_i &= i\pi c_0/l \\ \Omega_i &= p_i \eta_i \\ \eta_i &= \left[1 + \left(\frac{\sigma k \pi i}{l}\right)^2\right]^{1/2}. \end{aligned}$$

The corresponding strain expression becomes

$$e(x, t) = \frac{\partial u}{\partial x} = \frac{1}{M}\cdot\frac{\pi}{l} \sum_{i=1}^{\infty} (-1)^i \frac{i \sin \frac{i\pi x}{l}}{\eta_i{}^2\Omega_i} \int_0^t P(z) \sin \Omega_i(t - z) dz \tag{5}$$

The displacement and strain can thus be calculated by integration once $P(t)$ is prescribed. When a large number of terms of the series is desired for convergence both the second-degree

[1] By H. D. Conway and M. Jakubowski, published in the December, 1969, issue of the Journal of Applied Mechanics, Vol. 36, Trans. ASME, Vol. 91, Series E, pp. 809–813.

[2] Research Structural Engineer, Army Waterways Experiment Station, Vicksburg, Miss.

[3] Timoshenko, S., and Young, D. H., *Vibration Problems in Engineering*, D. Van Nostrand Co., Inc., New York, 1955.



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