Normal Ridge Flux
\[ \Delta n_p \left\{ -\frac{h_r \partial_p}{12 \mu \partial \zeta} - \frac{(\omega_1 + \omega_2)}{2} h_r \sin \beta \right\} \] (28)

Requiring the previous expressions to be equal, one finds that at a groove edge,
\[ h_s \frac{\partial p_r}{\partial \zeta} - h_r \frac{\partial p_r}{\partial \zeta} = 6 \mu (\omega_1 - \omega_2) (h_s - h_r) \sin \beta \] (29)

If the surfaces are parallel and the grooving geometry is perfectly congruent, the pressure must be periodic in each groove-ridge pair; consequently,
\[ 0 = \int_{\theta}^{\theta+\alpha \theta} \frac{\partial p}{\partial \theta} d\theta = \int_{\theta}^{\theta+\alpha \theta} \left( -\frac{\partial p \sin \beta}{\partial \zeta} + \frac{\partial p \cos \beta}{\partial \eta} \right) d\theta \] (30)

Summing foregoing terms and substituting for \( \Delta \) and \( p \) by expressions given by equations (25), (32), and (33):

**Total Radial Flux**
\[ -\Delta \frac{\partial p}{\partial \zeta} \left\{ -\alpha (1 - \alpha) h_s + \frac{\partial p_r}{\partial r} \right\} \]

**Groove Flux**
\[ -\Delta \frac{\partial p}{\partial \zeta} \left\{ -\alpha (1 - \alpha) h_s + \frac{\partial p_r}{\partial r} \right\} \]

Solving for \( \partial p_r / \partial \zeta \) and \( \partial p_r / \partial \zeta \)
\[ \frac{\partial p_r}{\partial \zeta} = \frac{-\alpha \mu (\omega_1 - \omega_2) (h_s - h_r) \sin \beta + h_r \frac{\partial p}{\partial \eta} \text{ctn} \beta}{\alpha h_s + (1 - \alpha) h_r} \] (32)

Also, assuming \( \Delta \theta \) to be infinitesimal, equations (23) and (24) reduce to

**Ridge Flux**
\[ -\Delta \frac{\partial p}{\partial \zeta} \left\{ \frac{\partial p_r}{\partial r} \right\} \]

Defining \( W^{(a)} \)
\[ \frac{\partial p}{\partial r} \left\{ \frac{\partial p_r}{\partial \zeta} \right\} = \frac{\partial}{\partial \eta} \left[ \frac{1}{2} \mu (\omega_1 + \omega_2) \right] \]

Then
\[ \text{Total Radial Flux} = \Delta \frac{\partial p}{\partial \zeta} \]

For computational ease, it is desirable to revert to the original cylindrical coordinates and replace \( \partial p / \partial \zeta \) by
\[ \sin \beta \frac{\partial p}{\partial r} \]

\( \partial p / \partial r \) is not precisely the actual radial pressure gradient since it does not contain the effects of discontinuity in \( \partial p / \partial \zeta \) at the groove edge. Insofar as these spatial fluctuations of pressure caused by the grooves are bounded and reduces to negligible magnitude when \( \alpha \) is very small, they will no longer be of any significance in the present work and no further distinction will be made between \( p \) and \( \rho \).

The mass contained inside the element of control volume is, again assuming \( \Delta \theta \) to be small,
\[ \text{Mass Element} = \Delta r \int_{\theta}^{\theta+\alpha \theta} \rho r d\theta = \Delta r \left[ \alpha h_s + (1 - \alpha) h_r \right] \rho \]

Finally, since circumferential influx and efflux will cancel each other, continuity consideration leads to
\[ \frac{\partial}{\partial r} \left( \text{Total Radial Flux} \right) + \frac{\partial}{\partial \eta} \left( \text{Mass Element} \right) = 0 \]

or
\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ W^{(a)} \right] + \frac{\partial}{\partial \eta} \left[ \alpha h_s + (1 - \alpha) h_r \right] \rho = 0 \] (1)

**DISCUSSION**

E. A. Muijderman

In their report, Malanoski and Pan take as the starting point for their calculations a differential equation previously reported by Voehr and Pan for the pressure buildup in a spiral groove bearing. This differential equation contains the illogical ratio \( (\omega_1 - \omega_2)/(\omega_1 + \omega_2) \). In Fig. 2, however, which represents the dimensionless load-carrying capacity calculated from this differential equation, only the term \( (\omega_1 + \omega_2) \) is found, which is certainly entirely logical. After closer inspection of the formulas, I believe that I have found in equation (27) in the Appendix the origin of the ratio \( (\omega_1 - \omega_2)/(\omega_1 + \omega_2) \); namely, in the term \( \frac{\omega_1 + \omega_2}{2} h_r \) which occurs in that formula. In my opinion, however, this formula should not properly contain the latter term.

In deriving the differential equation concerned, the assertion by Voehr and Pan that the pressure buildup is linear is less "serious" than might perhaps be thought at first sight. After all, it is immediately evident from the Reynolds equation used in lubrication theory that the pressure above a ridge and a groove must satisfy the potential equation, at least under steady-state conditions.

A linear pressure buildup thus satisfies the differential equation for a linear strip and, apart from correlation for the effects at the ends of the bearing, constitutes a solution. In this connection, reference may be made to the present author's thesis published in March, 1964, entitled "Spiral Groove Bearings."
in which the foregoing comments are dealt with at greater length. Some remarks might be appropriate regarding the dimensionless load-carrying capacities represented in Fig. 7. It is stated that the figure holds for linear strips, and the optimum conditions are indicated. But on p. 555 we read: "It is judged that the actual deviations from the optimum condition (in a true spiral groove bearing) are of secondary magnitude and will not invalidate the major conclusions that can be inferred." With this latter sentence I find myself not in complete agreement, because, for true spiral groove bearings, different optimum conditions apply which, it may be admitted, do not differ so very radically from the values given in the report. The values which I calculated for the same type of spiral groove bearings as in Fig. 2 were mentioned in the thesis referred to. It is shown there, from calculations made for true spiral groove bearings (that is to say, not for linear strips) working with incompressible media, that in "corresponding" cases the true spiral groove bearing has somewhat more load-carrying capacity; the optimum conditions differ too. The latter is particularly the case in regard to bearings with a large Rs/R ratio. For practical purposes it also seems to me simpler to calculate for each bearing width both the inflow and the outflow bearing and to mention for this straightforward the load-carrying capacities and the coefficient of friction optimum conditions. Calculations have shown that it is not correct to take the same optimum values of $\alpha$, $\eta$, and $\beta$ for an inflow and an outflow design. Although the order of magnitude of the result is not incorrect, discrepancies of 10 to 20 percent may be found in the load-carrying capacity in certain circumstances.

Of particular interest are the calculations for spiral groove bearings with compressible media. In this connection, I wish to remark that for spiral groove bearings with no net radial flow the compressibility of the medium is merely of no influence if one uses a large number of grooves. This might be of practical importance, since it is also found that in the case where some radial flow exists much higher load-carrying capacities could be achieved than with the more conventional self-acting thrust bearings, which as is well known, at high values of $A$, reach an almost constant load-carrying capacity. In the latter context, I should like to comment on Fig. 4. I regret that I have not read Cooper's report and therefore have no idea of the accuracy of his measurements or the conditions in which he carried out his experiments. Fig. 4 gives the results of calculations by Cooper and Malanoski and Pan, and measurements were made by Cooper which are in fairly good agreement with the calculations. When I calculated the value of $F_p/R$ for this case from my formulas—the latter are only applicable to incompressible viscous media, but they take into account the effects at the ends of the grooves due to the finite number of grooves—my result coincided with the following. A difficulty here which has been that the meaning of the letter $C$ in the report of Malanoski and Pan was not accurately defined. I take $C$ equal to $b$, (but then $H = b/C$ equals one). The value of $F_p/R$ is found to be (from my formulas) equal to 2.21. This means a difference of 8 percent with the experimental value of 2.04 found by Cooper. This may of course be due to the fact that my formulas hold only for incompressible media, but considering the still-low values of $A$, this is not so probable. It may also be that the discrepancy is due to errors in my calculation, and it is also possible that Cooper's measurement is inaccurate. I look forward to seeing this question cleared up.

What the report has to say about self-excited vibrations of spiral groove bearings is highly interesting. I myself have gone into this subject for a time but was unable to find any instabilities from simple theoretical considerations. Since we had not in any case experienced trouble from vibrations of this kind in practice, we decided to leave it for what it was. Although I can agree with the suggestion that a spiral groove bearing resembles a pressurized bearing, I am very curious to hear about practical cases where instability has in fact been encountered.

To conclude this discussion, I should like to point to the two main areas where spiral groove bearings, lubricated with oil and grease (1), can find practical application. It is because spiral groove bearings, size for size, possess greater load-carrying capacities than tilting pads and step bearings that they are so attractive from the point of view of small dimensions and relatively low friction. This applies not only to larger constructions where self-acting tilting-pad thrust bearings are used (in ships' engines, turbines, etc.) but also to fields where until now it has not been possible to use self-acting bearings, and porous metal bearings have had to be used instead. For the latter applications in particular, where small shaft diameters are found (from 1 to 6 mm), it seems that spiral groove bearings, lubricated with grease, can fulfill the long-cherished ideal of producing self-acting bearings (i.e., quiet and not wearing out) on an economic scale. Experiences which we have carried out give strong indications in this direction.

**Authors' Closure**

The authors thank Dr. Muijderman for his discussion on our paper. We became aware of his thesis work through Prof. Ir. H. Blok just about the time we released our manuscript for publication.

Another paper concerned with this type of bearing appeared recently. While these two studies have treated the same subject as our paper, different aspects are emphasized so that the three pieces of work compliment each other, and, together, they provide much useful information. For convenient reference, the following chart summarizes the essential contents of the respective studies which are identified by the initials of the authors:

<table>
<thead>
<tr>
<th>Compressible</th>
<th>Incompressible</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-P, M, W</td>
<td>M-P, W</td>
</tr>
<tr>
<td>M-P</td>
<td>M-P</td>
</tr>
<tr>
<td>W</td>
<td>W</td>
</tr>
<tr>
<td>M-P</td>
<td>M-P</td>
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<tr>
<td>M-P</td>
<td>M-P</td>
</tr>
<tr>
<td>M-P</td>
<td>M-P</td>
</tr>
</tbody>
</table>

**Dr. Muijderman's Thesis**

Dr. Muijderman's thesis also reported on his experience with manufacturing techniques and experimental verification of his own theoretical prediction. In general, both Dr. Muijderman's thesis and Mr. Wildmann's paper are aimed to supply information directly usable in design work, whereas our effort was directed toward discovery and prediction of new phenomena.

In the following, we shall attempt to clarify the questions raised by Dr. Muijderman in the same order as they were presented in his discussion.

The first point is really a matter of notation. The authors have used the symbol $A$ consistent with the convention commonly followed in the analysis of self-acting gas bearings. This symbol is defined to contain the sum of the rotational speeds of both members of the bearing and, as such, is related to the wedge

---


effect of a fluid-film in self-acting bearings. In a spiral-grooved, parallel thrust bearing, so long as the bearing surfaces remain parallel, the dominant process is the interaction between shearing of the fluid-film and the geometry of the inclined, shallow grooves; therefore, in the present work, \( \Lambda \) always appears together with \( f = \left( \frac{\omega_2 - \omega_2}{2} \right) \). In this connection, \( \Omega \) also should be contained in the definition of \( \Lambda \), i.e.,

\[
\Lambda = \frac{\Omega \Lambda}{2} \left[ 1 - \left( \frac{R_i}{R_e} \right) \right].
\]

The report by Vohr and Pan \([4]\) derived the differential equation applicable also to nonparallel surfaces, e.g., the cases of radially displaced journal bearings and misaligned thrust bearings. Under the latter circumstances, the effects of both shear, \( \Omega \), and wedge action, \( \Lambda \), exist simultaneously. Regardless of the physical meanings of \( \Lambda \) and \( \Omega \), our notation gives an unambiguous convention for defining the spiral angle, irrespective of whether the grooves are stationary or rotating and whether the bearing is an in-flow or an out-flow design. We wish to assure Dr. Muijderman that equations \((27), (28), \) and \((29)\) are correct and that the derivations of Vohr and Pan are free from illogical concepts.

The potential equation applies to the fluid film pressure in constant gap regions only when density variation can be neglected. Although an infinite number of grooves is implied in our study so that circumferential density variation within one width of the groove or the ridge should be neglected correspondingly, it is not automatically clear whether the resulting theory is applicable to real bearings where the number of grooves is always finite. Our comparison with Cooper's calculation and experiment provided some calibration for eighteen grooves at \( \Lambda = 70 \). It remains to be shown whether or not the theory for infinite number of grooves is applicable to eighteen-groove bearing at much larger values of \( \Lambda \). Fortunately, we have a check for self-consistency in equations \((32)\) and \((33)\). Having done this for a variety of bearings, we recommend that the number of grooves, \( N \), be selected to satisfy the following inequality:

\[
\frac{\Lambda}{2N} < 1.
\]

Deficiency in the bearing performance, as compared with the present theoretical prediction, can be considerable when this inequality is violated. Of course, as stated in the text, degradation of bearing performance will take place whenever \( N \) is small even if the foregoing inequality is satisfied. This is because the circumferential pressure gradients within the respective widths of grooves and ridges, be they linear or otherwise, cannot be maintained at the ambient periphery, nor are they compatible with the pressure in the seal region at the mutual border of the grooved and seal regions.

Fig. 2 is intended to show the effect of radius ratio and, in particular, the relative advantage of an in-flow design over an out-flow design. In this comparison, we chose to keep the geometrical parameters of grooving constant, as determined in \([2]\), to yield maximum load for an incompressible linear strip. Realizing, however, the results in Fig. 2 do not represent optimized conditions at all radius ratios, we uttered the precautionary statement that "... deviations ... of secondary magnitude ... will not invalidate the major conclusions ..." Since adjectives tend to inject subjectivity, numbers tabulated in Table 1 are offered to answer Dr. Muijderman's remark:

### Table 1

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Malanoski-Pan</th>
<th>Muijderman</th>
<th>Wildmann</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>In-flow</td>
<td>Linear strip</td>
<td>Out-flow</td>
</tr>
<tr>
<td>( \frac{R_i}{R_e} )</td>
<td>0.4</td>
<td>0.73</td>
<td>4.05</td>
</tr>
<tr>
<td>( F/\mu \Delta (R_e^2 - R_i^2) )</td>
<td>0.090</td>
<td>0.069</td>
<td>0.046 to 0.043</td>
</tr>
</tbody>
</table>

We concur with Dr. Muijderman that an important practical advantage of spiral-grooved thrust bearings is their insensitivity to compressibility. True independence of compressibility applies only in the static sense for designs with no transverse flow and only if equation \((38)\) is satisfied. For partially-grooved thrust bearings, which are the only cases considered in this paper, there is also an apparent independence of compressibility in their static behavior. This apparent independence of compressibility comes about because compressibility causes the fluid pressure in the grooved region to reduce and that in the seal region to increase at the same time. In the limit, compressibility effect would set in, the load would be carried entirely by the seal region, and it would become proportional to \( \sqrt{\Lambda} \).

To clarify the definitions of \( C \) and \( k_2 \), we wish to refer to Fig. 1, equation \((8)\), as well as the Nomenclature section.

On the basis of our calculation and the reported conditions \(( \Lambda \approx 70 \) for Cooper's bearing, the incompressible prediction of
its static performance should be quite satisfactory. However, equation (36) is not quite satisfied and end correction alone would amount to more than 10 percent according to our estimate. We consider our present agreement with the experimental data as better than can ever be expected. We are not in a position to satisfy Dr. Muijderman's curiosity with respect to the 8-percent discrepancy according to his own calculation, since we do not wish to second guess the exact circumstances of Cooper's experiment. It is pertinent to point out that accurate measurement of the operating gap and the possibility of thermal distortion are most difficult to keep under control in such experiments, and both factors can introduce considerable anomaly in the performance of these bearings.

Similarity between compressible and incompressible spiral-grooved thrust bearings disappears under dynamical conditions. Our results indicate that self-excited instability will not occur unless \( \Lambda \) is sufficiently large. For the bearing which gives the results shown in Table 2 of the paper, it appears that instability would not occur for \( \Lambda \) much smaller than twenty according to the theorem derived in the text. We do not know of any experience which exactly corroborates the type of instability that has been discussed. This lack of experimental corroboration is quite understandable since the particular dynamic system is highly idealized to exclude many extraneous effects which would exist in a real situation. On the other hand, we know of several experiences in the form of self-excited angular oscillation in systems containing spiral-grooved gas-lubricated thrust bearings while operating at a reduced ambient pressure. These latter phenomena often have been attributed to "slip flow" on the basis of purely circumstantial arguments. Presuming that some physical processes associated with the type of instability with which our paper deals can also contribute to angular motions, then we have offered an alternative explanation. For this reason, we feel it would be worthwhile to extend our analysis to consider angular motions. It is also valuable that controlled experiments be carried out to verify the theoretically predicted dynamical behavior of these bearings in both translational and angular modes.

We share Dr. Muijderman's enthusiasm in the versatility of spiral-grooved thrust bearings. Their inherent advantages over other types of thrust bearings are less obvious with incompressible lubricants. Their principle of operation certainly would not exclude use of grease as the lubricant. Non-Newtonian effects of grease on such a bearing no doubt will be an interesting subject to a curious investigator.