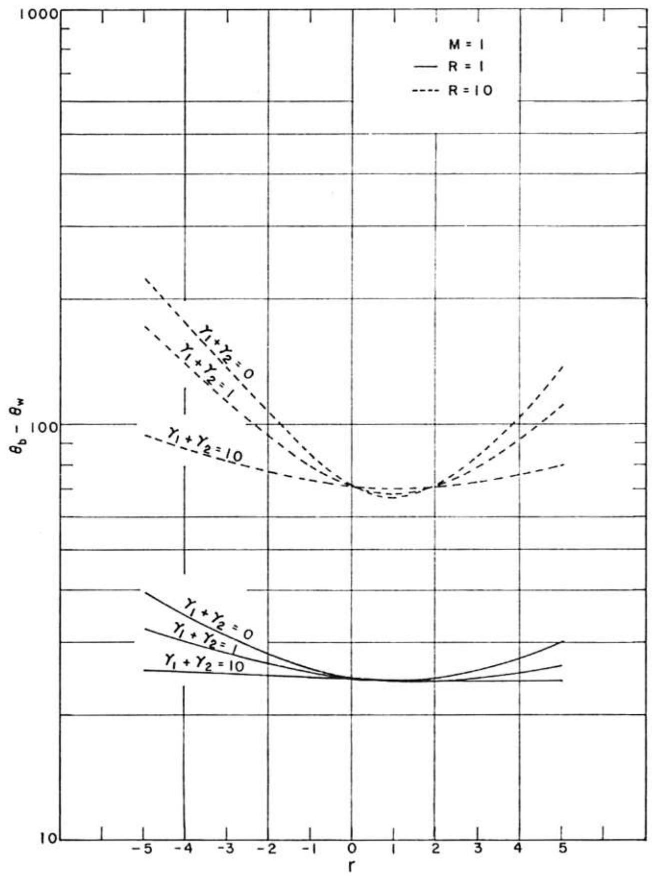

Fig. 2 Bulk to wall temperature difference for M = 1

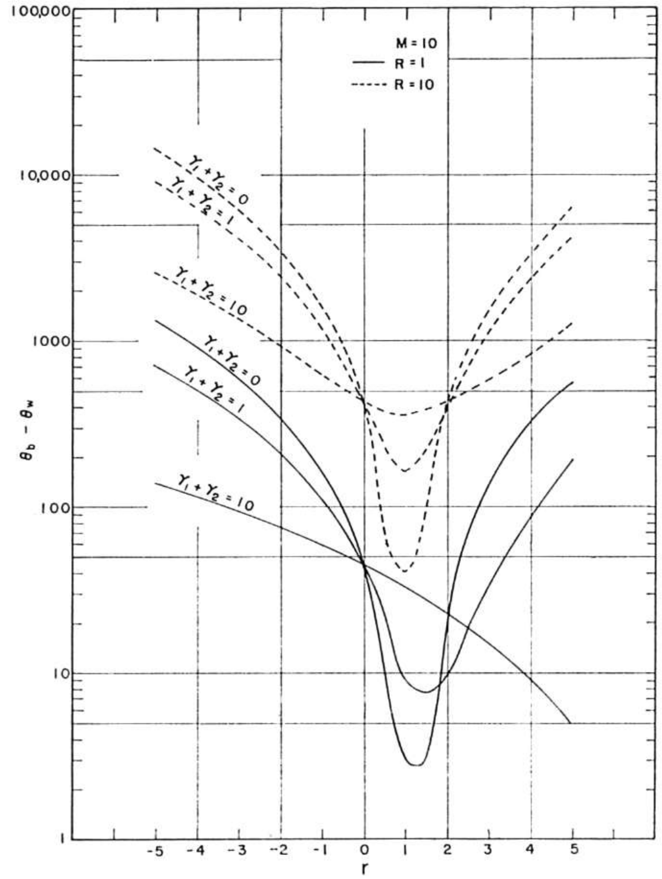

Fig. 3 Bulk to wall temperature difference for M = 10

celerator mode, the temperature difference decreases with increasing wall conductance.

Although the curve for M = 10, $R = 1$, $\gamma_1 + \gamma_2 = 10$ does not appear to be approaching a minimum value, an examination of this curve for larger values of r showed that it did not reach a minimum value for $r = 6.54$.

A Numerical Example

As an indication of the actual temperatures involved in Figs. 2 and 3, a representative calculation will be made for the following conditions.

$$M = 10,\ r = 1,\ R = 10,\ \bar{u} = 100 \text{ ft/sec},$$

$$\mu = 2.6 \times 10^{-5} \frac{\text{lb}_f \text{ sec}}{\text{ft}^2}, \qquad K = 6 \frac{\text{Btu}}{\text{hr ft F}}$$

The values of μ and K chosen correspond to mercury at 200 deg F. With these values, the resulting temperature differences between the bulk fluid temperature and the wall are shown below.

$\gamma_1 + \gamma_2$	$(T_b - T_w)$, deg F
0	8.2
1	32.8
10	72.6

In conclusion, it may be said that in considering the heat transfer in MHD channel flows, one must exercise caution in assessing the influence of the wall conductances on the heat transfer. For walls which are nominally insulators at low temperatures, the conductivity may become sufficient at elevated temperatures to significantly affect the heat transfer. The mode of operation, i.e., generator or accelerator, is significant since the trend of the heat-transfer dependence on wall conductance is opposite for the two modes.

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DISCUSSION

W. Tripp,[2] U. P. Hwang,[2] C. L. Hwang,[2,3] and L. T. Fan[2,3]

The analysis of the complicated heat transfer channel flow problem presented in the paper is valid. However, the author's concluding remarks regarding the correspondence between the range of voltage ratio, r, and the operating model appear to be in error.

It is stated that: (*a*) "the range $0 < r < 1$ corresponds to the generator mode of operation," (*b*) "the range $r < 0$ corresponds

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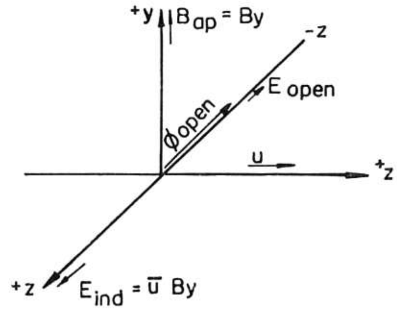

Fig. 4

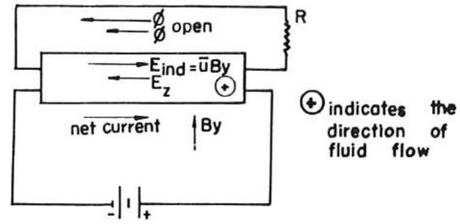

Fig. 5

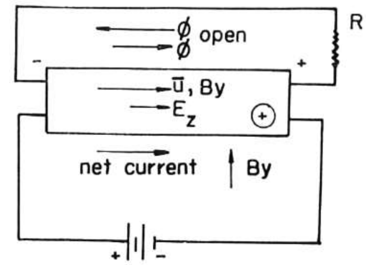

Fig. 6

to the accelerator mode of operation," and (c) "the range $r > 1$ corresponds to the application of external voltage larger than the induced open circuit voltage and in the same direction as the induced voltage." Let us consider Fig. 4.

The same nomenclature as that in the paper is employed in this figure. ϕ_{open} is defined by equation (14) in the paper.

From equation (13a) it can be seen that when the net current is equal to zero, i.e., when $I = 0$, $E_{open} \equiv (E_z)_{I=0}$ and $|E_{open}| \leqslant \bar{u}B_y$, where E_{open} is in the negative z-direction [see equation (14)]. Furthermore, the equality relation in $|E_{open}| \leqslant \bar{u}B_y$ holds when $\gamma_1 + \gamma_2 = 0$, and the inequality relation holds when $(\gamma_1 + \gamma_2) > 0$, noting that $\gamma_1 \geqslant 0$ and $\gamma_2 \geqslant 0$.

With reference to Fig. 4 and the above relationship, the following correspondences between the range of voltage ratio and the operating mode exit.

1 For the generator mode of operation for which the conditions that E_z is in negative direction and that $|E_z| < |E_{open}|$ (Fig. 5), the following relation is obtained with the help of equation (10):

$$\frac{E_z}{E_{open}} = \frac{\phi}{\phi_{open}} < 1 \text{ but } > 0, \qquad \therefore 0 < r < 1$$

[see equation (15)], which agrees with that of the author.

2 For the generator mode of operation under the condition that E_z is in positive direction (Fig. 6),

$$\frac{E_z}{E_{open}} = \frac{\phi}{\phi_{open}} < 0, \qquad \therefore r < 0,$$

which the author states is for the accelerator mode.

3 For the accelerator mode of operation under the conditions that E_z is in negative direction and $|E_z| > |E_{open}|$ (Fig. 7),

$$\frac{E_z}{E_{open}} = \frac{\phi}{\phi_{open}} > 1, \qquad \therefore r > 1,$$

which the author states as corresponding "to the application of an external voltage larger than the induced open circuit voltage and in the same direction as the induced voltage."

This discussion may further be clarified by reference to Figs. 5 6, and 7.

J. T. Yen[4]

The analysis of this paper is incomplete. The principal results, Figs. 2 and 3, may be misleading, because the parameter R, which is not a basic parameter, has been kept constant in each of the figures. These figures should be recalculated and replotted. The wall heat flux parameter $\kappa_1 + \kappa_2$, not the parameter R, should be kept constant, as it has been done by the authors of the references quoted by this paper.

By the definition (37) of R, it is seen that R is inversely proportional to the square of another parameter r, representing external electrical loading. In other words,

$$R \text{ is proportional to } (\kappa_1 + \kappa_2)/r^2.$$

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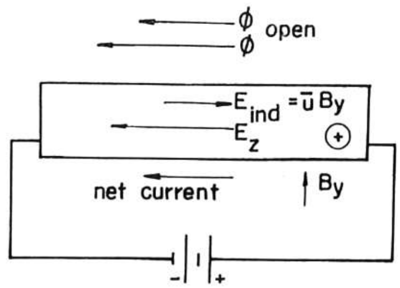

Fig. 7

Hence, by keeping R constant, $\theta_b - \theta_w$ has to increase when r^2 is increased, simply because the wall heat flux $\kappa_1 + \kappa_2$ is increased. This is the picture of Figs. 2 and 3, since r is the abscissa. However, this is not what the author purported to show, nor what the discusser would like to know, that is, how $\theta_b - \theta_w$ is affected by external electrical loading r alone, when all other basic parameters, including the wall heat flux, are kept constant.

Author's Closure

The author wishes to thank Professors Tripp, U.P. Hwang, C.L. Hwang, Fan, and Yen for their discussions. The discussion of the first group of discussers will be considered first.

The discussers correctly point out an error in the author's interpretation of the generator mode of operation. Two inequalities in the conclusion section of the paper should be interchanged. First of all, it should be noted that the direction of the induced electric field depends on the direction of the applied magnetic field. If the magnetic field is applied in the positive y direction as shown in Fig. 1 of the paper, then the induced E_z will be negative. The induced E_z may lie between the limits $E_z = 0$, corresponding to short circuit operation, and $E_z = -\bar{u}B_y\left(\frac{2}{2 + \gamma_1 + \gamma_2}\right)$ corresponding to open circuit conditions. For the generator mode of operation, E_z will be negative and will lie within the above range. Thus the conclusion of the paper and the discussers that $0 < r < 1$ corresponds to the generator mode of operation is correct. The condition $r < 0$ corresponds to the application of an external electric field in a direction opposite that of the induced field and does not correspond to the accelerator mode as stated in the paper.

The accelerator mode of operation can be determined by considering the x-component of the body force, $-J_zB_y$. Since J_z varies with y and may have a sign change across the channel, the local body force may be positive or negative. For acceleration, however, the total body force, obtained by integrating the local body force across the channel, must be positive. The total body force may be written

$$F_{total} = -2\sigma B_y{}^2 w\bar{u}(1 + \phi)$$

or from equation (16) of the paper,

Table 1 $\theta_b - \theta_w$ **for** $M = 1$

$\kappa_1 + \kappa_2 = 0$

r / $\gamma_1 + \gamma_2$	0	1	10
0	19.57	19.52	14.80
10	19.57	19.51	13.30

$\kappa_1 + \kappa_2 = 100$

r / $\gamma_1 + \gamma_2$	0	1	10
0	63.7	93.9	38.9
10	63.7	82.8	44.8

$$F_{\text{total}} = -2\sigma B_y^2 w\bar{u}\left(1 - \frac{2r}{2 + \gamma_1 + \gamma_2}\right)$$

For the acceleration mode, F_{total} must be positive which requires the term in parentheses to be negative. Thus the criterion for acceleration is

$$\frac{2r}{2 + \gamma_1 + \gamma_2} > 1$$

Dr. Yen correctly points out that the total wall heat flux is not constant along the curves of Figs. 2 and 3. The ratio of total heat flux to total dissipation is constant for each curve. Since the total dissipation depends on r, the total heat flux is actually a variable along the R = constant curves. Dr. Yen's suggestion is that $\kappa_1 + \kappa_2$ should be held constant. A considerable amount of numerical work, requiring the use of a computer, is involved in determining the $\theta_b - \theta_w$ curves. A set of hand calculations is shown in Table 1 for two values of $\kappa_1 + \kappa_2$ and for a range of r including short circuit conditions ($r = 0$) at one extreme and accelerator conditions ($r = 10$) at the other extreme. The effect of external loading, r, on $\theta_b - \theta_w$ is clearly shown for these conditions.