$$C6\ V_p^2 + C12\ V_p + C13 = 0 \quad (27)$$

where

$$C12 = C7\ \alpha_2 - 2C1$$

$$C13 = C8\ \alpha_2^2 + C3 - C2$$

Equation (27) is a simple quadratic and can be solved for V_p easily. With the value of the discharge through the unit known, substitution back into equations (18) and (19) yields the values of the heads on either side of the unit.

Thus, with the speed, discharge, head, and torque at the unit known, the computations can continue along the pipeline using equations (3) and (4) until the head and discharge at each point in the line is found. Then the time can be incremented and the process repeated until the transient period is completed. This entire solution is done on a digital computer.

DISCUSSION

L. Wozniak[2]

The problem of offspeed and pressure prediction in a closed loop analysis of governed pumps or turbines is, in the main, the problem of adequate representation of their characteristics. It is not clear why Mr. DeFazio used the homologous curves

$$\frac{h}{\alpha^2} = F\left(\frac{v}{\alpha}\right) \quad \text{or} \quad \frac{h}{v^2} = F'\left(\frac{\alpha}{v}\right) \text{ for head}$$

and

$$\frac{\beta}{\alpha^2} = G\left(\frac{v}{\alpha}\right) \quad \text{or} \quad \frac{\beta}{v^2} = G'\left(\frac{\alpha}{v}\right) \text{ for torque}$$

where

α = actual speed/rated speed
v = actual flow/rated flow

for his computer calculations. His approach requires duplication of data storage, h/α^2 as well as h/v^2, and switching from one representation to the other, depending on whether α or v is approaching zero. Marchal, Flesch, and Suter[3] use the parameters $\dfrac{h}{\alpha^2 + v^2}$ and $\dfrac{\beta}{\alpha^2 + v^2}$ versus $\tan^{-1} \dfrac{v}{\alpha}$. This choice seems especially well suited for analysis of pump-turbines where three-quadrant (pump, dissipation, turbine) representation of characteristics is essential. The main advantage is continuity of the arctangent function as well as of the head and torque expressions, thus requiring only one curve for the representation of each. Furthermore, the claim is made that the homologous-curves representation has the distinct advantage of being able to represent a fixed gate, variable-blade angle situation as well as a fixed angle variable-gate case with one computer program. This is true with most other representations of characteristics, and depends on the programming of interpolations.

The discusser has been able to find appropriate transformations that render triple-quadrant simulation of pump-turbine characteristics both single-valued and amenable to mathematical representation in closed form, thus making unnecessary the computer storage of characteristic data. As an example, Fig. 16 shows typical pump-turbine curves for three positions of wicket gates, z_1, z_2, and z_3 on the flow (v) versus speed (α) plane, all data being given for unity head. Fig. 17 displays the same information but on the speed times gate (αz) versus flow (v) plane. It is seen from this second figure that the flow is approximately linear with gate position z, and quadratic with the angular coordinate $\tan^{-1} \dfrac{\alpha z}{v}$. A functional representation may be found by the method of least square error approximation. This function may be used to solve for flow as a function of speed and gate position at unity head.

[2] University of Illinois, Urbana, Ill. Discussion prepared in cooperation with the Woodward Governor Company, Rockford, Ill.

[3] M. Marchal, G. Flesch, and P. Suter, "The Calculation of Waterhammer Problems by Means of the Digital Computer," International Symposium on Waterhammer in Pumped Storage Projects, ASME, November, 1965.

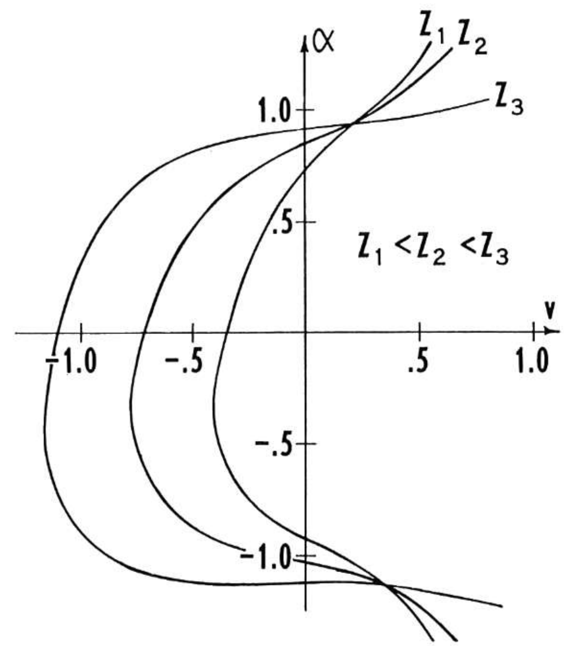

Fig. 16 Flow/speed representation of pump-turbine data for variable gate (z) at unity head

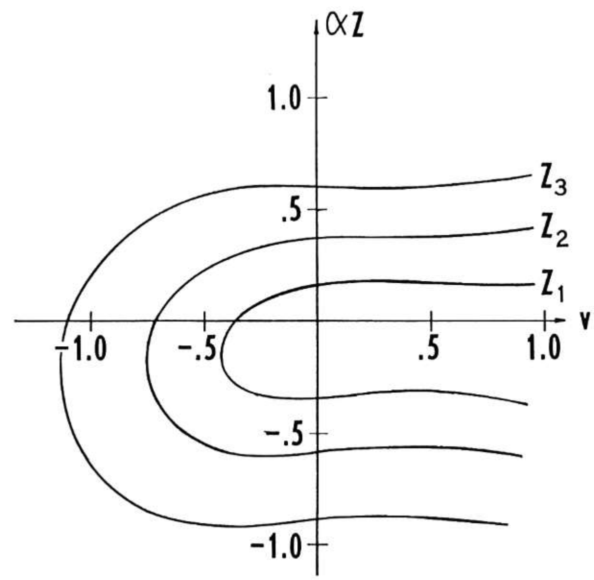

Fig. 17 Pump-turbine data with transformed ordinate speed × gate (αz)

Author's Closure

The author thanks Mr. Wozniak for his discussion. His remarks point out the need for further clarification of the representation and usage of the characteristic curves for computers.

Figs. 8–11 do not require any duplication of data storage. Also, the h/α^2 and h/v^2 curves do not overlap or duplicate each other. The uniqueness of the values of h/α^2 and h/v^2 can be realized by drawing the v/α and α/v lines for the values -1.0

to 1.0 on the flow representation of Fig. 16. The intersection of these lines with the characteristics represent the values shown in Figs. 8–11.

The choice of $h/(\alpha^2 + v^2)$ and $\beta/(\alpha^2 + v^2)$ versus $\text{TAN}^{-1}\ v/\alpha$ appears to be well suited for representation of the characteristics. However, since the principal values of the arctangent function are between 0 and $\pi/2$ one must still use the values of v and α to determine which portion of the characteristics the function is referring to.

One can usually find some function to represent portions of the characteristics. These functions will provide adequate solutions in many cases. The author questions the use of these functions where the characteristics have unstable or inverse slopes since they may introduce a smoothing effect in these zones.

The flow/speed or unit head representation shown in Figs. 16 and 18 do not have the capacity to represent a unit when the value of h is negative. Since the negative head region may determine the minimum possible head [6], and therefore possible water column separation, the author has concluded that any unit head representation of the characteristics is not general enough.