The parameters describing the system were reduced from seven or more dimensional parameters to three nondimensional parameters (λ,δ,Ω). The reduction to a few manageable constants permits a better understanding of how the performance varies with design changes. The figures presented are intended to facilitate such insight. They are not primarily intended for direct use by a designer. Rather, the conclusion of the paper is that a steady-state solution can be rather easily obtained for any particular set of conditions. A designer can therefore generate his own set of response curves for any particular specific variable of interest.

Several questions not addressed in this paper have been dealt with in detail in previous work [8]. In combination with that work, all aspects of the design process have been considered, transient simulation, nonlinear, steady-state response, and stability of steady-state response.

Continuing efforts are aimed at an analysis of the nonlinear stability of the solutions. This is particularly interesting because a stable, asynchronous, noncircular, steady-state orbit seems to exist for a wide range of parameters. Other efforts are addressed at (*a*) effects of external, unidirectional forces such as gravity, (*b*) optimization with competing objectives, and (*c*) multi-degree-of-freedom systems with multiple dampers.

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DISCUSSION

E. J. Hahn[1]

The major portion of this paper deals with the determination of the steady-state unbalance response of centrally preloaded symmetric rigid rotors in squeeze film dampers, and it is instructive to note the relevance of previous investigations in this area. Though circular orbit solutions had been utilized even earlier [11], documentation for centrally preloaded situations appeared around 1974, as evidenced by [4], wherein the desirability of assuming synchronous circular steady-state solutions, leading to equations (6), rather than arriving at such solutions via the full transient solution of the equations of motion (3) (whether such transient solutions be via hybrid or digital computer solution procedures) was demonstrated. The regions of bistable operation were more extensively investigated and extended to include supply pressure effects in [13]. For the assumed pressure distributions in this paper corresponding to the special case of zero supply pressure parameter P in [13], the conditions for bistable operation are summarized in Fig. 14, where the nondimensional parameters Ω_j/ω, U, and B refer to Ω, δ, and $\lambda/2$ in the notation of this paper. Thus, Fig. 14 predicts jump up at nondimensional speeds of 2.3 and 4.5 for unbalance parameter $\delta=0.3$ and bearing parameter $\lambda=0.072$, in agreement with Fig. 9 of this paper. To avoid ambiguity, jump up or jump down refers to whether there is a (theoretically) possible orbit increase or decrease, should one enter a bistable operating speed range from a monostable operating speed range. Whether such a jump to the alternative stable solution is likely to occur is, of course, another question. That the complicated bistable operating regimes predicted in Fig. 14 are practically realizable was verified in [14] and is illustrated in Figs. 15 and 16. Figure 15 shows a typical frequency response test, corresponding to $\delta=0.372$ and $\lambda=0.240$, used to obtain the jump speeds p, q, and r necessary for Fig. 16. Considering the sensitivity of the predicted jump speeds to the unbalance parameter, U, the agreement between theory and experiment was surprisingly good.

With regard to the stability of these steady-state circular orbit solutions, it is noted by the authors that studies (in [8]?) have shown that branch 3 in Fig. 9 is stable and branch 4 is unstable. According to stability investigations in [15, 16], whenever three such synchronous orbit solutions were obtained, the in-between orbit solution was unstable. Thus, not only is branch 4 in Fig. 9 predicted to be unstable, but also that portion or branch 2 in Fig. 9 for which $\epsilon_c>0.64$. Of particular interest is the comment of the authors regarding the apparent possibility of stable, asynchronous, noncircular, steady-state orbit solutions for a wide range of parameters. This discussor is somewhat sceptical of such solutions, for, as noted in the discussion to [17], such so-called nonsynchronous steady-state solutions can result from numerical inaccuracy in the case of very lightly damped circular orbit solutions. Further comments by the authors on such behaviour would be appreciated.

While it is agreed that the circular orbit steady-state solution approach is generally preferable to the transient solution approach, and hence has, circumstances permitting, been generally preferred, even for more compex rotor bearing systems [7], the authors appear to somewhat overstate the case for the solution procedure used. One of the advantages of transient solution approaches has surely been that *only* stable solutions can be found. Indeed, it is one of the disadvantages of the steady-state solution approach that the solutions obtained, in many instances, provide no indication at all as to whether they are stable; and one has to resort to transient solutions, linear stability analysis, or experimental evidence, to determine stability. Also, it is not clear how a noniterative solution procedure would have fared in the analysis of the more complex systems in [7].

Regarding solution procedures for synchronous circular orbit steady-state solutions, one is ultimately confronted with a nonlinear equation in the orbit radius in terms of the relevant system parameters; i.e., in the notation used in this paper, this involves the solution of equation (8) for ϵ_c in terms

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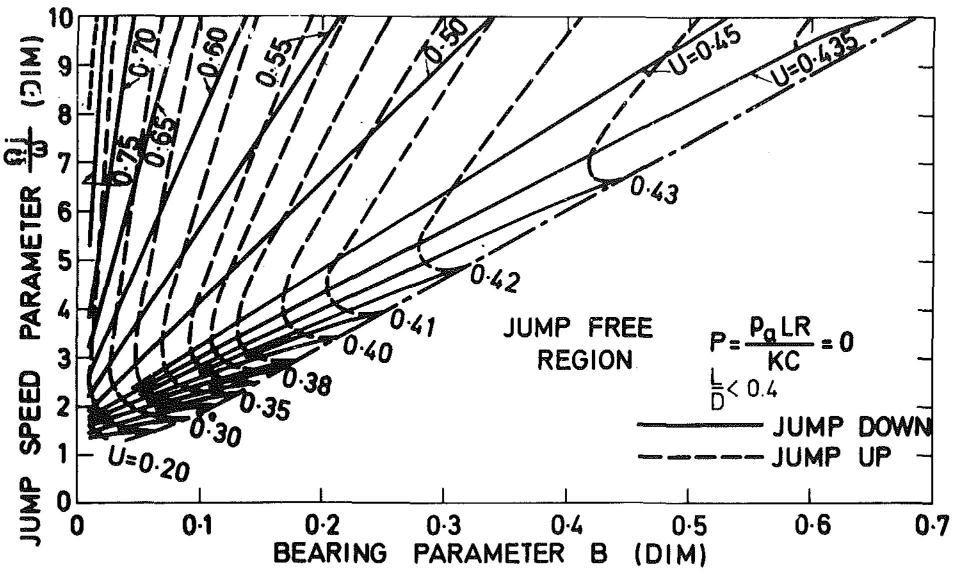

Fig. 14 Variation of jump speed with bearing parameter and unbalance parameter for unpressurized bearings (Fig. 4 of [13])

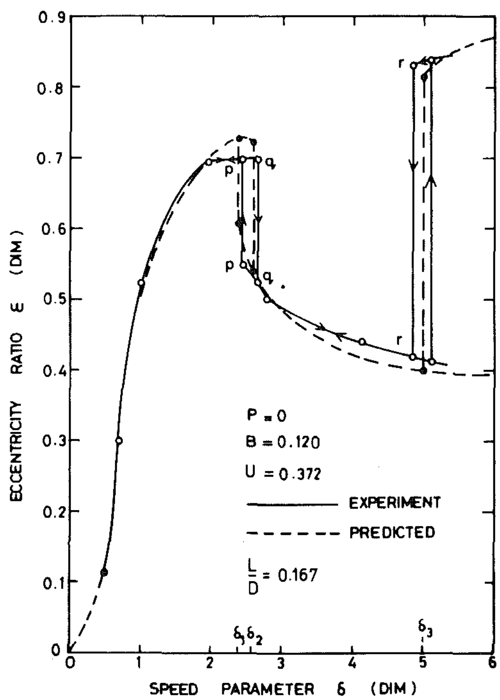

Fig. 15 Multiequilibrium frequency response of eccentricity ratio (Fig. 11 of [14])

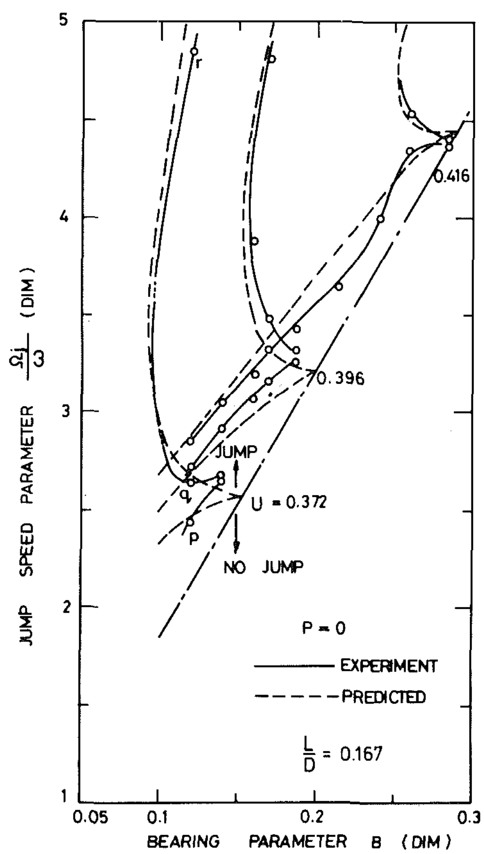

Fig. 16 Variation of jump speed with bearing and unbalance parameter for unpressurized bearings (Fig. 12 of [14])

of the three parameters Ω, λ, and δ. There are a variety of ways of solving equation (8) for physically meaningful values of ϵ_c (i.e., for $0<\epsilon_c<1$). Whether one adopts an iterative procedure, e.g., a simple search procedure, to find the value(s) of ϵ_c for some given values of Ω, λ, and δ (and this is the ultimate goal), or one avoids iteration by assuming values for the dependent parameter, ϵ_c, and for only two of the three system parameters Ω, λ and δ, and then solving for the third system parameter directly, is generally dictated by the amount of effort needed to achieve the required end results. Noniterative procedures, as adopted by the authors, solve equation (8) by an inverse-type approach. To obtain the actual value(s) of ϵ_c for, say, some specified Ω_o, λ_o, and δ_o requires successive solutions of equation (8) for Ω over a sufficient range of ϵ_c values to ensure that all values of ϵ_c likely to result in $\Omega \approx \Omega_o$, have been covered. The final values of ϵ_c at Ω_o then need to be determined by interpolation (e.g., by graphing). Thus, if one is merely seeking the ϵ_c values corresponding to a single set of parameters Ω_o, λ_o, and δ_o, such a noniterative procedure, which involves finding solutions over a range of Ω, is generally more time consuming than an iterative approach. On the other hand, if one is seeking to present solutions for ϵ_c over a range of the system parameters, the noniterative approach would generally be

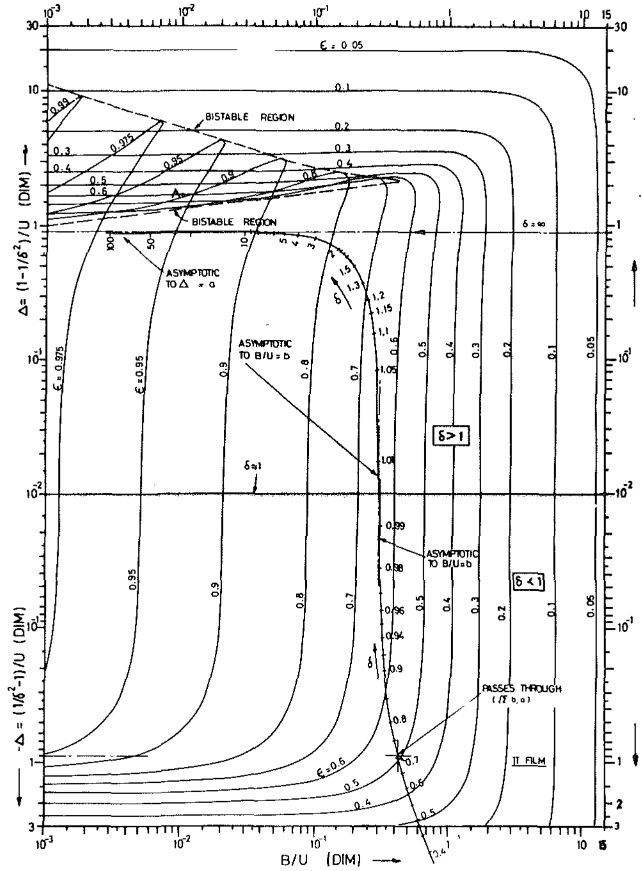

Fig. 17 Equilibrium orbit eccentricity data for unpressurized squeeze film supported rotors (Fig. 2 of [19] or [20])

preferable. In any case, it is felt misleading to refer to such noniterative numerical solution procedures as "closed form," a term which implies that an analytical expression is available for the dependent variable, ϵ_c, in terms of the system parameters Ω, λ, and δ.

Equation (8) of the paper corresponds to equation (13) in [18] or equation (6) in [19, 20] viz.,

$$(g_t B/U)^2 + (g_r B/U - \epsilon\Delta)^2 = 1 \tag{18}$$

where

$$\Delta = (1 - 1/\delta^2)/U \tag{19}$$

in the notation of [18–20].

In the notation of this paper, g_t corresponds to $\epsilon_c J_3^{20}|_c$, g_r to $\epsilon_c J_3^{11}|_c$, δ to Ω, U to δ, B to λ/Ω, and ϵ to ϵ_c. Equation (18) shows that in fact only two systems parameters, viz., B/U and Δ(or$\lambda/(\Omega\delta)$ and $(1-1/\Omega^2)/\delta$ in the notation used by the authors) are sufficient to uniquely determine the steady-state orbit radius ϵ_c. Hence, by solving equation (18) either using an iterative or noniterative procedure, it is possible to display all possible equilibrium solutions for the orbit radius, ϵ_c, on a single diagram. For reasons already discussed above, equation (18), being a quadratic in the parameter B/U, was solved for B/U directly over a physically meaningful range of ϵ_c and Δ, as shown in Fig. 17. Frequency response curves such as Figs. 3–7 and Fig. 9 of this paper may be all obtained from Fig. 17 by following the operating line, the positioning of which is explained in [19] or [20]. Once the ϵ_c solution has been obtained, corresponding transmissibility and phase angle values may be obtained as explained in [18–20], or in this paper. Unfortunately, it was not found possible to portray transmissibility data as conveniently. Figure 17 illustrates the point made in the paper about the disappearance of bistable operation for sufficiently large unbalance. Thus, once Δ in Fig. 17 is less than unity, bistable operation is not possible. This is not, however, a recommended operating condition, since such low values of Δ, resulting from values of unbalance parameter in excess of unity, always result in transmissibilities in excess of 1, as explained in [19, 20]

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Authors' Closure

The authors are certain that readers will appreciate the additional references provided (also cited in [1, 8]). In response to the discussion, it seems appropriate to review why this approach was chosen. The basic goal is to determine the overall structure of the dynamic response of this well-known system. An associated problem is how much to distill (for presentation) the response of a four-state system with three physically meaningful parameters. To this end, a single graph is not as meaningful as a family of curves in understanding why multiple responses may or may not occur in one or more speed ranges. It would have been even better to present a series of phase plane plots at various speeds in place of each figure such as Fig. 3, but space is prohibitive. The interested reader is referred to [8].

The value of knowing the overall structure can be shown in that the stable, high-speed, high-ϵ response (branch 3,4 in Fig. 9) was deduced from purely structural reasoning prior to any confirmation of its existence. Given the goal of a total response characterization, it becomes clear that both stable and unstable responses must be known to understand the overall topology, of the system. (A stable point cannot simply become unstable or split into two but must follow well-known bifurcation laws.)

The desire for overall topology of *all possible solutions* drives the analysis to closed form. The determination of the governing algebraic equations is one step closer than the governing O.D.E. The further reduction from nonlinear to polynomial form assures that all fixed points are known (vital for topology), independent of how they are actually determined.

Based on discussions after this paper was written, it is clear that curve 4 in Fig. 9 must be unstable. However, this is not for the seemingly obvious reasoning that 2 and 3 are stable. The reader is cautioned that this is a four-state system and so many familar phase plane rules do not hold. Rather, curve 4 must be unstable because a parameter variation in λ will cause the two "noses" of the curve to meet and merge, and the stability characteristics before and after must be consistent. The stability of each branch has been determined in [8].

Although the discussion documents a jump at higher speeds (δ_3 in Fig. 15), the authors have no analytical basis for initiation of the jump, unlike the inevitability of the lower speed jump/drops shown during speed scans. The smaller response is stable and should be maintained unless greatly disturbed. It is a purely subjective conclusion of the authors that the larger response is less stable in a global sense and so should not dominate except for a particular range of initial conditions.

The existence or nonexistence of jump drop regions as seen from Fig. 17 can be confusing. The authors have found Fig. 17 easier to understand in light of Figs. 3–7. For instance, every damper, for any particular unbalance, has one (and possibly two) speed ranges for which multivalued response is possible. Thus, even larger values of unbalance lead to multivalued reponse (contrary to the discussion). Consider curve 2 in Fig. 5. The lower half of the disconnected branch is stable, although during a speed scan a "drop" will not occur without external disturbance. In terms of Fig. 17, the bistable region monotonically expands with decreasing B/U, and thus any operating line will eventually intersect it at sufficiently high speed.

As noted, the nonsynchronous, high-speed, high-eccentricity orbits are mentioned in [13, 1], and the accompanying discussion submitted by the first author of this paper. It is true that the high-speed responses are very lightly damped [8], but this appears as a beat phenomenon of slowly decaying transient vibrations at the natural frequency superimposed on the forced response at the running speed. The nonsynchronous response (precessing loops) is numerically stable under changes in integration step size. The authors are currently trying to determine topologically if such a response is a valid response. Until then, any comments are welcomed as to how to determine if the "simulation response" is a valid "physical response."