quency roughness components, are almost flattened completely. When rough surface moves (with constant rolling speed), the roughness of the modified surface profile in the contact zone increases with its moving speed. When the moving speed of the rough surface approaches or exceeds the rolling speed, the modified roughness is nearly the same as the roughness of the undeformed roughness profile.

4 Due to the nonsynchronization of pressure fluctuation and the roughness profile, the minimum film thickness (i.e., the average minimum peak value) is notably smaller in the cases where the rough surface moves at a speed equal to or in excess of the rolling speed than in the cases where the rough surface moves at a speed slower than the rolling speed.

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DISCUSSION

J. A. Greenwood[2] and G. E. Morales-Espejel[3]

This is a most interesting and valuable contribution to our understanding of the behavior of rough surfaces in EHL. We read it, however, with some dismay: for its results are completely at variance with the predictions of a theory [A] to which we have devoted much time and effort.

Like the authors, we have been attempting to separate out the behavior of the roughness from that of the overall EHL contact; but not having the author's skill in EHL computations, we have been forced to the simplification of idealizing the overall geometry as a long parallel channel of given film thickness $\bar{h}$ and ambient pressure p_0—the maximum Hertzian pressure. That is, we extract the heart of the EHL contact from its surrounding inlet and outlet, and examine what happens to transverse roughness in this channel.

We find, like the authors in their previous paper [B], that when the rough surface is stationary the roughness amplitude is greatly diminished, and that each harmonic component behaves independently of the others—the viscosity is so high that the nonlinear, viscous term disappears from the equations. Our equation for the pressure fluctuations is rather different from the authors': in their notation

$$\frac{\Delta P}{A}=\left[Cp_0+\frac{2L(\alpha p_0)}{\pi H_0 G}\right]^{-1} \qquad \text{(D-1)}$$

where C is the compressibility $(\gamma-\beta)/((1+\beta p_0)(1+\gamma p_0))$; this states that the original surface roughness is ironed out except to the extent that the lubricant compressibility allows it to persist. Comparing this with the authors'

$$\Delta P = 0.0164\, W^{-.839} U^{.407} G^{.666} A^{.768} L^{-.806} \qquad \text{(D-2)}$$

the only certain conflict is the non-linear relation between ΔP and A: the other terms may well represent the dependence on G, W, and U of our terms αP_0 and H_0. We believe that the possibility of superposing roughnesses of different wavelengths, which both groups accept, implies the possibility of superposing roughness of the same wavelength—i.e., that the relation should be linear.

The relevance of this is that when both surfaces are moving, our theory suggests that the pressure fluctuations barely change, being still largely determined by the same mechanism. In contrast, a fluctuating oil supply is carried into the parallel channel by the roughness in the inlet, and causes fluctuations in the film thickness to accommodate it—but these move down the channel at the *rolling* speed $\bar{u}$ and so, except for the case of pure rolling ($u_1=u_2=\bar{u}$), have a wavelength differing from that of the roughness which stimulated them. The basic pressure fluctuations, of course, move with the roughness and have the "proper" speed (u_1 or u_2) and wavelength. To complete the picture, these film thickness fluctuations are accompanied by their associated pressure fluctuations: but these are relatively small and (again except for pure rolling), are not coherent with the basic pressure fluctuations, and so with a random roughness should increase the amplitude, not decrease it.

The theory seems to us to be consistent with the behavior reported by other workers—and indeed is largely based on results and ideas developed by Venner and Lubrecht [C, D]. In particular, they (and we) find for a wavy surface that the pressure oscillations are largely independent of the slide/roll

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ratio, except that for pure rolling the amplitude is somewhat smaller. But the present results, exemplified by Figs. 7 to 11, seem to demolish our theory completely. As we said above, we are dismayed.

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L. Chang[1]

The authors present transient results of EHL between a perfectly smooth surface and a rough surface of random roughness. This is a significant step forward from earlier steady-state analyses with a moving smooth surface and a stationary rough surface. Figures 7 to 11 show that abrupt pressure rippling is generated (except in pure rolling), and the magnitudes of the pressure ripples increase with sliding. This discusser speculates that different micro-EHL results might have been obtained had the rheological model included the shear-thinning characteristics of the lubricant. A brief analysis follows.

Consider two locations inside the Hertzian region, x_1 and x_2 which, along with the two surface segments in between, define a control volume. Neglect lubricant compressibility, flow continuity in this volume gives:

$$\left(uh-\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right)\Bigg|_{x_1}-\left(uh-\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right)\Bigg|_{x_2}=\int_{x_1}^{x_2}\frac{\partial h}{\partial t}\,dx \qquad \text{(D1)}$$

The first term on the left is the rate of lubricant inflow and the second term outflow. The right hand side is the rate of accumulation of the lubricant in the control volume. Rearrange Eq. (D1) as:

$$u(h_{x_1}-h_{x_2})-\left(\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right)\Bigg|_{x_1}-\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right)\Bigg|_{x_2}\right)=\int_{x_2}^{x_2}\frac{\partial h}{\partial t}\,dx \qquad \text{(D2)}$$

Let h_{x_1} be a local maximum and h_{x_2} the adjacent local minimum downstream. If the rough surface is stationary while the smooth surface in motion (i.e., steady-state sliding, SR $= -2$), the right-hand side of Eq. (D2) is zero. Then the second term on the left-hand side must be large enough to balance the first term. Since h^3 is small and η large in the Hertzian region, $|\partial p/\partial x|$ needs to be large to maintain flow continuity, generating sharp pressure ripples. The large pressure ripples deform the roughness in such a way to reduce the difference between h_{x_1} and h_{x_2}. In the limit, $h_{x_1}\approx h_{x_2}$, or the roughness is flattened out (Fig. 7). Consider next the case of SR $= -1$ (Fig. 8) where the rough surface is also in motion but moves more slowly than the smooth surface. The right-hand side of Eq. (D2) is positive but can be shown to be smaller than the first term on the left-hand side. A smaller $|\partial p/\partial x|$ is needed in this case than in the zero-right-hand-side case to maintain flow continuity. In the case of pure rolling (Fig. 9), the right-hand side of Eq. (D2) is larger than the previous case of SR $= -1$ and is about equal to the first term on the left-hand side (authors' results suggest exact equal). Consequently, there is little need to generate pressure ripples to maintain flow continuity. As SR further increases, the right hand side of Eq. (D2) becomes greater than the first term on the left, pressure ripples are then generated to balance the difference between these two terms, the larger the difference, the larger the pressure ripples.

If the shear-thinning behavior of the lubricant is modeled, another competing factor enters the system which may significantly change the pressure-ripple generation. Consider again the case of steady-state sliding (Fig. 7). With SR $= -2$ (or SR $= 2$), the lubricant exhibits the strongest shear thinning, which substantially reduces the effective viscosity of the lubricant. For the given problem (i.e., Fig. 7), the effective viscosity with shear thinning (by Eyring viscous law) is about two to three orders of magnitude smaller than the (two-slope-law) viscosity. Therefore, a much smaller $|\partial p/\partial x|$ is needed to generate significant pressure-induced flow of lubricant to maintain flow continuity. Smaller $|\partial p/\partial x|$ means smaller pressure ripples and thus smaller roughness deformation. The roughness does not have to be flattened out and flow continuity can still be satisfied. Next, consider the case of SR $= -1$ again (Fig. 8). The shear thinning is weaker and therefore the effective viscosity is larger in this case than in the case of SR $= -2$. Whether larger or smaller pressure ripples will be generated depends on the changes in the following two competing mechanisms as SR varies. One is the change in the difference between the first term on the left-hand side of Eq. (D2) and the right-hand side. The other is the change in the effective viscosity due to shear thinning. The maximum pressure rippling may be generated at a slide-to-roll ratio at which the shear thinning is weak while the different between the first term on the left-hand side of Eq. (D2) and the right-hand side is sufficiently large. Most important of all is that, in any case, the magnitudes of pressure ripples seem to be limited by one of these two competing mechanisms.

Since EHL lubricants can exhibit strong shear-thinning behavior which may substantially affect micro-EHL conditions, it is important to incorporate this behavior into the rheological model in micro-EHL analyses.

Authors' Closure

The authors are grateful for the constructive discussions by Drs. Greenwood and Morales-Espejel and Dr. Chang. We would like to respond to the discussions respectively.

Response to Drs. Greenwood and Morales-Espejel

As mentioned by the discusser, the pressure prediction models developed by the two groups are not in a very good agreement, at least for Eqs. (D-1) and (D-2) in their appearances; the former is strictly linear in A, the amplitude of roughness component, while the latter shows some nonlinearity in A. However, the difference could be well understood by examining the basis upon which the models are developed.

We all agree that for stationary roughness under heavily loaded conditions the roughness amplitude is greatly diminished, particularly for those low frequency components, and that the Reynolds equation shows a great deal of linearity which indicates the possible applicability of superposition. However, it by no means implies that the EHL is a strict linear system. Since Eq. (D-2) is regressed directly from numerical simulation results for a relatively wide range of operation conditions, the nonlinearity in A is expected. It reflects the nonlinearity of EHL system.

Obviously, the exponent of A in Eq. (D-2) depends on the range of load used in the regression. For Eq. (D-2) the regression covered 41 sets of simulation results with the load ranges form $W=4.33\times10^{-4}$ to 2.17×10^{-5}. If only high load ($W=4.33\times10^{-4}$) results are used, regression yields

$$\Delta P=C\bullet A^{0.955}L^{-1.01}$$

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