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same initial flow conditions and the four boundary layer representations. The choice of the $^{1}/_{7}$ or $^{1}/_{5}$ profile has negligible effect. The no boundary layer and no wall shear stress solution is poorer in region II but slightly better in region III.

However, the inclusion of the boundary layer does have a significant effect on the theoretical pressure distribution (Fig. 6). The no boundary layer and no wall shear representation yields considerably different results from that of the $^{1}/_{7}$ power boundary layer solution or the experimental data. Also, this pressure distribution never exhibits the favorable pressure gradient which is present in the final stages of flow development. The inclusion of the wall shear (but no boundary layer) decreases the error and does predict the final negative pressure gradient. The best results are obtained with the $^{1}/_{7}$ and $^{1}/_{5}$ models.

Figs. 7 and 8 shows a comparison of the experimental results of Razinsky and Brighton [17], the present analysis ($^{1}/_{7}$ boundary layer) and the theoretical works of Mikhail [4] and Abramovich [8] for a velocity ratio of 2 and radius ratio of $^{1}/_{6}$. The pressure distribution once again, demonstrates the importance of the inclusion of the wall boundary layer.

Conclusions

This investigation covered an analytical study of nonseparated jet mixing in ducts of constant diameter. Predictions were made for the flow development from the initial step change in velocity profile to the final fully developed conditions, and included the effects of the wall boundary layer. Two important conclusions are:

1 The analytical predictions indicate that the jet spreads more rapidly (both inward and outward) for larger velocity ratios. The influence of velocity ratio on the boundary layer growth is as pronounced as it is for the jet spread rate.

2 Including the boundary layer in the model gave results which were significantly different and in better agreement with experiment. This conclusion was established for the velocity ratios of this investigation. For higher velocity ratios the boundary layer becomes progressively less significant.

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DISCUSSION

H. A. Becker[3]

I assume that this paper represents the authors' ideas about the theoretical interpretation of their earlier experimental study,[4] in addition to being a general contribution to the calculation of the mean-flow characteristics of confined jets. One of my comments in the discussion of that work was that the authors had produced some new and apparently excellent data on confined-jet turbulence characteristics, and that their work complemented that of other investigators, providing the first comprehensive examination of the post-jet-mixing zone of transition to turbulent pipe flow.

The principal conclusion of the present paper, to me at least, is the finding that the wall boundary layer cannot, in general, be neglected if fairly exact predictions of the mean flow field, and particularly the static pressure distribution, are required. The analysis demonstrates one approach, among the several that might be adopted, to the inclusion of this often significant feature of the flow. The other novel feature of the analysis is the calculation of the post-jet-mixing zone (Region III) which other authors have treated scantily or not at all.

The general approach in the paper is to begin with a set of rather simple assumptions about the shapes of velocity profiles and the nature of turbulent transport. The radially integrated equations of motion are then solved. Finally, some of the predictions are compared with experimental data. This approach leaves me asking questions of two kinds: (i) are some of the assumptions too simple, and (ii) how much has the theory contributed to understanding the experimental data? I will now put to the authors my queries and observations in this relation, and I hope they will comment on some of the points that I shall raise.

1 The boundary layer formulas used in the analysis are for flat plates without a longitudinal pressure gradient. Might not neglect of the effects of pressure gradients on velocity profile shape and wall shear introduce errors comparable to those produced by neglecting the boundary layer altogether?

2 The presence of the wall boundary layer, if significant, should be noticed in the velocity profiles, and the authors' experimental data[4] do indeed reveal it. It did not, therefore, require a theoretical analysis to deduce that wall boundary layers might be important. The effects on, e.g., the pressure distribution, can be estimated from the experimental data. Have the authors attempted this? Also, have they made a detailed examination of the data in respect to velocity profile shape, boundary layer thickness, and the effects of longitudinal pressure gradients on these? In my own work,[5] though in a different confined-jet flow regime, the boundary layer thickening and change in profile shape resulting from adverse pressure gradients was very evident.

3 In addition to the boundary layer data, there is a mass of material in the authors' experimental study[4] that is left un-

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touched in the present paper, and of which a more detailed examination would be useful. Measurements were made, for example, of the profiles of mean velocity and turbulent shear stress, from which eddy transport coefficients could be calculated. The turbulence and mean-flow data are sufficient to evaluate most items in the turbulent energy balance and allow a study of turbulent energy decay in the post-jet-mixing zone. Such further interpretative work would provide more useful information about transport processes in confined jets, and suitable transport hypotheses for the calculation of confined jets, than does the approach taken by the authors in the present paper. Are the authors doing, or contemplating, any further work on the data in this direction?

4 In my discussion of the earlier paper,[4] I noted that the flow similarity parameters adopted by the authors—the velocity ratio and the diameter ratio—are not the best. Among several forms that have been proposed for a more revealing criterion, the Craya-Curtet number, *Ct*, appears to be fundamentally right, being a direct consequence of the conservation of mass and momentum. In my remarks then [17], I gave, and discussed, a generalized definition of *Ct* which becomes, for the present case, simply

$$Ct = \left(\frac{\dot{m}_2}{\dot{m}_1}\frac{U_2}{U_1 - U_2}\right)^{1/2},$$

where $\dot{m}_1$ is the primary stream (jet nozzle) mass flux, $\dot{m}_2$ is the secondary stream mass flux, and U_1 and U_2 are, respectively, the primary and secondary feed stream velocities. For point-source jets, the criterion *Ct* is sufficient by itself. When, as in the present case, point-source behavior is not sufficiently approximated in a significant region of the system, the diameter ratio D_1/D_2 must be introduced as a second parameter to account for the inadequacy of *Ct* alone. The authors should find that the effect, in their work, of going from $D_1/D_2 = {}^1/_6$ to $D_1/D_2 = {}^1/_3$ is much more clearly rationalized by using the parameter pair (Ct, D_1/D_2) than by (U_1/U_2, D_1/D_2).

S. Wolf[6]

The authors have followed their previously reported experimental work with a useful analysis for problems associated with confined jet mixing.

A shortcoming of the presented analysis is that the model does not include the flow region in which the jet and wall boundary layer have merged while the jet still has a potential core. This condition exists for the authors' experimental data having a $^1/_3$ radius ratio, r_j/r_s, with velocity ratios, u_{j0}/u_{p0}, of 2, 5, and 9, as discussed by the authors. These cases, corresponding to secondary-to-primary mass flow rate ratios of 2.67, 1.33, and 0.8, have significant industrial application in jet pump work. Consequently, the extension of the theory to cover these cases should improve its usefulness.

The authors' theoretical pressure distribution for a radius ratio of $^1/_6$ and velocity ratio of 2, Fig. 8, shows good agreement with experimental data in the initial region of $x/r_s \leq 2$; however, for $x/r_s > 2$, significant disagreement exists. The predictions from the theory of Abramovich and Mikhail are presented and show even greater disagreement. I have found the theory of Hill, authors' references [5 and 6] useful in jet pump investigations. The jet region is replaced by a point source at the nozzle exit so that agreement with experimental data is not expected in the "near field" region. Wall shear stress is taken into account with a friction coefficient which can be adjusted on the basis of experience to obtain good agreement between experimental data and analysis for static pressure distributions beyond the near field region. Two predictions of pressure distribution based on Hill's theory with wall friction coefficient, $C_f = 2\tau_w/\rho u_p^2$, of 0.005 and 0.008 are compared in Fig. 9 with the authors' experimental data and their new analysis. The best agreement with experimental data in the "far field" region is obtained with the prediction based on Hill's theory having $C_f = 0.008$.

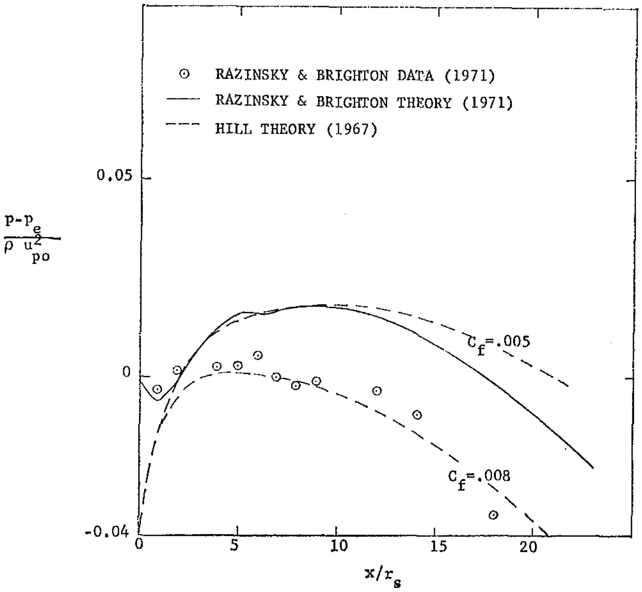

Fig. 9 Pressure distribution for jet mixing in a constant area duct, $u_{j0}/u_{p0} = 2$, $r_j/r_s = {}^1/_6$

Authors' Closure

The authors would like to thank Professor Becker and Dr. Wolf for their comments.

With regard to Professor Becker's suggestions for further interpretation of the experimental data presented in our previous paper [17] and the authors' reference [18], present work schedules and commitments preclude analysis and publication in the immediate future. As stated previously [17] we are in general agreement with Professor Becker's point that an adequate description of the wide range of entrance conditions (jet and secondary stream velocity and dimensions) is not provided by the Craya-Curtet number alone. The work of Exley [16] gives a more detailed discussion of the necessity of a two parameter flow specification. The authors agree that the revised definition of the Craya-Curtet number presented by the discusser in a critique of the author's previous paper [17] would be preferred in some flow situations beyond those considered in the present work, for example, mixing of streams with different densities or with nonuniformities.

Dr. Wolf's comparison of the authors' data, theoretical predictions, and the predictions of the Hill theory for the case of a velocity ratio of 2 and a radius ratio of $^1/_6$ show the sensitivity of pressure distribution calculations on the selection of the magnitude of stress model constants (friction coefficient in the case of the Hill analysis and the jet eddy viscosity Reynolds number and wall stress constants in the author's work). Our analysis has shown that adjustment of R_T for each of the three flow regions considered results in noticeable differences in the predictions of region length, center line velocity magnitude, and boundary layer growth, in addition to the pressure distribution. The importance of these other variables should not be overlooked, as, for example, a decrease in the length of the initial region changes the local flow model with its appropriate differential equations and, most significantly, the location of the

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initialization of the sharp decrease in center line velocity characteristic of the second region. The present work presents results in which the values of R_T were selected to give the best agreement for the foregoing variables and for the flow range covered by our data. Early work on the authors' analysis indicated that the use of $R_T = 39$ in the three flow regions gave very good agreement for the pressure distribution for the first 15 radii, but was somewhat poorer in its center-line velocity predictions. A more detailed treatment of the thinking that went into the present analysis can be found in [18]. The authors feel that presentations of Dr. Wolf's analysis with the Hill theory for flow variables in addition to the pressure distribution are necessary before full evaluation of the theoretical results could be made.

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