

ratio, except that for pure rolling the amplitude is somewhat smaller. But the present results, exemplified by Figs. 7 to 11, seem to demolish our theory completely. As we said above, we are dismayed.

Additional References

A Greenwood, J. A., and Morales-Espejel, G. E., "The Behavior of Transverse Roughness in EHL Contacts," to appear in *I.Mech.E. Journal of Engineering Tribology*, Apr. 1994.

B Ai, X., and Cheng, H. S., "A Fast Model for Pressure Profile in Rough EHL Line Contacts," *ASME JOURNAL OF TRIBOLOGY*, Vol. 115, 1993, pp. 460-465.

C Venner, C. H., and Lubrecht, A. A., 1992, "Transient Analysis of Surface Features in an EHL Line Contact in the Case of Sliding," submitted to *ASME JOURNAL OF TRIBOLOGY*.

D Venner, C. H., "Multilevel Solution of the EHL Line and Point Contact Problems," Ph.D. thesis, University of Twente, Netherlands, 1991.

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The authors present transient results of EHL between a perfectly smooth surface and a rough surface of random roughness. This is a significant step forward from earlier steady-state analyses with a moving smooth surface and a stationary rough surface. Figures 7 to 11 show that abrupt pressure rippling is generated (except in pure rolling), and the magnitudes of the pressure ripples increase with sliding. This discussor speculates that different micro-EHL results might have been obtained had the rheological model included the shear-thinning characteristics of the lubricant. A brief analysis follows.

Consider two locations inside the Hertzian region, x_1 and x_2 which, along with the two surface segments in between, define a control volume. Neglect lubricant compressibility, flow continuity in this volume gives:

$$\left(u h - \frac{h^3}{12\eta} \frac{\partial p}{\partial x} \right) \Big|_{x_1} - \left(u h - \frac{h^3}{12\eta} \frac{\partial p}{\partial x} \right) \Big|_{x_2} = \int_{x_1}^{x_2} \frac{\partial h}{\partial t} dx \quad (D1)$$

The first term on the left is the rate of lubricant inflow and the second term outflow. The right hand side is the rate of accumulation of the lubricant in the control volume. Rearrange Eq. (D1) as:

$$u(h_{x_1} - h_{x_2}) - \left(\left(\frac{h^3}{12\eta} \frac{\partial p}{\partial x} \right) \Big|_{x_1} - \left(\frac{h^3}{12\eta} \frac{\partial p}{\partial x} \right) \Big|_{x_2} \right) = \int_{x_2}^{x_1} \frac{\partial h}{\partial t} dx \quad (D2)$$

Let h_{x_1} be a local maximum and h_{x_2} the adjacent local minimum downstream. If the rough surface is stationary while the smooth surface in motion (i.e., steady-state sliding, $SR = -2$), the right-hand side of Eq. (D2) is zero. Then the second term on the left-hand side must be large enough to balance the first term. Since h^3 is small and η large in the Hertzian region, $|\partial p / \partial x|$ needs to be large to maintain flow continuity, generating sharp pressure ripples. The large pressure ripples deform the roughness in such a way to reduce the difference between h_{x_1} and h_{x_2} . In the limit, $h_{x_1} \approx h_{x_2}$, or the roughness is flattened out (Fig. 7). Consider next the case of $SR = -1$ (Fig. 8) where the rough surface is also in motion but moves more slowly than the smooth surface. The right-hand side of Eq. (D2) is positive but can be shown to be smaller than the first term on the left-hand side. A smaller $|\partial p / \partial x|$ is needed in this case than in the zero-right-hand-side case to maintain flow continuity. In the case of pure rolling (Fig. 9), the right-hand side of Eq. (D2) is larger than the previous case of $SR = -1$ and is about equal to the first term on the left-hand side (authors' results suggest exact equal). Consequently, there is little need to generate pressure ripples to maintain flow continuity. As SR fur-

ther increases, the right hand side of Eq. (D2) becomes greater than the first term on the left, pressure ripples are then generated to balance the difference between these two terms, the larger the difference, the larger the pressure ripples.

If the shear-thinning behavior of the lubricant is modeled, another competing factor enters the system which may significantly change the pressure-ripple generation. Consider again the case of steady-state sliding (Fig. 7). With $SR = -2$ (or $SR = 2$), the lubricant exhibits the strongest shear thinning, which substantially reduces the effective viscosity of the lubricant. For the given problem (i.e., Fig. 7), the effective viscosity with shear thinning (by Eyring viscous law) is about two to three orders of magnitude smaller than the (two-slope-law) viscosity. Therefore, a much smaller $|\partial p / \partial x|$ is needed to generate significant pressure-induced flow of lubricant to maintain flow continuity. Smaller $|\partial p / \partial x|$ means smaller pressure ripples and thus smaller roughness deformation. The roughness does not have to be flattened out and flow continuity can still be satisfied. Next, consider the case of $SR = -1$ again (Fig. 8). The shear thinning is weaker and therefore the effective viscosity is larger in this case than in the case of $SR = -2$. Whether larger or smaller pressure ripples will be generated depends on the changes in the following two competing mechanisms as SR varies. One is the change in the difference between the first term on the left-hand side of Eq. (D2) and the right-hand side. The other is the change in the effective viscosity due to shear thinning. The maximum pressure rippling may be generated at a slide-to-roll ratio at which the shear thinning is weak while the difference between the first term on the left-hand side of Eq. (D2) and the right-hand side is sufficiently large. Most important of all is that, in any case, the magnitudes of pressure ripples seem to be limited by one of these two competing mechanisms.

Since EHL lubricants can exhibit strong shear-thinning behavior which may substantially affect micro-EHL conditions, it is important to incorporate this behavior into the rheological model in micro-EHL analyses.

Authors' Closure

The authors are grateful for the constructive discussions by Drs. Greenwood and Morales-Espejel and Dr. Chang. We would like to respond to the discussions respectively.

Response to Drs. Greenwood and Morales-Espejel

As mentioned by the discussor, the pressure prediction models developed by the two groups are not in a very good agreement, at least for Eqs. (D-1) and (D-2) in their appearances; the former is strictly linear in A , the amplitude of roughness component, while the latter shows some nonlinearity in A . However, the difference could be well understood by examining the basis upon which the models are developed.

We all agree that for stationary roughness under heavily loaded conditions the roughness amplitude is greatly diminished, particularly for those low frequency components, and that the Reynolds equation shows a great deal of linearity which indicates the possible applicability of superposition. However, it by no means implies that the EHL is a strict linear system. Since Eq. (D-2) is regressed directly from numerical simulation results for a relatively wide range of operation conditions, the nonlinearity in A is expected. It reflects the nonlinearity of EHL system.

Obviously, the exponent of A in Eq. (D-2) depends on the range of load used in the regression. For Eq. (D-2) the regression covered 41 sets of simulation results with the load ranges from $W = 4.33 \times 10^{-4}$ to 2.17×10^{-5} . If only high load ($W = 4.33 \times 10^{-4}$) results are used, regression yields

$$\Delta P = C \cdot A^{0.955} L^{-1.01}$$

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which is very much close to the idealized linear relationship.

The linear relation in A is only obtainable in dry contacts, as mentioned in [B], or in EHL contacts under extremely heavy load conditions where the viscosity of lubricant is infinitely high. The diffusive terms (pressure flow terms) vanish and the EHL system reduces to a linear system. This of course, is a highly idealized case. Under most circumstances, EHL system does exhibit some extent of nonlinear behavior. Numerical evidence shows that the induced surface feature is always smaller in amplitude as compared to the undeformed original surface feature [C, D, E, F] and that the induced surface feature has a tendency to spread out along the sliding direction [D, E]. These diffusive behaviors are governed by the nonlinear diffusive terms in the Reynolds equation, and clearly indicate the existence of these terms. Noticeable diffusive behaviors are also evidenced in the experimental investigations conducted by Wedeven and Kaneta [G, H].

Even though the Reynolds equation shows a great deal of linearity, as stated above, it still possesses nonlinear aspects. In developing our model, as the discussers did in theirs, the superposition which holds for linear system is used only as an approximation. Such approximation does not necessarily require a further approximation that Eq. (D-2) should be linear in A . In fact for most of the realistic values of A , Eq. (D-2) is very close to Eq. (D-1) assuming that other parameters are well represented.

To the authors's knowledge, aside from the assumption of superposition of pressure fluctuation, an additional restriction which requires that

$$\rho H = \text{const}$$

has been imposed by the discussers in their model [A]. This restriction leads to a linear relation between ΔP and A , and confines the validity of the equation to extreme cases.

With respect to the moving roughness, Eq. (15) shows a continuous influence of slid-to-roll ratio SR on pressure distribution. The dependence of pressure fluctuation ΔP on SR varies with the operation conditions. For extreme high loads with stiff viscosity-pressure relationships (such as Barus), such dependence may become weak.

Response to Dr. L. Chang

The present study is based on Newtonian fluid. For lubricants which exhibit strong non-Newtonian behavior, the pressure rippling induced by surface roughness may be largely suppressed by rheological properties of the lubricants, particularly for lightly loaded cases. [E] compared the pressure fluctuations induced by sinusoidal waviness between Newtonian and non-Newtonian lubricants. The operation conditions are $W = 4.33 \times 10^{-5}$, $U = 1.3 \times 10^{-4}$, $G = 4616$. Results showed a reduction of 40 percent in pressure fluctuation with the non-Newtonian lubricant. Again, under heavily loaded conditions, the difference between Newtonian and non-Newtonian is expected to be smaller.

Additional References

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