Combined particle swarm optimization and fuzzy inference system model for estimation of current-induced scour beneath marine pipelines

M. Zanganeh, A. Yeganeh-Bakhtiary and R. Bakhtyar

ABSTRACT

In this paper the capability of Particle Swarm Optimization (PSO) is employed to deal with an Adaptive Network based Fuzzy Inference System (ANFIS) model's inherent shortcomings to extract optimum fuzzy if–then rules in noisy areas arising from the application of nondimensional variables to estimate scour depth. In the model, a PSO algorithm is employed to optimize the clustering parameters controlling fuzzy if–then rules in subtractive clustering while another PSO algorithm is employed to tune the fuzzy rule parameters associated with the fuzzy if–then rules. The PSO model's objective function is the Root Mean Square (RMSE), by which the model attempts to minimize the error in scour depth estimation with respect to its generalization capability. To evaluate the model's performance, the experimental datasets are used as training, checking and testing datasets. Two-dimensional and nondimensional models are developed such that in the dimensional model the mean current velocity, mean grain size, water depth, pipe diameter and shear boundary velocity are used as input variables while in the nondimensional model the pipe, boundary Reynolds numbers, Froude number and normalized depth of water are set as input variables. The results show that the model provides an alternative approach to the conventional empirical formulae. It is evident that the developed PSO–FIS–PSO is superior to the ANFIS model in the noisy area in which the input and output variables are slightly related to each other.

Key words | ANFIS, clustering parameters, gradient-based algorithms, noisy area, PSO, scour estimation

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NOMENCLATURE

A_i, B_i, C_i, D_i , E_i:	fuzzy sets	Re:	pipe Reynolds number
D:	pipe diameter	S:	equilibrium scour depth
E:	mean square error	SI:	scatter index
Fr:	Froude number	$\overline{T}$:	average of target outputs
G_s:	sediment specific gravity	U:	mean current velocity
N:	number of data	μ:	fluid kinematic viscosity
O:	observed value	ρ:	fluid density
P:	estimated value	θ:	Shields parameter
P_o:	potential data point	α:	nonlinear antecedent parameter
Re_*:	boundary layer Reynolds number	η:	learning rate

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n	dimension
n_e:	number of training epochs
a:	standard deviation of Gaussian membership function
c:	the center of Gaussian membership function
c_1 and c_2:	cognitive and social parameters in the PSO algorithm
d:	distance between the candidate clusters
d_{50} :	mean grain size
g:	gravitational acceleration
h:	water depth
$iter_{\max}$:	maximum of iteration number
p_g:	gbest
$p_{b:}$:	pbest
o_i, p_i, q_i, r_i, s_i, t_i:	the linear consequent parameters
ra:	radii of clusters
ran_1, ran_2:	random numbers generated uniformly
v:	particle velocity
w:	firing strength
x:	particle position
z:	data point
γ:	squash factor
$\bar{\epsilon}$:	acceptance ratio
$\underline{\epsilon}$:	rejection ratio and the relative distance criterion
χ:	constriction factor
ω:	inertia weight
σ:	Gaussian membership function

INTRODUCTION

Local scouring beneath marine pipelines is a crucial problem that results in the rupture of pipelines and finally environmental and economical damages (Mao 1986). Estimation of scour in the vicinity of a pipeline laid on the seabed due to a current is of great importance in the design and maintenance of marine pipelines. Thus, recently the scouring mechanism beneath marine pipelines has generated considerable research interest. There are mostly two types of available approaches for the estimation of scour: (i) physical and empirical approaches based on experimental data and (ii) numerical modeling approaches. In the experimental studies, which are mainly conducted in two-dimensional flumes, it is attempted to find a relationship among more effective variables such as Shields parameter, current mean velocity, water depth and sediment properties below pipelines with the equilibrium scour depth (e.g. Kjeldsen *et al.* 1973; Leeuwenstein *et al.* 1985). Hence, a wide range of empirical formulae or parametric models have been proposed for the estimation of scour hole dimensions. Because of the complexity involved in the scour process and measurement difficulties, there is still much interest in investigating the scour process beneath marine pipelines.

Numerical models have become increasingly popular in the simulation of local scouring, and can be divided into potential flow and viscous/turbulent flow theory based models. The potential flow-based models are able neither to simulate the flow separation and vortex shedding nor to estimate the correct scour profile (Liang & Cheng 2001). To deal with the foregoing shortcomings researchers applied a viscous-based model in which the turbulence models such as $k-\epsilon$ and $k-\omega$ are subjected to a Reynolds Averaged Navier–Stokes (RANS) equation solver (Liang & Cheng 2004a, b). Recently, two-phase flow models have also been developed to simulate delicately the scour process, coupling the sediment and fluid phases in a two-way formulation (Zhao & Fernando 2007). Although the reviewed studies show that the numerical models are rather accurate in their estimation of scour, applying these kinds of sophisticated models are both expensive and very time-consuming despite their efficiency in simulating bed evolution.

Recently, soft computing tools such as Artificial Neural Networks (ANN) and Fuzzy Inference Systems (FIS) have been used due to their simplicity in order to predict different complex phenomena in river and coastal activities such as nearshore processes (Liriano & Day 2001; Cheng *et al.* 2002; Ruessink 2005; Muttil & Chau 2006; Chau & Muttil 2007; Bakhtyar *et al.* 2008, 2009; Shourian *et al.* 2008) and wind-driven wave prediction (e.g. Deo *et al.* 2001; Kazeminezhad *et al.* 2005; Mahjoobi *et al.* 2008; Zanganeh *et al.* 2009). ANN models are based on the mathematical mapping between input and output variables in a nonlinear system, while FIS is a model on the basis of linguistic functions, the so-called fuzzy if–then rules. ANN and the Adaptive-Network-Based Fuzzy Inference System (ANFIS) were employed to estimate the scour depth around piles (Kambekar & Deo 2003; Bateni & Jeng 2007; Bateni *et al.* 2007a, b). More recently, Kazeminezhad *et al.* (2008) applied ANN to estimate equilibrium wave-induced scour depth and pointed out that the ANN model can be a suitable alternative to empirical methods in estimating scour equilibrium depth in terms of their accuracy.

In addition there are numerous applications of genetic fuzzy systems in various engineering problems. A state-of-the-art review on genetic fuzzy systems can be found in Cordon *et al.* (2004). Lie *et al.* (2007) proposed a method in which Genetic Algorithms (GAs) and ANFIS are combined and applied to the fault diagnosis of rolling element bearings. Ho *et al.* (2008) used a genetic fuzzy inference system in which the parameters of membership functions and the consequent part of an FIS are optimized by GA for the prediction of surface roughness in an end milling process. Zanganeh *et al.* (2009) more recently used Genetic Algorithm–Adaptive-Network-based Fuzzy Inference Systems (GA-ANFIS) in the prediction of significant wave parameters in the duration-limited conditions at Lake Michigan. The model employed a GA to generate clustering parameters, whereas ANFIS is being employed to optimize the FIS associated with the clustering parameters. In the study it is concluded that the effective variables have got less clustering radii than the noneffective ones. The ANFIS model in their developed GA-ANFIS model is a gradient-based model; hence, applying another GA in order not to trap answers in a local optimum can be a suitable substitute to optimize fuzzy if–then rule parameters.

The GA is a kind of algorithm in which the number of the population and operators is much more than in the Particle Swarm Optimization (PSO) algorithm. Thus employing other evolutionary algorithm such as PSO commonly with lower population and less operators than the GA can be an alternative approach (Kennedy & Eberhart 1995). The newly common PSO algorithm has been used as an optimization tool in combination with simulation models in optimal design of some civil engineering problems. Chau (2006) used PSO in combination with ANN to train perceptrons and developed a model for the prediction of water level in the Shing Mun River.

Noting the deficiencies discussed in both GA and ANFIS models, it is still possible for the PSO algorithm to be investigated as an alternative tool for both GA and ANFIS models. To demonstrate the PSO capability, this model with a conjugated FIS model is employed to estimate the equilibrium current-induced scour depth as a complex phenomenon in which applying nondimensional variables to have a generalized model makes the estimation and relationships among involved inputs and output variables noisy. In the estimator model, a PSO algorithm is implemented to optimize the number and structure of FIS, while another PSO algorithm is used to tune the fuzzy if–then rule antecedent and consequent parameters with respect to minimization of the estimated error. For this reason, this paper can be divided into five sections in which the second section is in four subsections and outlines the main concepts of Fuzzy Inference Systems (FISs), PSO algorithm and the combined PSO–FIS–PSO model. In the remaining sections the capability of the model is employed to estimate the equilibrium scour depth underneath pipelines accompanied with sensitivity analysis against the involved dimensional and nondimensional variables, and a brief discussion on the developed model is provided for estimation of the phenomenon.

STRUCTURES OF PSO–FIS MODELS

Fuzzy Inference Systems (FIS)

FIS is a suitable substitute for the approximation of ill-defined nonlinear functions. In addition, the FIS model is able to subject qualitative aspects of human knowledge and reasoning processes by datasets without employing precise quantitative analysis through five functional components. Figure 1 schematically outlines the relation between these five functional components, whose explanation is given as follows (Jang 1993):

- A rule base containing a number of fuzzy if–then rules.
- A database defining the membership functions of fuzzy sets.
- A decision-making unit as the inference engine.
- A fuzzification interface which transforms crisp inputs into linguistic variables.
- A defuzzification interface converting fuzzy output to crisp outputs.

Since having an efficient fuzzy rule base to make decisions is a crucial issue in an FIS, clustering methods can be a suitable option to extract fuzzy if–then rules in terms of their

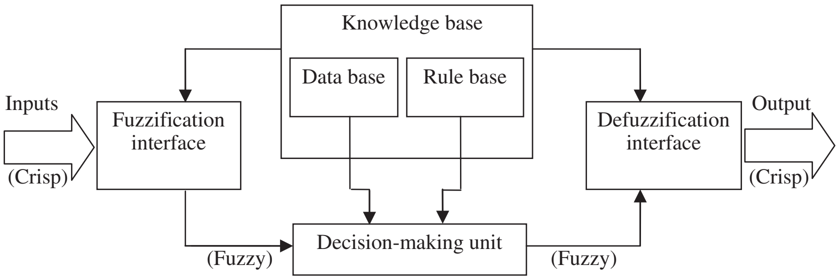


Figure 1 | FIS structure (Jang 1993).

simplicity and accessibility compared to expert methods. To do so, in this paper extracting of the fuzzy if–then rules is attempted by using the so-called subtractive clustering method, in which the radius of clustering and the squash factor as clustering parameters would control the number and structure of fuzzy rules. This model performs on the basis of the potential value of each data point to be chosen as a cluster center, given as follows for a collection of K data points (Chiu 1994):

$$P_{ok} = \sum_{j=1}^{K} \exp\left(-4\sqrt{\sum_{i=1}^{n}\left(\frac{z_k^i - z_j^i}{ra_i}\right)^2}\right) \quad (1)$$

where P_{ok} is the potential of the kth data point, ra_i is the cluster radius associated with the ith dimension of the point and z_k^i is the ith-dimensional data point. After calculation of the potential value of each data point, the point with the highest potential is selected as the first cluster center. Then, the potential value of each data point z_k is reduced by the following equation:

$$P'_{ok} = P_{ok} - P^*_{o1}\exp\left(-4\sqrt{\sum_{i=1}^{n}\left(\frac{z_k^i - z_{C1}^i}{\gamma ra_i}\right)^2}\right) \quad (2)$$

where P'_{ok} is the reduced potential value of the kth data point, $P^*{}_{o1}$ is the first center potential value and γ is a squash factor. New cluster centers are determined based on the acceptance ratio, $\bar{\epsilon}$ is the rejection ratio and $\underline{\epsilon}$ is the relative distance criterion. A data point with a potential greater than the acceptance threshold is directly chosen as a cluster center. The acceptance level of data points with potential values between the upper and lower thresholds depends on the relative distance calculated by the following formula:

$$d_{\min} + \frac{P^*_{ok}}{P^*_{o1}} \geq 1 \quad (3)$$

where $d_{\min}$ is the nearest distance between the candidate cluster center and all cluster centers previously found as follows:

$$d_{k,c} = \left(\sqrt{\sum_{i=1}^{n}\left(\frac{z_k^i - z_{C1}^i}{ra_i}\right)^2}\right) \quad (4)$$

where $d_{k,c}$ is the distance of the kth data point, in which c_i is the previously found cluster center.

In the ANFIS model each cluster center would represent a fuzzy if–then rule. Assuming the Gaussian membership function due to its efficiency in a noisy media (Kreinovich 1998), the cluster center i is considered as the mean value (c_i) of the membership function, while the deviation variables of the membership function are calculated by

$$a_i = ra_i\left(\frac{\max(z_i) - \min(z_i)}{\sqrt{8}}\right) \quad (5)$$

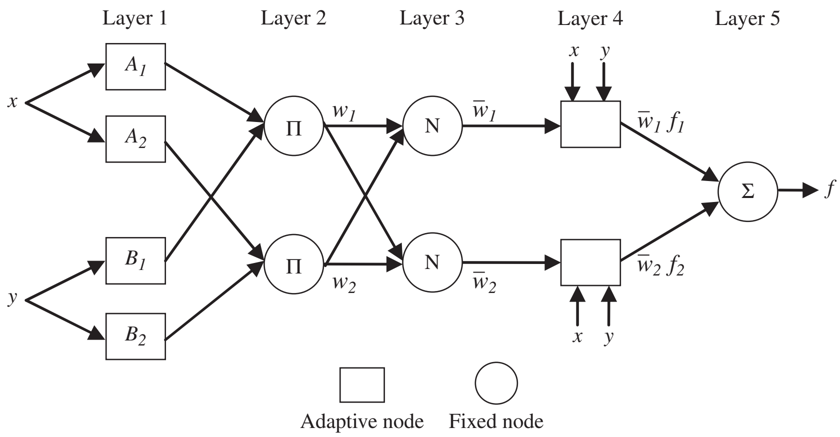
.

Following the above discussion, the cluster centers and squash factors may be considered as the parameters controlling the number and structure of the initial FIS. Figure 2 schematically sketches an initial FIS with two fuzzy if–then rules in a Takagi–Sugeno–Kang (TSK) form fuzzy rule base including two inputs x and y and one output f. The fuzzy if–then rules can be expressed as follows (Takagi & Sugeno 1985):

$$\text{Rule 1: If } x \text{ is } A_1 \text{ and } y \text{ is } B_1 \text{ then } f_1 = p_1x + q_1y + r_1,$$

$$\text{Rule 2: If } x \text{ is } A_2 \text{ and } y \text{ is } B_2 \text{ then } f_2 = p_2x + q_2y + r_2,$$

where A_1, A_2 and B_1, B_2 are fuzzy sets of input premise variables x and y, respectively, and p_1, q_1, r_1 and p_2, q_2, r_2 are the consequent parameters. According to Figure 2, the FIS model contains the following five layers with respect to its conjunction with tuning algorithms such as PSO and/or ANN.



Fgure 2 | FIS model structure coupled with PSO and ANN (Jang 1993).

Layer 1. All the nodes in this layer are adaptive. It contains membership functions of input variables. Each node i in this layer is represented by a function as

$$O_i^1 = \sigma_{A_i}(x) \tag{6}$$

$$O_i^1 = \sigma_{B_i}(x) \tag{7}$$

where A_i is the linguistic variable, x is the input to node i and O_i^1 is the membership of A_i, which is usually defined by a bell-shape function with maximum and minimum values equal to 1 and 0 as follows:

$$\sigma_{A_i}(x) = \exp\left[-\left(\frac{x - c_i}{a_i}\right)^2\right] \quad i = 1, 2 \tag{8}$$

where a_i is the standard deviation (SD) and c_i is the center of the above Gaussian membership function obtained from subtractive clustering.

Layer 2: The fixed nodes in this layer are T-norm operators like the AND operator. The output of each node in this layer represents the firing strength of the associated rule as follows:

$$w_i = \sigma_{A_i}(x) \times \sigma_{B_i}(x), \quad i = 1, 2. \tag{9}$$

Layer 3: Nodes in this layer are fixed nodes and the normalized ratio of the i^{th} rule's firing strength to the sum of all rules' firing strength is calculated in this layer as

$$\overline{w} = \frac{w_i}{w_1 + w_2} \quad i = 1, 2. \tag{10}$$

Layer 4: All nodes in this layer are adaptive. The output of each node (rule) is simply the product of normalized firing strength and a first-order polynomial:

$$O_i^4 = \overline{w}_i f_i = \overline{w_i}(p_i x + q_i y + r_i) \tag{11}$$

where p_i, q_i, r_i are parameters of the consequent part of rule i.

Layer 5: This layer has only one node labeled $\sum$ to indicate that it performs as a simple sum over all outputs coming from layer 4.

Particle Swarm Optimization (PSO) algorithm

The PSO algorithm is a member of the wide category of swarm intelligence methods for solving global optimization problems (Kennedy & Eberhart 1995). It was originally proposed as an optimization method in the simulation of social behavior. PSO is related to swarm theories and evolutionary computing, especially evolutionary strategies and genetic algorithms. PSO is inspired by the metaphor of social interaction observed among insects and animals. The kind of social interaction modeled within a PSO is used to guide a population of individuals (namely particles) moving towards the most promising area of the search space. According to the global PSO algorithm, each particle moves towards its best previous position and toward the best particle in the whole swarm (Kennedy & Eberhart 1995).

In a PSO algorithm, each particle is a candidate solution equivalent to a point in an n-dimensional space, so the ith particle can be represented as $x_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$. Each particle flies through the search space, depending on two important positions, $p_b = (p_{b1}, p_{b2}, \ldots, p_{bD})$, the best position the current particle has found so far (pbest), and $p_g = (p_{g1}, p_{g2}, \ldots, p_{gD})$, the global best position identified from the entire population or within a neighborhood (gbest). The rate of the ith particle's position change is given by its velocity $v_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$. Equation (12) updates the velocity for each particle in the next iteration step, whereas Equation (13) updates each particle's position in the search space (Kennedy & Eberhart 1995):

$$v_{id}^{n+1} = \chi\left(\omega v_{id}^{n} + c_1 \times ran_1^n(p_{bd}^n - x_{id}^n) + c_2 \times ran_2^n(p_{gd}^n - x_{id}^n)\right) \tag{12}$$

$$x_{id}^{n+1} = x_{id}^{n} + v_{id}^{n+1} \tag{13}$$

where $d = 1, 2, \ldots, D$; $i = 1, 2, \ldots, N$, in which N is the size of the swarm, χ is a constriction factor used in constrained optimization problems in order to control the magnitude of the velocity (in unconstrained optimization problems it is usually set equal to 1.0), ω is called the inertia weight, and c_1 and c_2 are two positive constants, called cognitive and social parameters. Carlisle & Dozier (2001) recommended that it was better to choose a larger cognitive parameter than the social parameter, but with the $c_1 + c_2 \leq 4$ limitation. A PSO model starts with a set of ran_1, ran_2 random numbers uniformly distributed in [0, 1], and $n = 1, 2, \ldots$ determines the iteration number.

Experiments with PSO indicated that it had better initially set the inertia weight to a large value to promote global exploration of the search space, and then gradually decrease it to get more refined solutions (Shi & Eberhart 1998a, b). Hence, an initial value around 1.2 and a gradual decline towards zero can be considered as a good choice. PSO updates the inertia weight using the following equation in each iteration:

$$\omega_{iter} = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{iter_{\max}} \times iter \tag{14}$$

where ω_{iter} is the iteration's inertia weight, $iter_{\max}$ is the maximum iteration number, and $\omega_{\max}$ and $\omega_{\min}$ are the maximum and minimum inertia weights, respectively. The PSO model starts with a set of randomly generated solutions, then it updates the swarm using Equations (12) and (13) in every iteration. This process is continued until the stopping criteria are met.

Combined fuzzy inference systems–particle swarm optimization

Since in the ANFIS model, the antecedent and consequent parameters of fuzzy if–then rules are tuned by gradient-based methods, answers getting stuck in the local optimum may be possible (Zanganeh *et al.* 2009). Applying an evolutionary algorithm such as PSO can be helpful as an alternative approach to cope with this deficiency. In addition, in the ANFIS model, finding the best cluster parameters is either an experience-based or a trial-and-error process that is somewhat tedious, especially when a phenomenon is influenced by a bunch of variables. Therefore, applying a random search method considering estimating the phenomenon with less error is recommended. On the basis of the above explanation, improvement of the ANFIS model in the following two views is considerable:

- Optimization of the clustering parameters appropriately by subtractive clustering in the ANFIS model.
- Using an evolutionary algorithm such as PSO instead of Steep Descend Error (SDE) and Least Square Error (LSE) methods in optimization of the fuzzy if–then rule antecedent and consequent parameters extracted by subtractive clustering.

Embedded PSO–FIS–PSO model

In this section the proposed PSO–FIS–PSO model is described in detail. This model is one in which the clustering parameters are optimized via a PSO, while another PSO model is incorporated to optimize the fuzzy if–then rule antecedent and consequent parameters. The parameters of an FIS designed for mapping input values to the desired output are optimized by the PSO–FIS–PSO model in order to minimize the total estimation error of the resulting final FIS. Figure 3 displays the flow diagram of the proposed PSO–FIS–PSO model in which the methodology of employing both embedded PSO algorithms is described. The stopping criteria is dependent on having an unchanged answer for some subsequent iterations and the objective function of the PSO optimizer would be the minimization of the RMSE of the estimations.

In solving the above optimization problem using the proposed PSO–FIS–PSO model, the weights w_i resulting

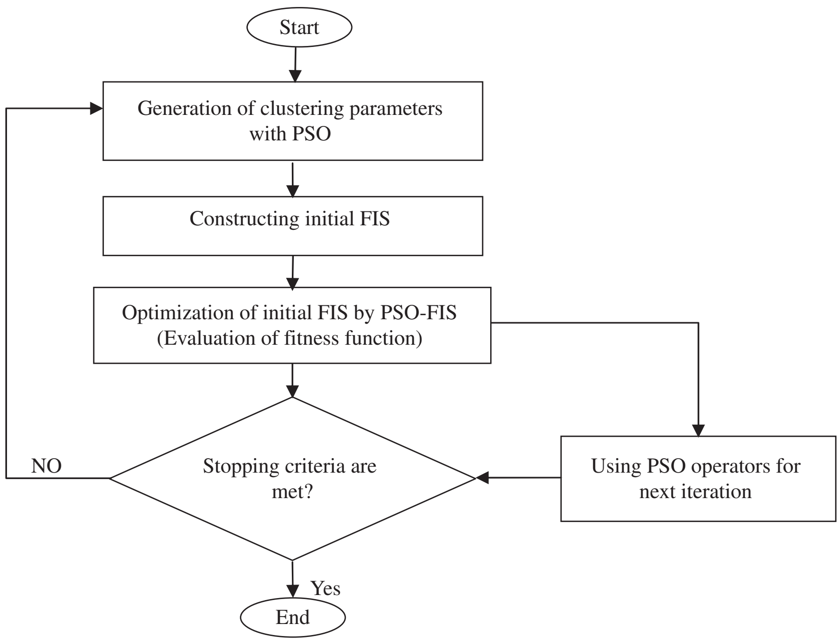


Figure 3 | Flowchart of the hybrid PSO–FIS–PSO model.

from the fuzzy antecedent parameters as well as the linear parameters such as p_1, q_1 and r_1 are tuned through a PSO learning algorithm. Therefore, the first PSO, as a global optimization method, searches for the optimal clustering parameters, while the second one is seeking for the optimal parameters of the first generated FIS by the first PSO algorithm. This embedded model may enhance the performance of the FIS model in terms of increasing exploration and exploitation capabilities for solving complicated nonlinear problems. It should be noted that the PSO–FIS–PSO model, like the other FIS-based methods, uses two sets of data. The former set is the training dataset directly used in the learning and optimization process and the latter one is known as the checking data used to avoid overtraining. An over-trained model does not have a generalization capability and performs poorly when faced with data that are not among the training dataset (Jang 1993). In order to show the efficiency of the PSO algorithm in optimization of both clustering and fuzzy antecedent and consequent parameters, the FIS models including the PSO–FIS–PSO model should be compared with the ANFIS gradient-based model. In this model, the fuzzy antecedent and consequent parameters are optimized by SD and LSE methods, respectively, while the clustering parameters are generated randomly.

The following formulae show how an ANN method tunes the FIS parameters:

$$E = \frac{1}{N}\sum_{k=1}^{N_{tm}}(O^i - P^i)^2 \tag{15}$$

$$\Delta\alpha = -\eta\frac{\partial E}{\partial \alpha} \tag{16}$$

where E is the Mean Square Error (MSE), O^i is the ith network output at a given output node, P^i is the ith target output, N is the number of data, α is a nonlinear antecedent parameter and η is the learning rate expressed as follows:

$$\eta = \frac{n_e}{\sqrt{\sum_\alpha\left(\frac{\partial E}{\partial \alpha}\right)^2}} \tag{17}$$

where n_e is the number of training epochs. More details about ANFIS models can be found in Jang (1993).

APPLICATIONS OF THE DEVELOPED MODELS

Indicating any function estimator model efficiency depends on its assessment either in a real-life problem or its comparison with the existing empirical methods for the prediction of phenomenon. To do so in this section, it is attempted to employ the developed PSO–FIS–PSO model to estimate the current-induced scouring beneath marine pipelines in which the nondimensional variables make the observed dataset noisy.

The existing empirical methods

As marine pipelines are laid on a movable bed, the changes in the flow pattern around the pipelines causes local scouring beneath them. The local scoring at the onset of scour is mainly induced by a pressure gradient; however, at the tunnel erosion stage the bed–pipeline–fluid hydrodynamic interactions (Figure 4) becomes more significant. Therefore, local scouring beneath marine pipelines depends on several variables, where each variable represents one of the physical processes involved. In that regard, considerable experimental activities have been conducted to identify the involved variables. Kejeldsen et al. (1973) pointed out that the scour depth is a function of current mean velocity and pipe diameter and proposed the following formula:

$$S = 0.972\left(\frac{U^2}{2g}\right)^{0.2} D^{0.8} \tag{18}$$

where U is the mean current velocity, S is the scour depth, g is the gravitational acceleration, and D is the pipeline diameter. In the above formula many variables, despite their importance, are ignored, e.g. the grain size of the sand bed and boundary shear velocity. Leeuwenstein et al. (1985) added the effect of sediment grains to the scour depth formula and recommended their formula known as the Delft University formula:

$$S = 0.929\left(\frac{U^2}{2g}\right)^{0.26} D^{0.78} d_{50}^{-0.04} \tag{19}$$

where d_{50} is the mean grain size of the sand bed. In the Delft University formula, some variables such as boundary shear velocity, in spite of its important effect on scour process, are neglected.

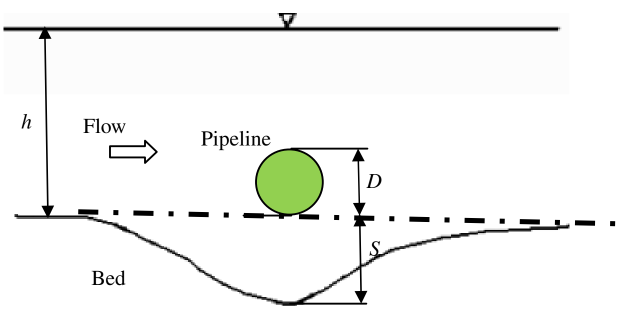


Figure 4 | Schematic sketch of pipelines along with scour hole.

Numerical illustration

To ensure that the developed PSO–FIS–PSO model is a suitable substitute for the ANFIS model, herein the model efficiency is investigated facing a real-life complex phenomenon including the estimation of scour depth beneath marine pipelines. On the other hand, to highlight the variables involved in the local scouring the results should be evaluated versus the previously explained empirical methods.

The most important issue in implementation of a soft computing model is the recognition of involved input and output variables, based on the previous experimental studies and the existing empirical methods, so that the FIS input and output variables are chosen as follows:

$$S = f(D, d_{50}, U, h, u_*, g, \mu, G_s, \rho) \tag{20}$$

where D is the pipe diameter, d_{50} is the mean grain size of the sand bed, U is the mean current velocity, h is the water depth and u_* is the shear velocity considered as the model input variables, while S as the scour depth is chosen as the only output variable of the FIS. Also g is the gravitational acceleration, μ is the fluid kinematic viscosity, ρ is the fluid density and G_s is the sediment's specific gravity. The above-mentioned parameters were chosen as the repeating parameters consistent with the theorem of Buckingham. The scour physical process either in a live-bed or in clear-water conditions is influenced by the Shields parameters, hence u_* is added to the input variables (Mao 1986).

Developing a comprehensive model for estimation of scour depth beneath marine pipelines would definitely rely on gathering a wide range of data representing the physical processes involved. To achieve this, herein the developed PSO–FIS–PSO and ANFIS estimator models are assessed using 110 data points gathered by Kejeldsen et al. (1973), Mao

(1986), Mousavi (2006) and Subhasish & Navneet (2008). Table 1 presents the statistical characteristics of the dataset such as the maximum, minimum, mean values and the range of data including the difference between maximum and minimum values of a variable. From the 110 selected data points, 60 data points were chosen as the training data, 10 data points as the checking data and the remaining 40 data points were used as the testing data to evaluate the model's performance. As shown in Table 1, the range of training data should completely cover the ranges of the checking and testing data points in order to have a well-trained model.

A great deal in the application of a FIS is the extraction of fuzzy if–then rules so that the following expression outlines the applied set of TSK (Takagi & Sugeno 1985) fuzzy if–then rules, which are implemented in the estimation of scour depth.

If D is $A_1 d_{50}$ is B_1 and U is C_1 and h is D_1 and u_* is E_1 then

$$S = o_1 D + p_1 d_{50} + q_1 U + r_1 h + s_1 u_* + t_1$$

If D is $A_2 d_{50}$ is B_2 and U is C_2 and h is D_2 and u_* is E_2 then

$$S = o_2 D + p_2 d_{50} + q_2 U + r_2 h + s_2 u_* + t_2$$

$$\ldots \tag{21}$$

If D is $A_i d_{50}$ is B_i and U is C_i and h is D_i and u_* is E_i then

$$S = o_i D + p_i d_{50} + q_i U + r_i h + s_i u_* + t_i$$

where A_i, B_i, C_i, D_i and E_i, respectively, are fuzzy values defined for pipe diameter, mean grain size of sand, mean current velocity, water depth and shear velocity, which are considered as the fuzzy antecedent parameters. Also o_i, p_i, q_i, r_i, s_i and t_i are the linear consequent parameters in the fuzzy rule-based system.

After surveying the dataset, according to the input and output variables the developed PSO–FIS–PSO model is applied in the estimation of scour depth. To understand the importance of the PSO algorithm the FISs model are categorized as the ANFIS and PSO–FIS–PSO models. Table 2 shows both employed PSO algorithm parameters in which stopping criteria only meet the number of iterations. The PSO parameters reported in the table are chosen based on the authors' experiences within a trial-and-error process. An advantage of the PSO algorithm is its performance with a population size equal to the decision variables that helps two embedded PSO algorithms to work with much less calculation load. Table 3 shows the result of the ANFIS and PSO–FIS–PSO models in which the RMSE of the training and checking datasets are considered as the objective function to tune the clustering and fuzzy antecedent and consequent parameters. It is noted that to control the robustness of the PSO model as an evolutionary algorithm in optimization of an FIS, ten runs have been performed to demonstrate their related statistical parameters such as minimum, maximum, mean and standard deviation of the optimized objective function, which is presented in Tables 3 and 4. In addition, Figure 5 shows the optimization process of the parameters

Table 1 | Statistical characteristics of datasets

	Training data (number = 60)					
	D(cm)	d_{50}(mm)	U(cm/s)	h(cm)	u_*(cm/s)	S(cm)
Mean.	7.3	0.88	34.84	21	1.89	6.31
Min.	3.0	0.07	22.0	6	0.98	1.8
Max.	50.0	3.0	86.9	143	4.40	30
Range	47	2.93	64.10	137	3.42	28.2
	Checking data (number = 10)					
	D(cm)	d_{50}(mm)	U(cm/s)	h(cm)	u_*(cm/s)	S(cm)
Mean	6.0	1.86	48.96	19	2.53	7.7
Min.	5.0	1.86	45.7	12	2.49	5.9
Max.	7.0	1.86	50.8	23	2.56	9.3
Range	2.0	0	5.10	11	0.07	3.4
	Testing data (number = 40)					
	D(cm)	d_{50}(mm)	U(cm/s)	h(cm)	u_*(cm/s)	S(cm)
Avg.	4.60	2.58	55.03	15	3.04	5.2
Min.	3.0	0.81	34.20	6	1.55	2.1
Max.	7.0	3.00	62.60	27	3.39	10.3
Range	4.0	2.19	28.4	21	1.84	8.2

Table 2 | The parameters associated with two embedded PSO algorithms

PSO	Population size	c_2	c_1	itmax	ω_{min}	ω_{max}
First PSO	20	0.5	0.4	30	0.4	0.9
Second PSO	200	0.5	0.5	500	0.4	0.9

Table 3 | The RMSE for training and checking data in the dimensional scour estimator models (cm)

ANFIS Statistical parameters	Training error 0.197 SD	Avg.	Max.	Min.	Checking error 0.671 SD	Avg.	Max.	Min.
PSO–FIS–PSO	0.051	0.162	0.172	0.1562	0.091	0.328	0.423	0.261

Table 4 | The RMSE for training and checking data in the nondimensional scour estimator model

Method Statistical parameters	Training error SD	Avg.	Max.	Min.	Checking error SD	Avg.	Max.	Min.
PSO–FIS–PSO	0.0005	0.024	0.029	0.023	0.0091	0.328	0.423	0.261

from the best run among the ten runs with six rules, in which the associated parameters are as follows:

$$\left[ra_D, ra_{d_{50}}, ra_U, ra_h, ra_{u_*}, ra_S, \gamma_f\right] = [1.3, 0.503, 0.1, 0.53, 0.734, 1.0076, 0.21].$$

As shown in the above expression the radii related to the mean current velocity (0.10) and the mean grain size of sand (0.503) are less than the radii of other variables. From Figure 5, it is apparent that not only in the PSO–FIS–PSO model is the first applied PSO model tuning the fuzzy clustering, i.e. the radii and squash factor have a more efficient role, but also the second PSO algorithm plays an important role in tuning the fuzzy antecedent and consequent parameters. It can be concluded that the first PSO that optimizes fuzzy clustering has an exploration role (global search), whereas the second PSO plays an exploitation role (local search that prevents an over-trained model).

In addition, to ensure the use of the clustering parameter optimization, with assumed randomly generated clustering parameters, it is attempted that the fuzzy antecedent and consequent parameters should be tuned by an ANN method (combined SDE and LSE), whose results are reported in Table 3. It is evident from Table 3 that the ANFIS model is trapped in the local optima at iteration 33 in which both the checking and training error are minimized (RMSE $_{\text{checking data}} =$ 0.68 and RMSE $_{\text{training data}} = 0.197$). It is further highlighted in Figures 6 and 7 in which there is no sign of having a trained model due to negligible changes in membership function after training. Hence, applying the PSO algorithm not only enhances the capability of the FIS but also it obtains better efficiency in the noisy area such as GA (Beyer 2000) as an evolutionary algorithm.

SENSITIVITY ANALYSIS AND GENERALIZED MODEL

One of the most important issues in the application of soft computing tools for estimation of local scour is the sensitivity analysis to determine the most effective variables in the scour depth estimator model. The effect of a variable on the model accuracy would be evaluated in such a way that a variable is eliminated from the FIS model inputs; then the model estimation error is calculated. If the model error increases the eliminated variable is effective; otherwise, it is not needed to be taken into account. The obtained results

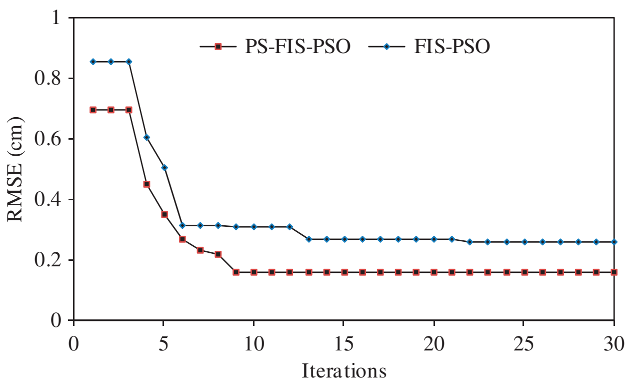


Figure 5 | Evolution of RMSE in both PSO–FIS–PSO and FIS–PSO models versus number of iterations in the estimation of scour depth.

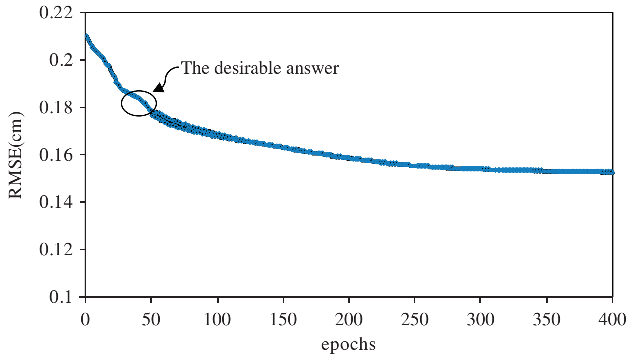


Figure 6 | Evolution of RMSE error in the ANFIS model versus epochs in the estimation of scour depth.

associated with each eliminated variable are shown in Figure 8. The figure clarifies that the current velocity, pipe diameter and sand size have the great influence on the scour depth, although the effect of the other variables is not completely negligible.

Another important issue in developing a soft computing model is its application in different physical conditions and flow patterns. Therefore, to enhance the PSO–FIS–PSO model applicability by different practitioners developing a nondimensional-based model can be a suitable remedy in which each variable stands for separate

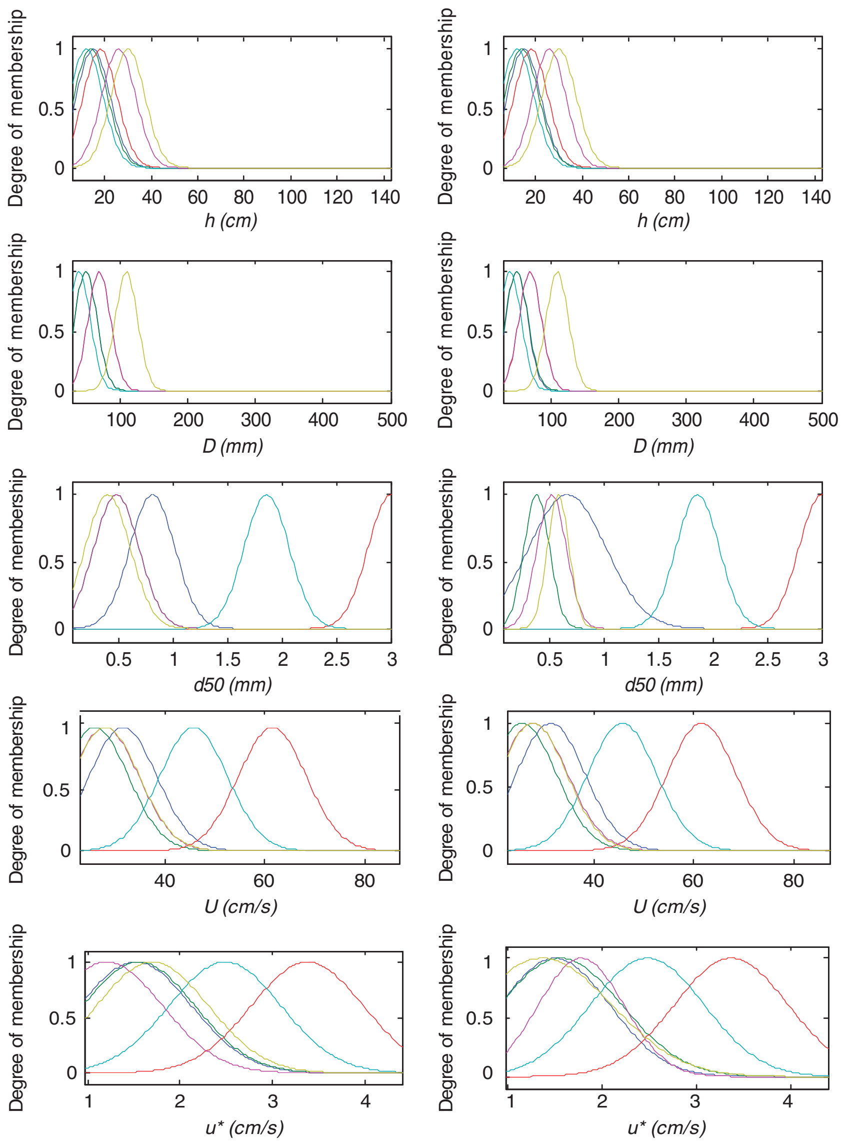


Figure 7 | Initial and tuned membership functions by ANN in the ANFIS model for random clustering parameters with six rules.

physical characteristics. The relevant nondimensional variables in the scour depth can be categorized by the following expression:

$$\frac{S}{D} = f\left(\mathrm{Re}, \mathrm{Re}_*, \frac{h}{D}, \theta, Fr\right) \tag{22}$$

where $\frac{S}{D}$ is the nondimensional scour depth, Re and Re_* are, respectively, the fluid and boundary layer Reynolds numbers, $\frac{h}{D}$ is the nondimesional water depth, θ is the Shields parameter and Fr is the Froude number. These nondimensional numbers are expressed as

$$\mathrm{Re} = \frac{\rho UD}{\mu}, \quad \theta = \frac{{u_*}^2}{\rho g(G_s - 1)}, \quad \mathrm{Re}_* = \frac{u_* d_{50}}{v},$$
$$Fr = \frac{U}{\sqrt{gh}}. \tag{23}$$

After selection of the nondimesional input and output variables the model is developed so that its associated training process is shown in Figure 9. The final tuned membership functions are provided in Figure 10. The number of rules associated with them is six rules. The most important issue about the ANFIS model is the selection of the nondimensional parameters that make the dataset noisier and consequently forcing the ANFIS model not to work properly. Therefore, this event proves the developed PSO–FIS–PSO model's superiority to the ANFIS model.

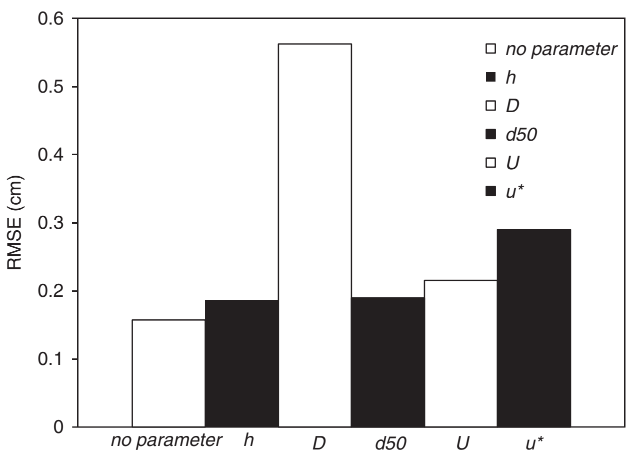


Figure 8 | The variation of training data RMSE error with elimination of the input variables with respect to the checking data RMSE error.

ASSESSMENT OF COMBINED PSO–FIS–PSO MODEL

Despite the development of the models by training and checking datasets, the models should be verified by testing data to show how the model performs in facing a dataset never used in the training process. This dataset is known as the testing data to ensure the generalization capability of the built model. To achieve this, a scatter diagram (Figures 11 and 12) indicates the accuracy of the models compared to the observed and estimated values. To quantitatively evaluate the models, three statistical indexes are used.

The first index is *bias*, showing the mean error caused by overestimating and underestimating the observed value and the second one is the scatter index (SI), showing how scattered are the data around $y = x$, in which y and x stand for the observed and estimated values, respectively. This index is indeed a normalized RMSE divided by mean values of the observed data. Since RMSE is a PSO objective function this index is more important than the *bias* index. The third index is the correlation coefficient, extensively used to show the correlation between estimated and observed values. These three indexes can be calculated with the following equations:

$$bias = \frac{1}{N}\sum_{k=1}^{N}(P^i - O^i) \tag{24}$$

$$SI = \frac{RMSE}{\overline{T}} \times 100 \tag{25}$$

$$R^2 = 1 - \frac{\sum_{k=1}^{N}(P^i - O^i)^2}{\sum_{k=1}^{N}(P^i - \overline{T})^2} \tag{26}$$

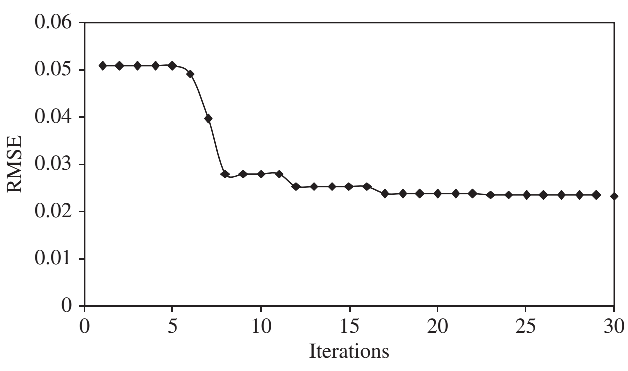


Figure 9 | Evolution of RMSE by nondimensional PSO–FIS–PSO model versus number of iterations in the estimation of scour depth.

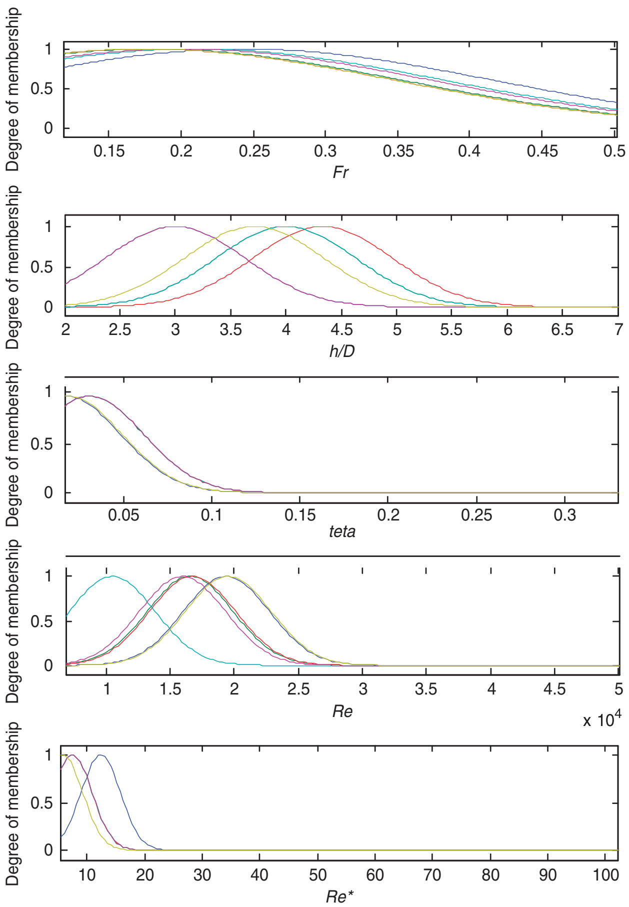


Figure 10 | Final membership functions extracted by nondimensional PSO–FIS–PSO model.

where O^i is the is observed value, P^i is the estimated value by network, N is the number of testing data points and $\overline{T}$ is the average of the target outputs.

The results of the estimation of scour via the PSO–FIS–PSO, ANFIS and empirical models have been reported in Tables 5 and 6. As shown in Tables 5 and 6, the combined models with PSO have greater accuracy ($SI = 15.75\%$) than the models in which ANFIS is used ($SI = 36.43\%$) to optimize the fuzzy if–then rule antecedent and consequents parameters. In addition, the ANFIS model overestimates the scour depth ($bias = 1.44$ cm) as the PSO–FIS–PSO model estimates scour with satisfactory accuracy ($bias = -0.19$ cm). It is also evident that the empirical methods underestimate the scour depth ($bias = -1.84$ cm and $bias = -1.86$ cm for the Delft University and Kejeldsen formulae, respectively) compared to the PSO–FIS–PSO model. From the above results, it can be concluded that

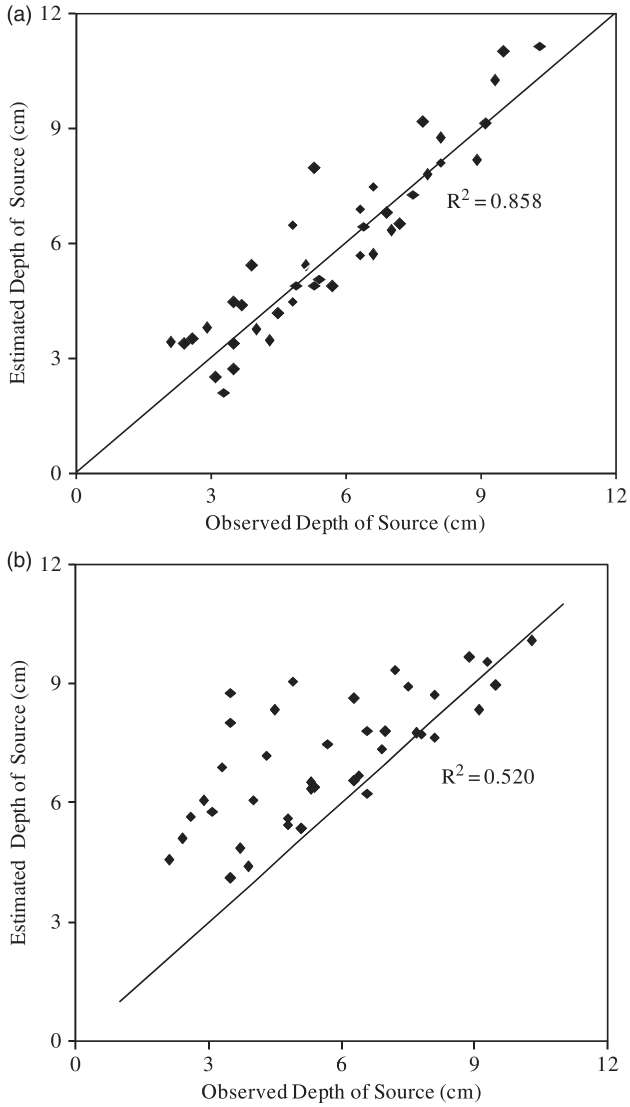


Figure 11 | Comparison between estimated scour depths by PSO–FIS–PSO (a) and the ANFIS (b) dimensional models versus observed ones associated with testing data.

the PSO–FIS–PSO performs better than the ANFIS model in the noisy media.

The main deficiency of the existing empirical methods is the lack of consistency among them, which decreases their reliability under different hydrodynamic conditions. Another point that should be highlighted here is the lower correlation coefficient in the nondimensional PSO–FIS–PSO model (which is shown in Figure 12) in comparison with the dimensional one as reported in Figure 11 (a, b). The lower correlation coefficient shows that using the nondimensional data increases noise in the dataset and

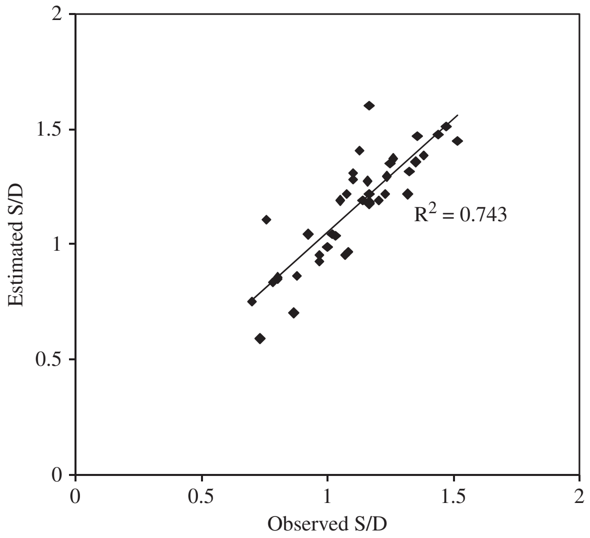


Figure 12 | Comparison between estimated scour depths by PSO–FIS–PSO nondimensional versus observed ones associated with testing data.

the relation between the inputs and output variables so that the ANFIS model is not able to tune fuzzy if–then rule antecedent and consequent parameters. Referring to Figure 11 (a, b) again, we see that the nondimensional model regression is slightly lower ($R^2 = 0.743$) in comparison with the previous activities. But note that this model can estimate the scour phenomenon in different experimental conditions, namely the live-bed and clear-water conditions. Therefore, this slightly lower regression can be justified in terms of a penalty for improvement of the model generalization capability.

SUMMARY AND CONCLUSIONS

In this paper, an evolutionary algorithm known as PSO was used to improve the ANFIS shortcomings by using a PSO as a random search method instead of ANN. This algorithm has also been applied for extracting fuzzy if–then rules in subtractive clustering methods (optimizing radii of

Table 5 | The error of the developed dimensional FIS models

ANFIS		PSO–FIS–PSO		Scour depth
bias (cm)	*SI*(%)	*bias* (cm)	*SI*(%)	
1.44	36.43	−0.19	15.75	*S* (cm)

clustering and squash factor). To achieve this, several combined fuzzy inference systems and PSO models were developed for estimation of the equilibrium scour depth that, in evaluating the different sceneries, concluded the following:

- Two embedded PSO algorithms enhanced the FIS generalization capability, while the first PSO is exploiting the answer and the second one is trying to explore a near-optimum answer, such as a hill-climbing method.
- The result shows that applying nondimensional models exerts some noise on the system so that ANFIS is not able to tune the fuzzy if–then rule parameters.
- The sensitivity analysis shows that the velocity of the current and diameter of the pipe are more significant variables in estimation of scour depth.

The main shortcoming with the developed model is its high calculational load during execution, especially at the time when numerous training data points or variables are involved. Although the developed model herein has shown more accuracy and efficiency than the common ANFIS model, to increase model efficiency by employing another ANFIS model as the hill-climbing model in the following second PSO algorithm to tune the fuzzy if–then rule antecedent and consequent parameters can be a suitable option for future work. The newly developed model may be applied for the prediction of the phenomenon in which the relationships among the involved variables are not that clear. In addition, in order to precisely understand the model efficiency versus ANN its comparison with an ANN model can be a suitable option for future work.

Table 6 | The error in the empirical methods (cm)

Kejeldsen *et al.* (1973)			Delft University (1985)			Scour depth *S*(cm)
R^2	*bias* (cm)	*SI*(%)	R^2	*bias* (cm)	*SI*(%)	
0.823	−1.84	41.18	0.831	−1.86	41.38	*S* (cm)

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